Fall 2021 CSAAPT Virtual Meeting

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MODIFIED ATWOOD MACHINE

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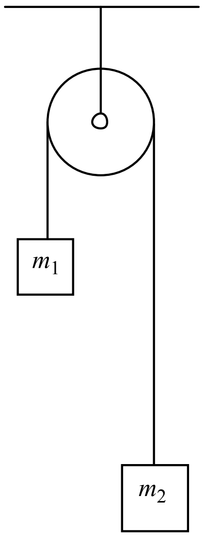
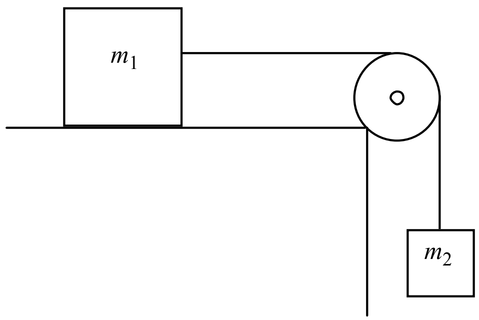
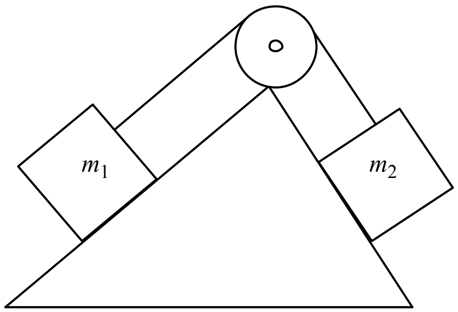
Definition of Atwood machine

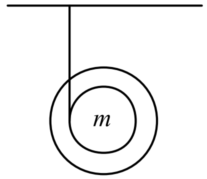
One or more masses falling
while connected by a string
and possibly constrained
to move along surfaces.

Examples: (Pulleys may have mass and surfaces may have kinetic friction.)



