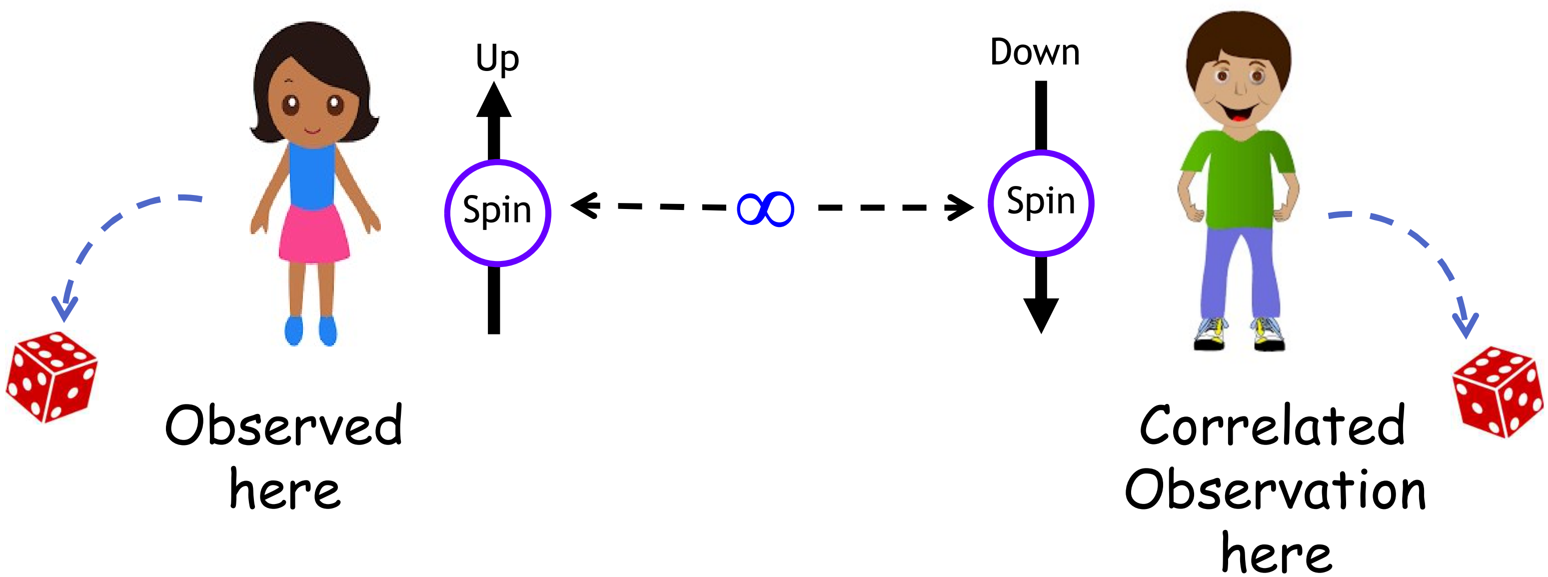
Interpreting Reality
Bell's Test



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The Albert Einstein - Neils Bohr Debates

Einstein – "God does not play dice with the universe…"

Bohr's response - "Don't tell God what to do."

Einstein – "Coincidence is God's way of remaining anonymous."

Quantum Theory (QT) of the Past

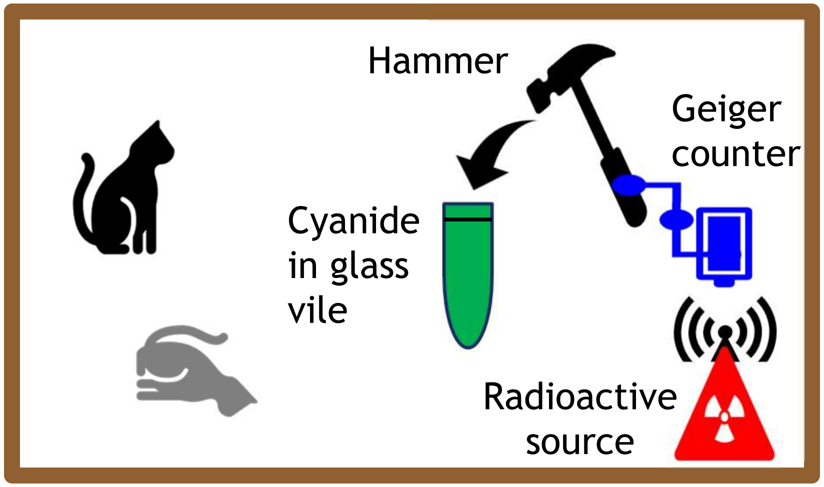
QT, founded in 1900 by Max Planck, divides the world into two parts: the **System** and the **Observer**.

