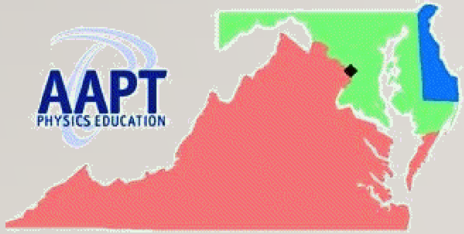
Chesapeake Section of the American Association of Physics Teachers

A ROCKET PROPELLED BY A PAIR OF SPRING-LOADED CANNONS



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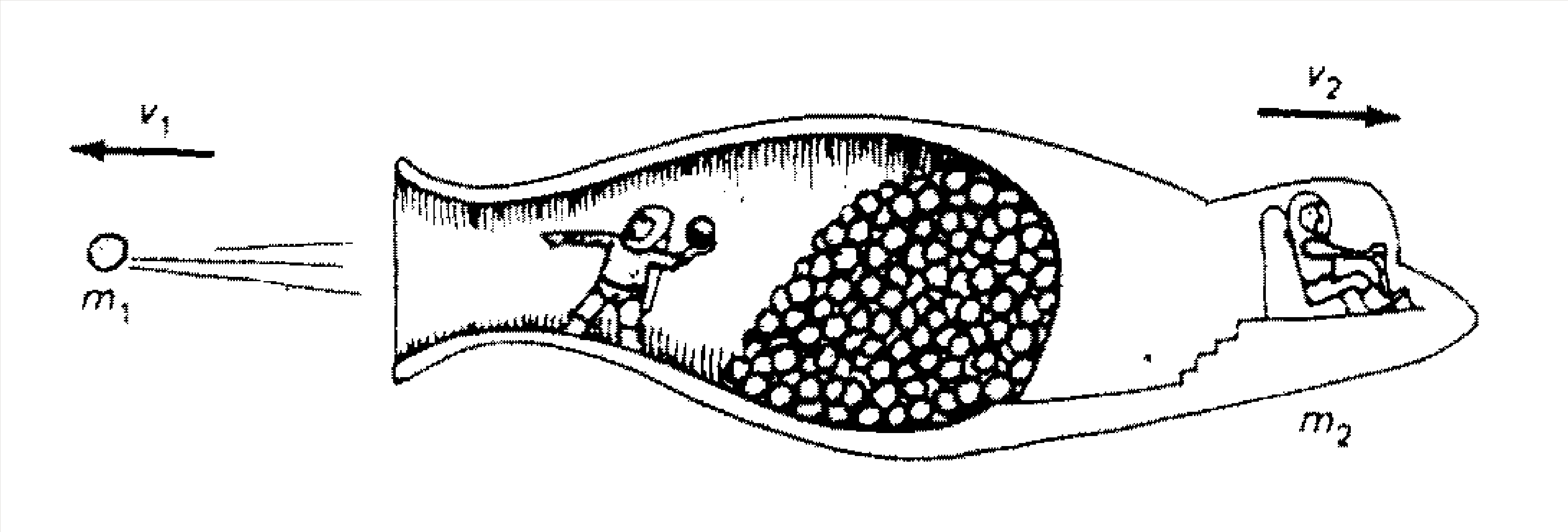
The "Fred Flintstones" rocket: Throw stones out the back end to get forward recoil of the spaceship.



reference: Wilson *Physics: A Practical and Conceptual Approach* (Saunders, 1989)

The "tennis-powered" rocket: Hit balls off the back end to get forward recoil of a trolley.

Motion of a trolley powered by ejecting balls

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Abstract

Rockets not subjected to gravitational forces are often considered in mechanics textbooks as an example of the use of the momentum conservation equation for an isolated system. We consider in this paper the momentum conservation in the motion of a trolley moving horizontally and powered by a tennis player with a racket and a given number of tennis balls.