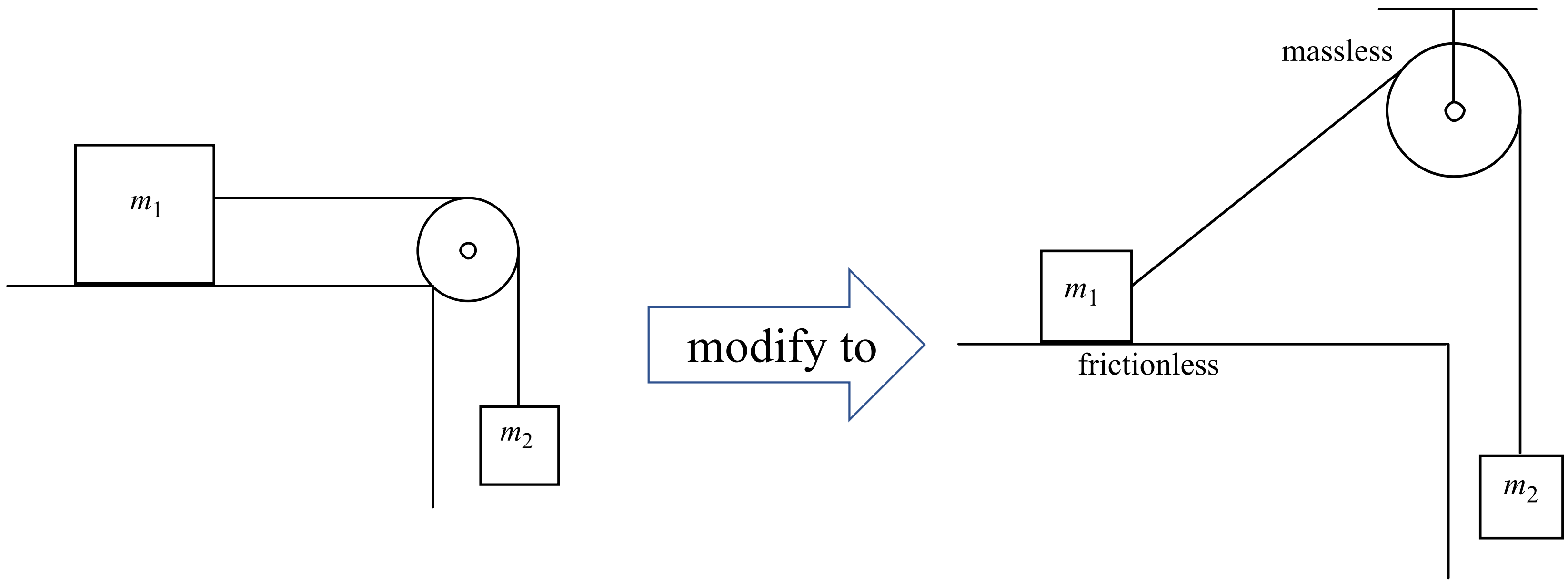




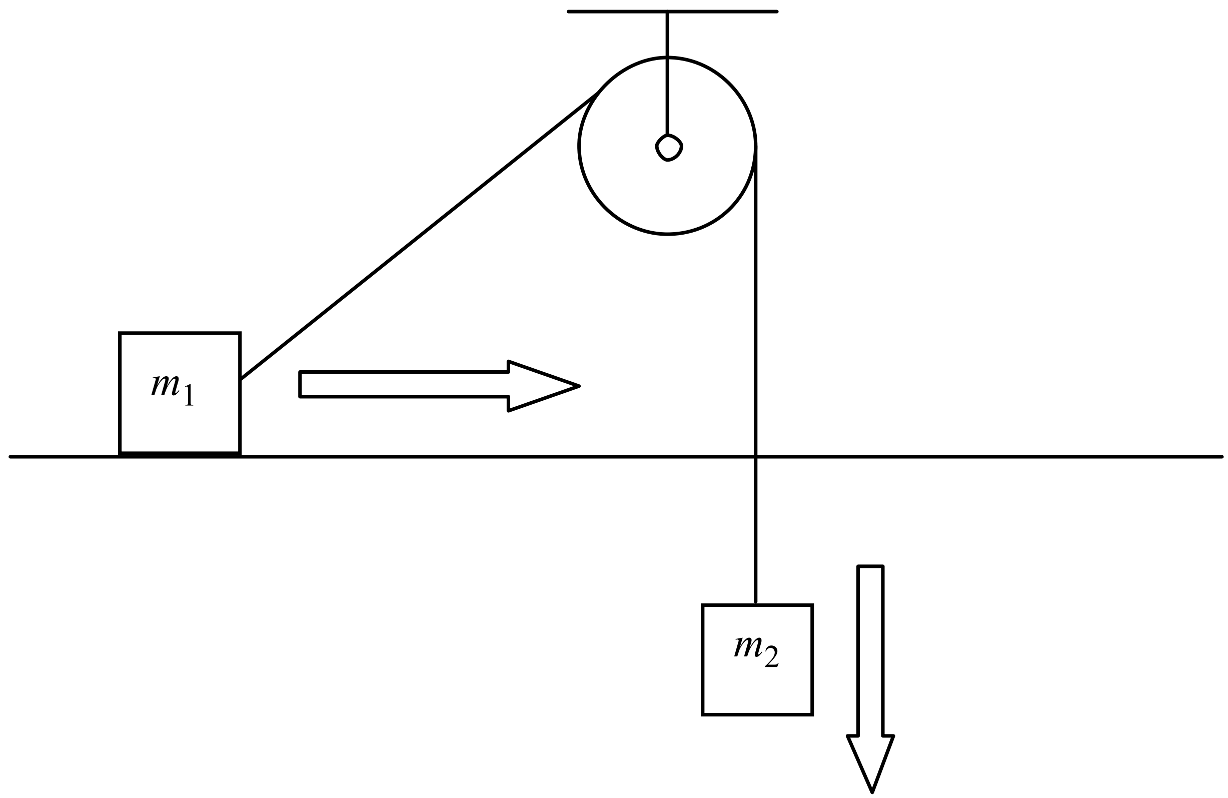


All of these machines have constant acceleration a and constant tension T.