- The **System** (properties of the world being modeled) is a wavefunction, ψ, represented as a "superposition" of states, ϕ, (properties):

$$\psi = a_1\phi_1 + a_2\phi_2 + a_3\phi_3 + \text{etc..}$$

- The coefficients a_1, a_2, a_3 etc.(usually complex) are the "possibility amplitudes" of obtaining a particular property upon measurement.
  - "Probabilities" for each property are given as the "squares" of these coefficients: a_1^2, a_2^2, a_3^2 etc.
  - Atoms, sub-atomic particles, etc. act like **"particles"** and also have **"wave-like", properties** depending on how you "Observe" (i.e. Measure) them.

A Strange QT Conclusion – E. Schrodinger's Cat



A Geiger counter detects the decay of a radioactive source according to some probability distribution, P.

$$P(x,\mu) = \frac{\mu e^{-\mu}}{x!}$$

μ = mean decay rate
x = number of occurrences

A "probability", not a certainty.

$\psi = \frac{1}{\sqrt{2}}$ |  ⟩ + $\frac{1}{\sqrt{2}}$ |  ⟩

QT: The probability of cat being "*dead*" or "alive" is:

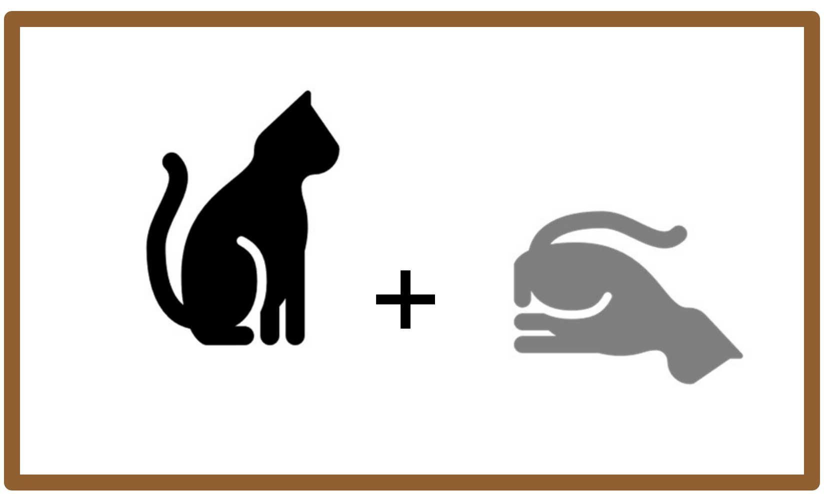
$|\psi^*\psi|$ = ($\frac{1}{\sqrt{2}}$ ⟨  | + $\frac{1}{\sqrt{2}}$ ⟨  |) ($\psi = \frac{1}{\sqrt{2}}$ |  ⟩ + $\frac{1}{\sqrt{2}}$ |  ⟩)

$$= \frac{1}{4} + \frac{1}{4} = 50\%$$

According to **Neils Bohr's QT** the measurement process (opening the box to look) "**collapses**" the state, ψ, of the cat (system) into an "**eigenstate** of that system and the "**eigenvalue**"(either "**alive**" or "***dead***") can be known.

So, what <u>was</u> the cat before it's state collapsed?

Reality?



Reality?