ISSUES WITH THESE TWO SCHEMES:

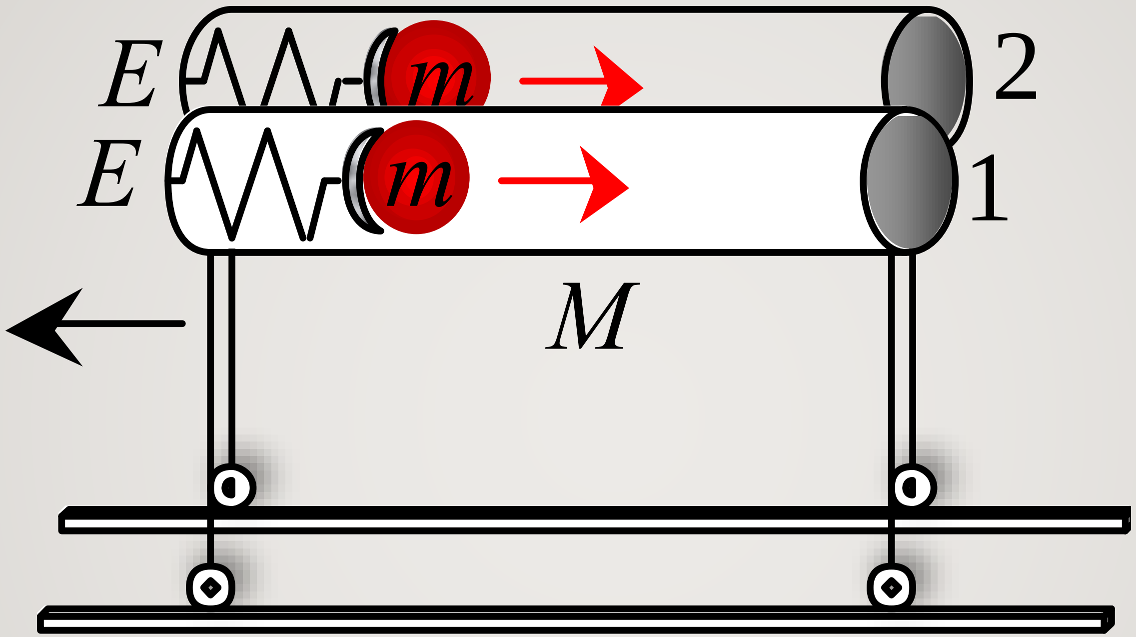
- The projectiles are too light compared to the vehicle to give significant recoil.
- The projectile launches are not reproducible from shot to shot.
- But most important of all, the projectile "exhaust" velocity is relative to what?

Even if we say it is relative to the vehicle, as is usual for rocket thrust, we still have an ambiguity because the exhaust mass is in intermittent finite increments *m* rather than in continuous infinitesimal amounts *dm*.

Namely, is the exhaust velocity relative to the initial or to the final velocity of the rocket?

The recoil velocity depends on which choice you make! So let's throw out these two schemes and choose an alternative launch mechanism without these issues.

FRICTIONLESS CART PROPELLED BY TWO SPRING CANNONS



cart is propelled leftward by the recoil of the balls

Two projectiles is enough to understand the issues.

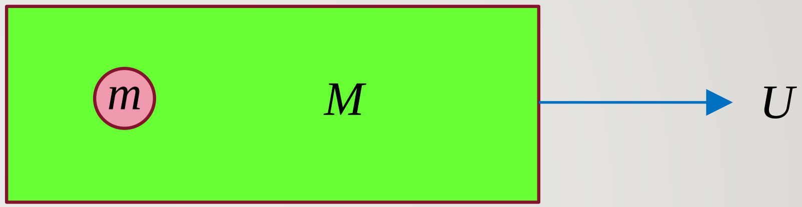
Each spring delivers a fixed amount of mechanical energy E to the kinetic energy of the system of cart and balls. No <u>assumption</u> is made about the exhaust velocities of the balls. Instead they are <u>calculated</u> by simultaneously conserving momentum and energy.

CLICKERS!