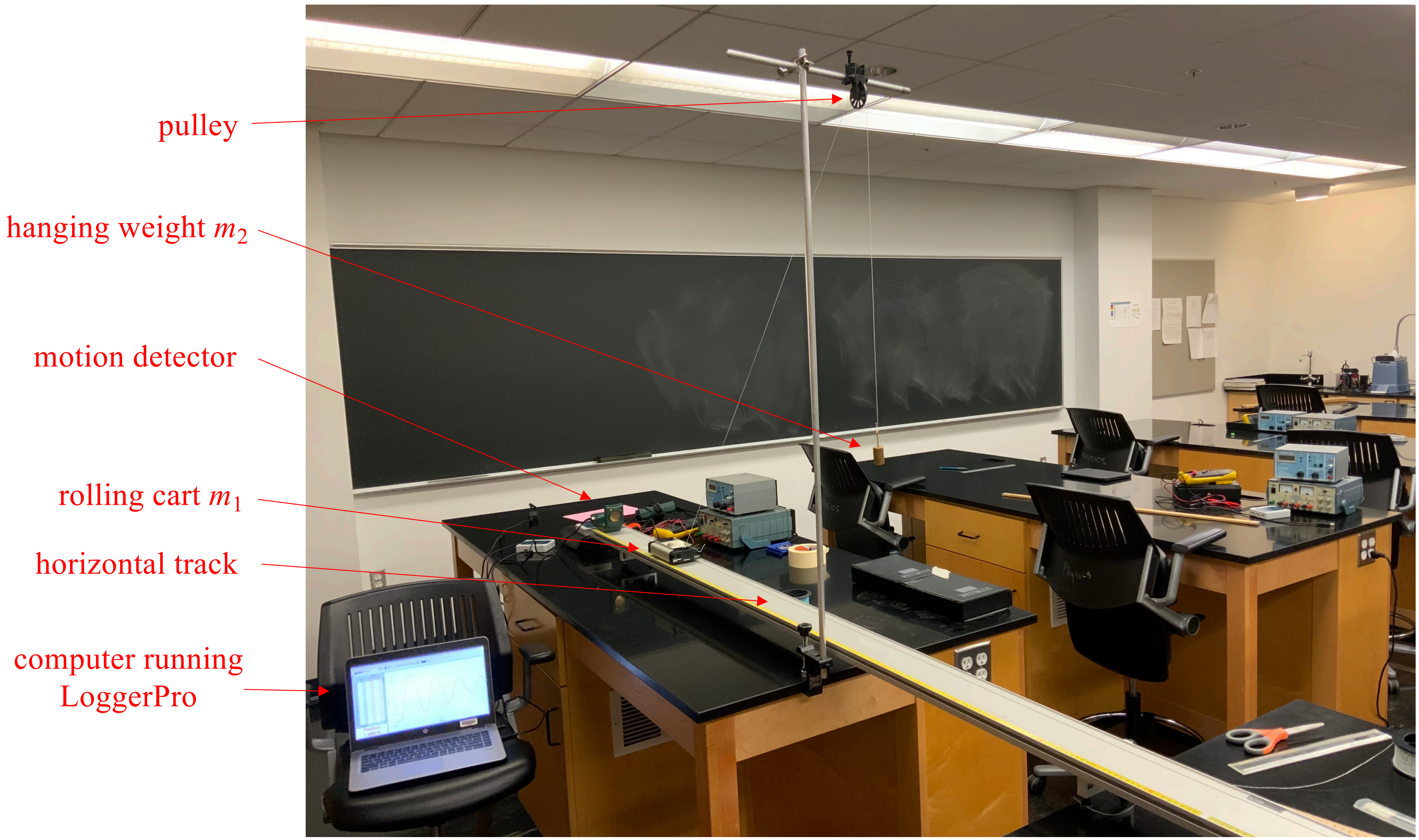
Raise the pulley vertically:



Also extend the track so m_1 does not fall over the edge:

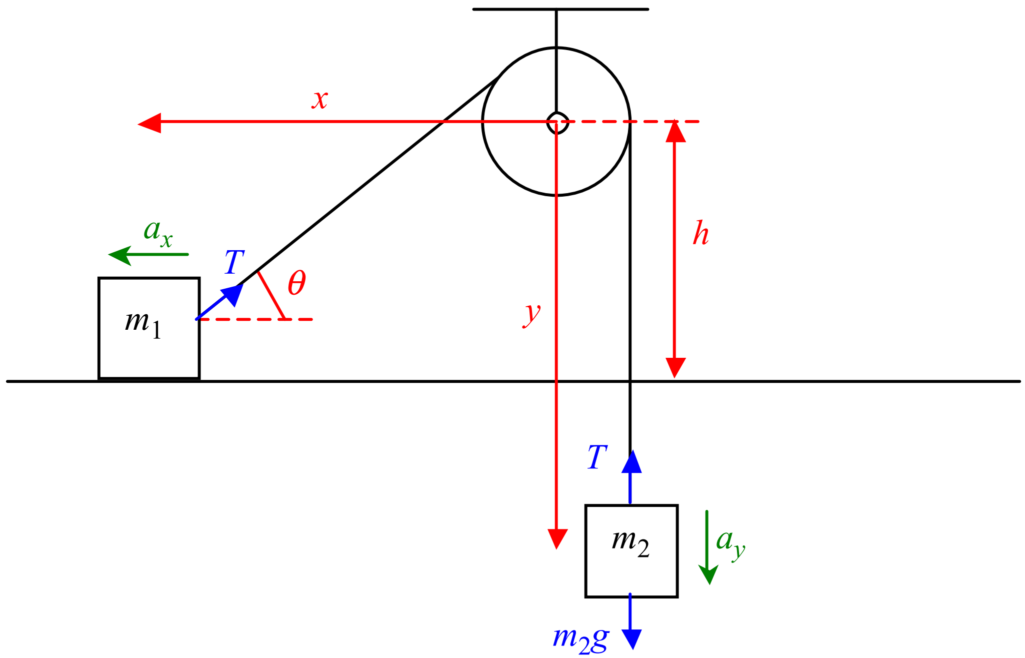


Photograph of Experimental Setup



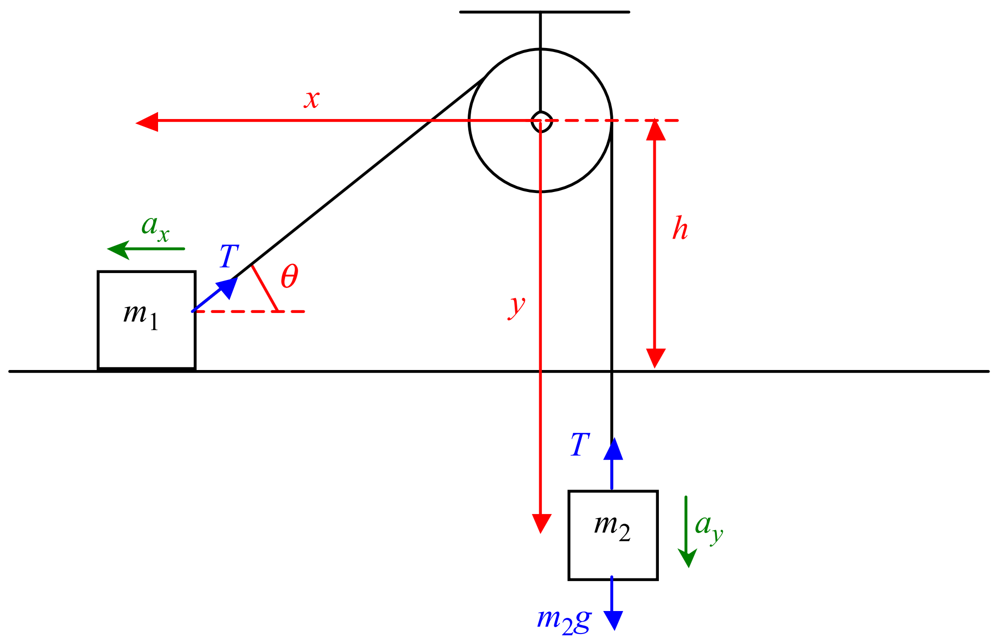
Theory: Phys. Educ. **56**, 028021 (2021)