Alive and well

Einstein's View of Quantum Theory

- Could not accept QT's "probabilistic view of the universe.
  - Recognized QT's ability to accurately predict the results when measuring properties of nature such as energy, momentum, and electron spin, but rejected the statistical statements describing the likelihood of different values.
    - He felt that any successful theory should also provide some understanding of the physical events responsible for those results.
- Believed there existed "**hidden variables**" that would support a more cogent world, and if found, would remove the statistical interpretation of nature by QT.
    - e.g., Knowing the microscopic behavior of atoms in thrown pair of die could predict the outcome, with certainty.
      - Impossible, in principle.

    - He thought QT was really Classical Statistics in disguise.

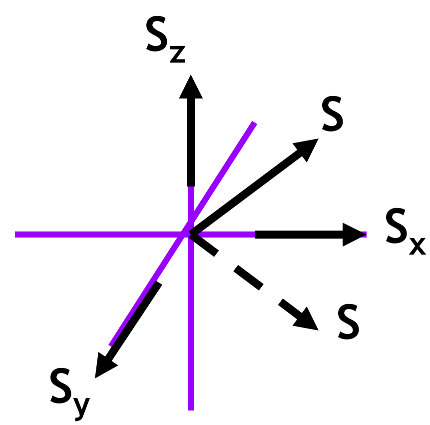
Einstein's World View

- **Realism** - Physical reality is independent of human observation or measurement.
- **Inductive reasoning** – Can be applied freely.
- **Locality** – No influence (signal) can propagate faster than the speed of light (186,000 miles/second)
  - The basis of Einstein's Special Theory of Relativity

These "local realistic" theories make predictions about the outcomes in certain experiments that are in conflict with **QT**.

The Uncertainty Principle and QT

- Werner Heisenberg's **"Uncertainty Principle"**, upon which **QT** is based, claims:
  - Physical "observables" of a system are not just unobservable, **they don't exist at all** outside of the context of an observation.
    - This deviates from our everyday "Reality".
- Consider the three components (observables), S_x, S_y, and S_z, of the spin, S, of an electron.
  - If you decide to measure S_x first, then S_y second, you get an answer.
  - However, if you measure S_y first, then S_x second you get a *different* answer.
  - We say S_x and S_y don't "commute".
    - **They cannot be known simultaneously.**



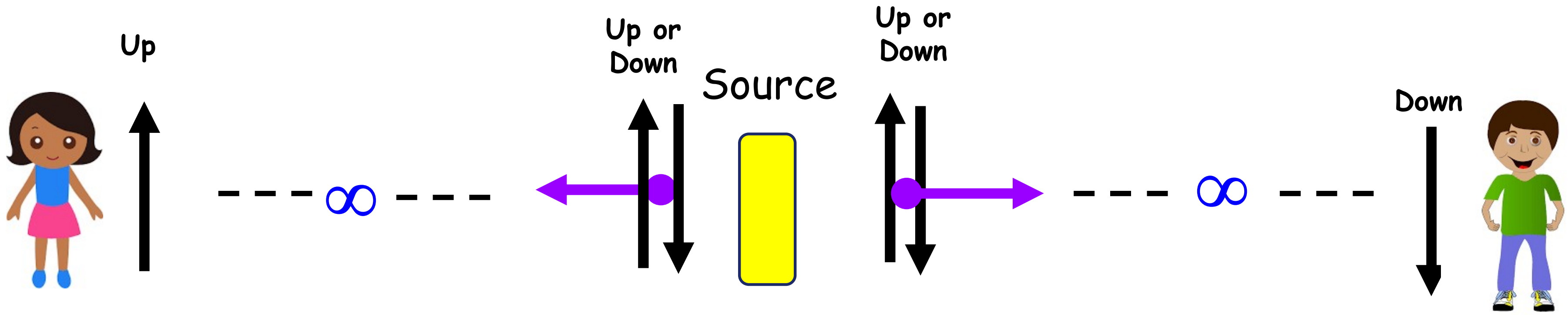
- Einstein believed **all** spin components could be predicted with certainty if the Hidden Variables could be uncovered.

The EPR (Gedankin) Paradox (1935)

A. Einstein, N. Rosen, and B. Podolsky, challenged

QT by publishing a "thought" experiment.