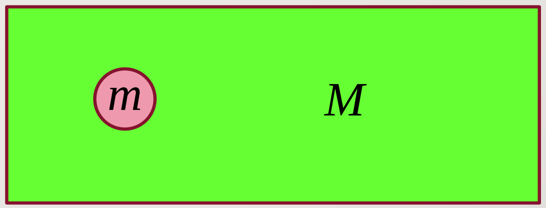
FIRST CONSIDER AN INITIALLY MOVING CART HAVING ONLY ONE CANNON:

<u>Before Firing Cannon</u>

in reference frame of ground



in reference frame moving at U



at rest

(where M does <u>not</u> include mass m of ball)

After Firing Cannon

in reference frame moving at U

conserve momentum: $m\upsilon = MV$

conserve energy: $E = \frac{1}{2}m\upsilon^2 + \frac{1}{2}MV^2$

simultaneous solution: $\upsilon = \sqrt{\dfrac{2EM}{(M+m)m}}$ and $V = \sqrt{\dfrac{2Em}{(M+m)M}}$

NOW CONSIDER AN INITIALLY STATIONARY CART HAVING TWO CANNONS:

<u>Simultaneously Fire Both Cannons</u>

put $U = 0$ and replace m with $2m$ and E with $2E$ to get

$$V_{\text{simultaneous}} = \sqrt{\frac{8Em}{(M+2m)M}}$$

for the final cart speed relative to the ground

Sequentially Fire One Cannon After Another

final cart speed V after first firing becomes initial speed U for second firing except that we must replace M with $M + m$ because one ball is left onboard:

$$U = \sqrt{\frac{2Em}{(M+2m)(M+m)}}$$

final cart speed after second firing is V relative to U and thus relative to ground it is:

$$V_{\text{sequential}} = V + U = \sqrt{\frac{2Em}{(M+m)M}} + \sqrt{\frac{2Em}{(M+2m)(M+m)}}$$

SO WHICH WINS: SIMULTANEOUS OR SEQUENTIAL FIRING?

ratio of results is $$\frac{V_{\text{sequential}}}{V_{\text{simultaneous}}} = \frac{\sqrt{M} + \sqrt{M + 2m}}{2\sqrt{M + m}}$$

This ratio equals 1 if $m << M$. The balls have to be heavy to see a difference!

Try $m = M$. Then the ratio is 96.6%.

SIMULTANEOUS WINS!

LET'S REVIEW HOW THIS HAPPENED!

final relative to initial cart speed for the one-cannon case is:

$$V = \sqrt{\frac{2Em}{(M+m)M}}$$

or in general velocity added to cart by a ball is:

$$V = \sqrt{\frac{2Em}{M_{\text{onboard,i}} M_{\text{onboard,f}}}}$$

and so:

$$V_{\text{simultaneous}} = \sqrt{\frac{2Em}{(M+2m)M}} + \sqrt{\frac{2Em}{(M+2m)M}}$$

due to second ball — due to first ball

$$V_{\text{sequential}} = \sqrt{\frac{2Em}{(M+m)M}} + \sqrt{\frac{2Em}{(M+2m)(M+m)}}$$

These two expressions are similar but <u>not</u> equal in value.

So you can maximize the "efficiency" of a rocket
by launching all the fuel out the tail at once,
rather than dribbling it out in sequential bits.

HOWEVER! That is only true if you can maintain
the same launch energy per unit mass of fuel.

Specifically we pay a price in our case because we
need TWO cannons to fire the balls simultaneously.
Our rocket would be lighter (and less costly)
if it had only ONE cannon and we fired
the two balls sequentially using it.

In reality, there are trade-offs in the weight and
power of a rocket engine, and it will be used sequentially.

QUESTIONS?

COMMENTS WELCOME BY EMAIL TO MUNGAN@USNA.EDU.

Acknowledgment: Seth Rittenhouse suggested this problem in the form of a physics teacher stranded on a frozen pond with two identical textbooks in her hands that she could throw with the goal of maximizing her recoil speed toward land.

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