$$\cos\theta = \frac{x}{\sqrt{x^2 + h^2}}$$



constant length of string $L = y + \sqrt{x^2 + h^2} \rightarrow$ solve for $y(x)$ and differentiate twice to get $a_y(x, v_x)$

Find a nonlinear ODE for block 1 alone:



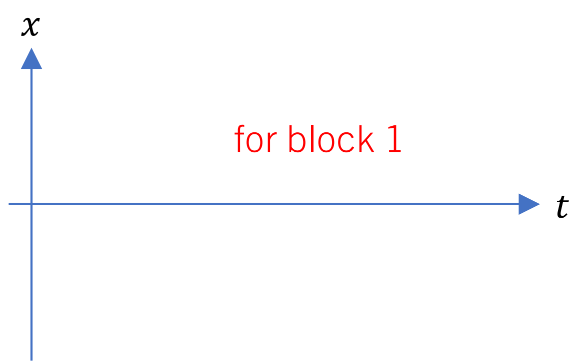
block 2: $m_2 g - T = m_2 a_y$ → sub in a_y and solve for $T(x, v_x)$

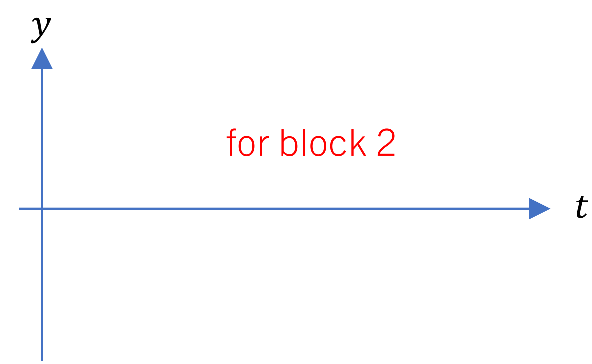
block 1: $-T cos\theta = m_1 a_x$ → sub in T and $cos\theta$ to get $\boxed{a_x(x, v_x)}$

Numerical Solution in Mathematica

$$L = 2.02 \text{ m}$$
$$h = 1.05 \text{ m}$$
$$x_0 = 1.16 \text{ m}$$
$$v_{x0} = 0$$