- Two particles in close proximity at their Source, are allowed to separate, reaching Alive and Bob infinite distances away.



- Being separately independent, each particle, leaving the source, should form a **Product state** $|\psi\rangle = |\uparrow\rangle|\downarrow\rangle$ of either a **spin-up** state or the **spin-down** state, each state being equally probable.
- Any measurement by Alice would immediately change the state of Bob spin (and vice-versa), no matter how far apart.
  - No matter what spin axis, S_x, S_y, or S_z is measured, there is this strict "negative" correlation between spin directions.
    - There is no classical explanation for this phenomenon

The EPR Paradox

- EPR argued that this "instantaneous" communication between Alice and Bob goes against Special Relativity that says no signal can travel faster than the speed of light, violating the **Locality** assumption.

- The EPR experiment then proved that either:
  - There is some "**hidden variables**" present before the two particles are released.
    - These variables would contain an **infinite amount of information**.

  OR

  - Particle's properties (position, velocity, polarization, etc.) **are not real and are undefined until observed.**
    - Therefore, QT is an ***incomplete* theory of nature**.

- They defined an 'element of reality":

IF an observable property of a system could be predicted with 100% absolute certainty without disturbing the system, THEN it must correspond to an element of reality.

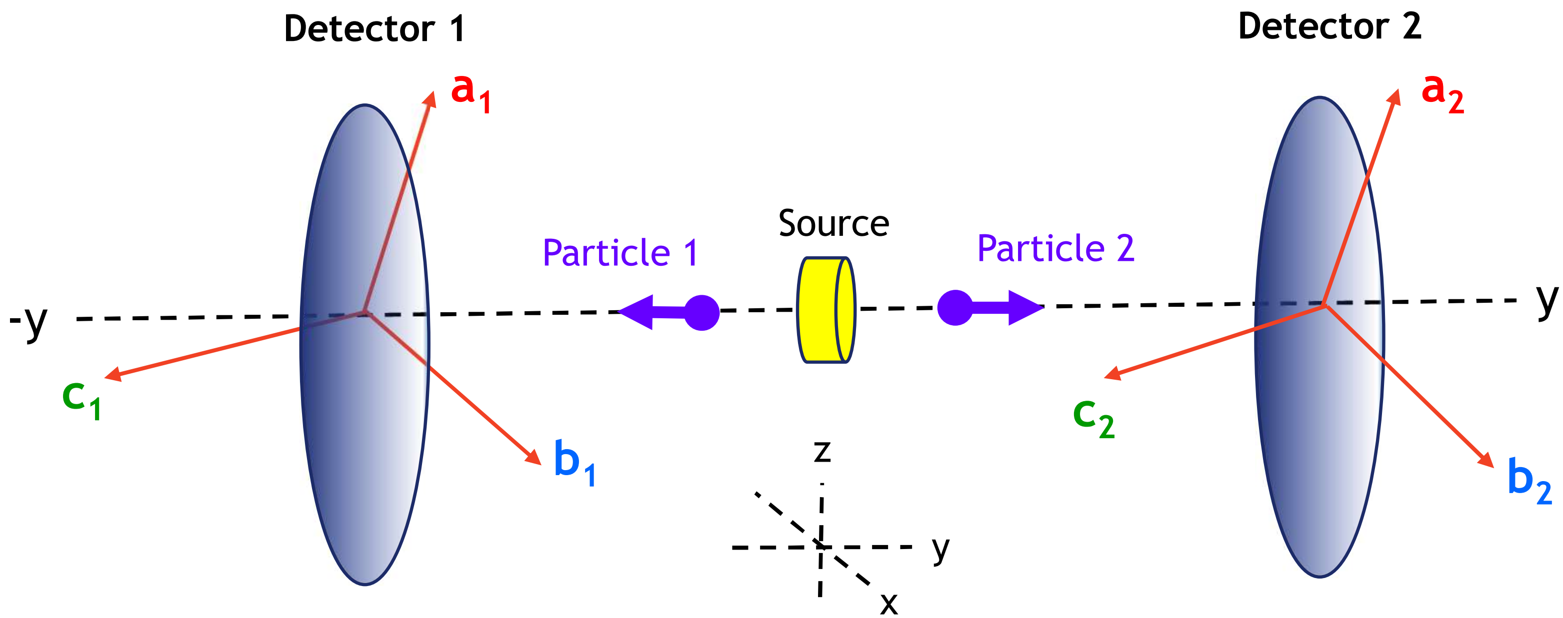
Neil Bohr's Views on QT

- "Realism", and "Locality" premises were incorrect.