$$m_1 = 574 \text{ g}$$
$$m_2 = 100 \text{ g}$$
$$g = 9.8 \text{ m/s}^2$$

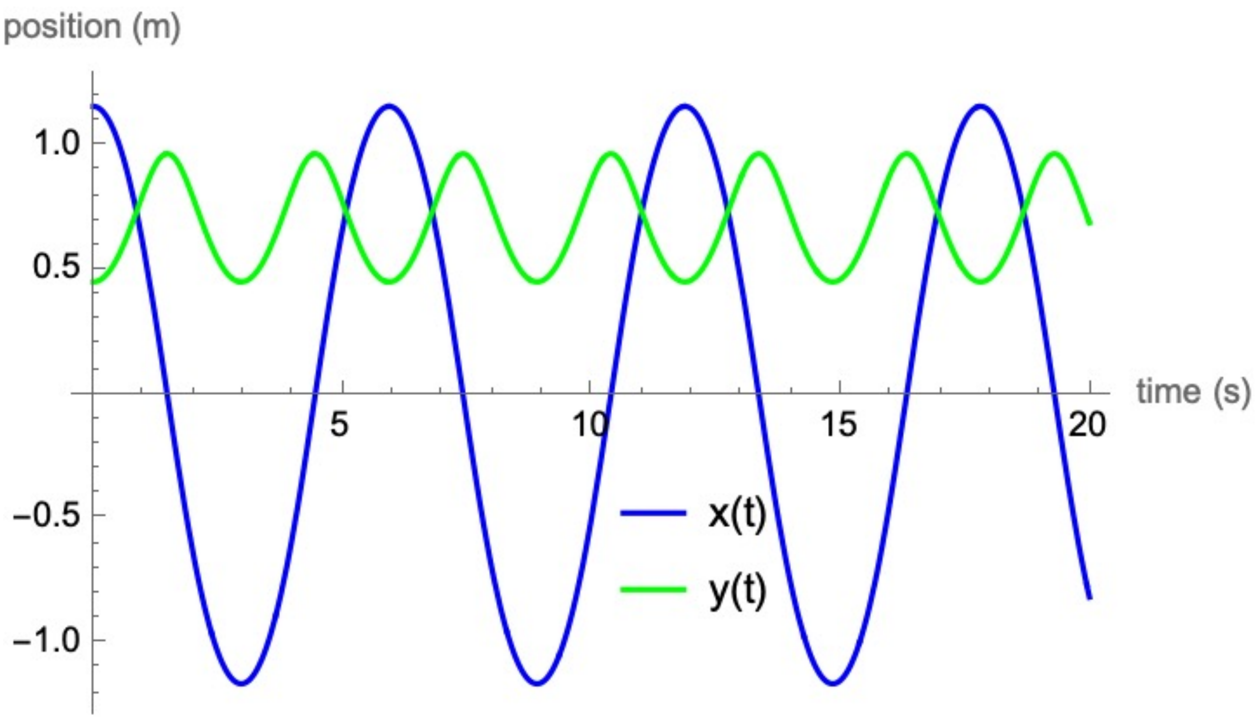


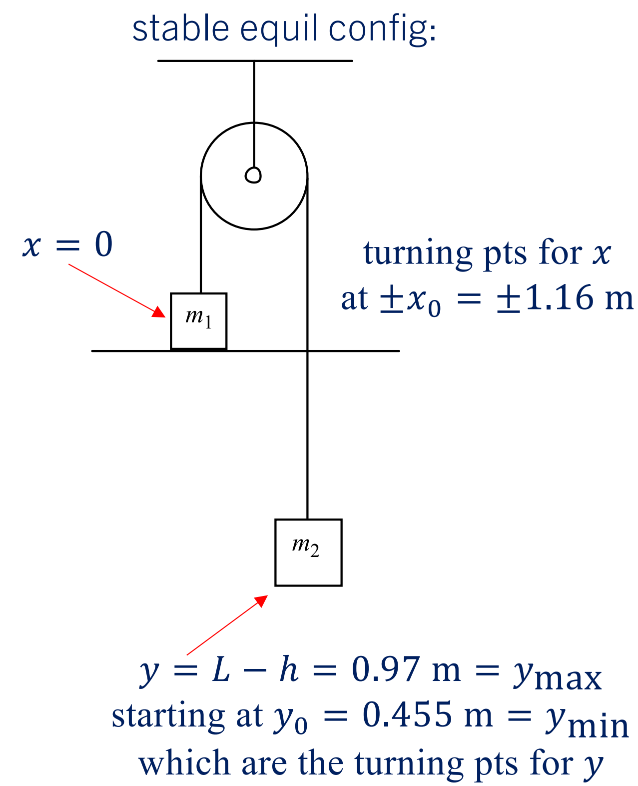


Predict what these two graphs look like!

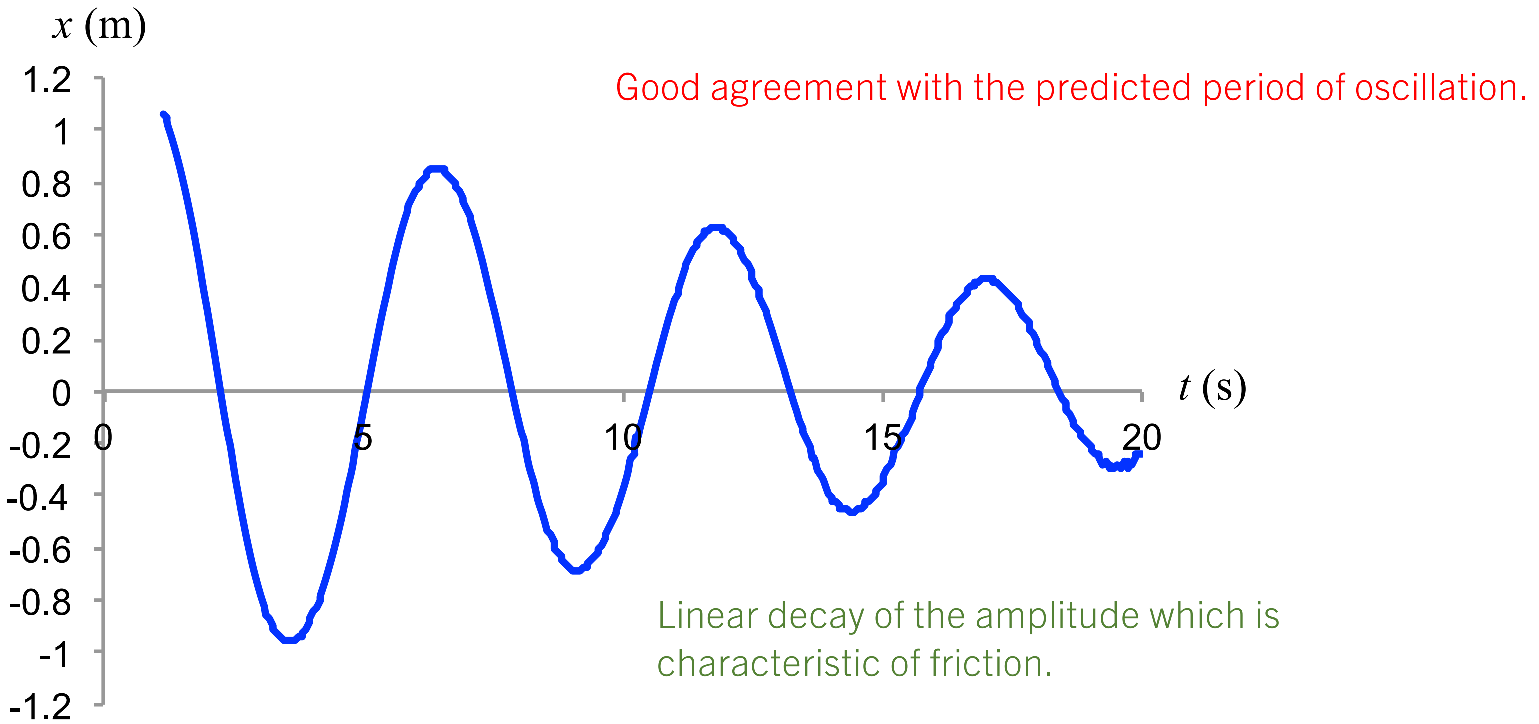
AUDIENCE RESPONSES?

Theoretical Predictions





Experimental Results



Suggested Student Extensions:

- add rolling friction in theory
- add pulley mass in theory
- mount force sensor on top of cart to measure T_x
- tilt the track
- replace hanging weight with a second cart on another tilted track

Thanks for your interest!

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