- In QT, measurement results are immediate, without any delay for signal communication.
  - When the particles' spins are measured, it in fact represents a measurement on the entire system of the two particles regardless of how far apart they had moved from one another.
    - Einstein called this "spooky".

- The two particles are in an "**Entangled**" state, $|\psi\rangle$ = **(**$|\uparrow\downarrow\rangle$ **-** $|\downarrow\uparrow\rangle$**)** right from their Source. Their wave functions cannot be separated and cannot be represented or talked about individually.
  - Upon measurement, the **Entangled** state "**collapses**" **(decoheres)** into one spin state or the other.
    - "**Reality**" (spin direction) only exists at the moment of measurement.
    - Observers "**subjectively**" create the properties of the entities they measure, rather than "**objectively**", as Einstein purported.

A Simplified EPR-Like Experiment



Detection angles: a, b, c

u = Upward spin ↑

d = Downward spin ↓

Particle 1	Particle 2	# of Observations, N
$a^u\ b^u\ c^u$	$a^d\ b^d\ c^d$	N_1
$a^u\ b^u\ c^d$	$a^d\ b^d\ c^u$	N_2
$a^u\ b^d\ c^u$	$a^d\ b^u\ c^d$	N_3
$a^u\ b^d\ c^d$	$a^d\ b^u\ c^u$	N_4
$a^d\ b^u\ c^u$	$a^u\ b^d\ c^d$	N_5
$a^d\ b^u\ c^d$	$a^u\ b^d\ c^u$	N_6
$a^d\ b^d\ c^u$	$a^u\ b^u\ c^d$	N_7
$a^d\ b^d\ c^d$	$a^u\ b^u\ c^u$	N_8

$2^3 = 8$ possible results

Bell's Inequality

A 1964 publication by Irish-Scottish physicist, John S. Bell mediated this debate with his famous "Inequality" which put a limitation on correlated measurement results.

- **If violated**, would **reject** the local realistic theory of nature with its hidden variables, and support Bohr's **QT**.
- Bell - "**No physical theory of local hidden variables can ever reproduce all of the predictions of quantum mechanics**."
- Quantum mechanical probabilities cannot arise from our ignorance of local pre-existing (hidden) variables.
  - Assigning hidden variables properties must then be **non-local**.
  - Bell – The issue can be settled experimentally.

Bell's Inequality

- **Actual experiments can only measure only one spin at a time.**
  - Therefore, must consider probabilities, P, of seeing different combinations of "paired" spins in N measurements. e.g. $a_1^u c_2^u = a^u c^u$ (dropping subscripts)

- Let $n_k = N_k/N$ = frequency of particular combinations.

Particle 1	Particle 2	# of Observations, N
$a^u\ b^u\ c^u$	$a^d\ b^d\ c^d$	N_1
$a^u\ b^u\ c^d$	$a^d\ b^d\ c^u$	N_2
$a^u\ b^d\ c^u$	$a^d\ b^u\ c^d$	N_3
$a^u\ b^d\ c^d$	$a^d\ b^u\ c^u$	N_4
$a^d\ b^u\ c^u$	$a^u\ b^d\ c^d$	N_5
$a^d\ b^u\ c^d$	$a^u\ b^d\ c^u$	N_6
$a^d\ b^d\ c^u$	$a^u\ b^u\ c^d$	N_7
$a^d\ b^d\ c^d$	$a^u\ b^u\ c^u$	N_8

e.g.s, $P(a^u c^u) = \frac{(N_2 + N_4)}{N} = n_2 + n_4$

$P(a^u b^u) = n_3 + n_4$

$P(b^u c^u) = n_2 + n_6$

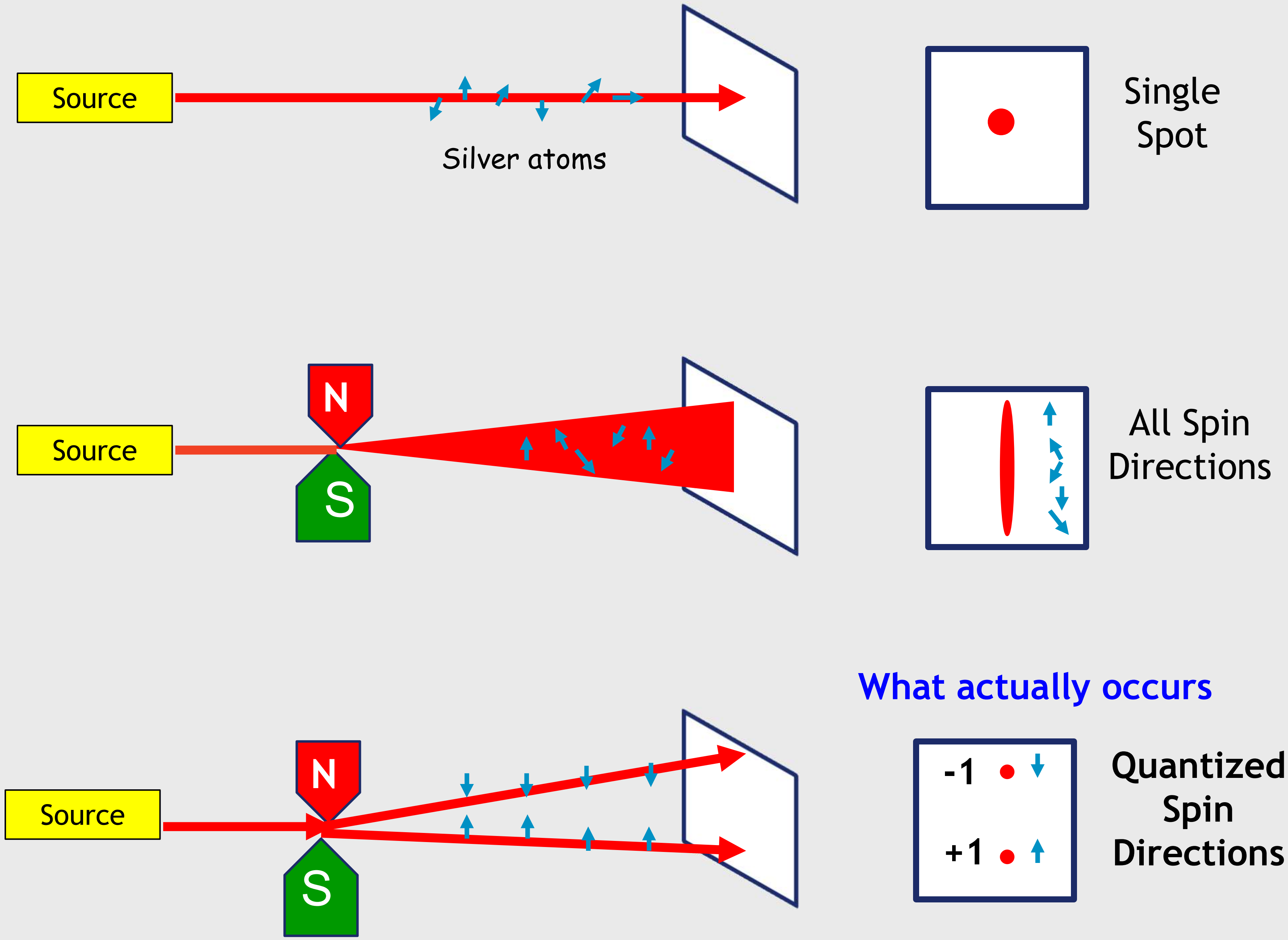
$$\therefore\ P(a^u b^u) + P(b^u c^u) = n_2 + n_4 + n_3 + n_6$$
$$= P(a^u c^u) + n_3 + n_6$$

- Since all the n_k are **positive** (or zero at the least), we have:

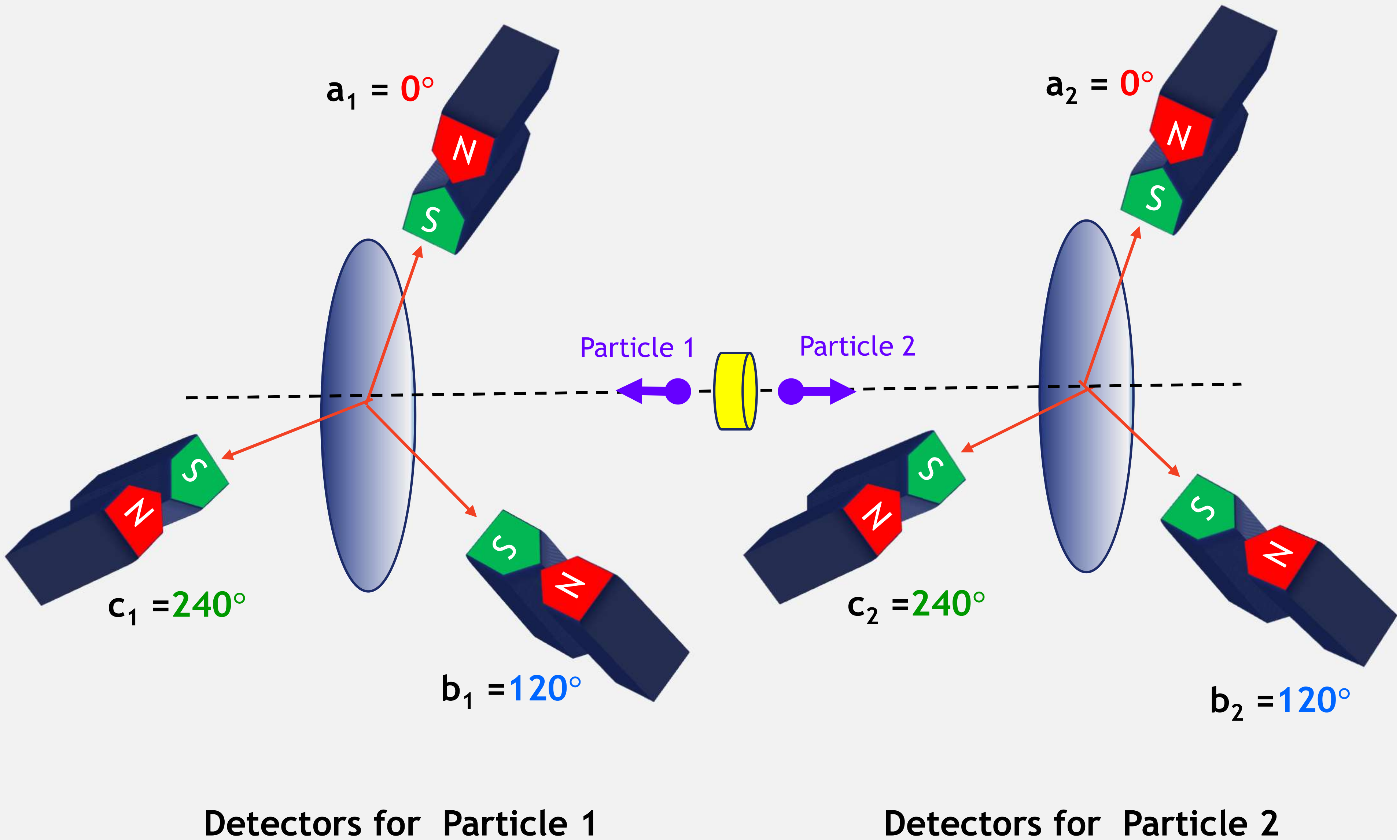
$$P(a^u c^u) \leq P(a^u b^u) + P(b^u c^u)$$

"**Bells Inequality**" (one version)

A STERN-GERLACH EXPERIMENT



A STERN-GERLACH EXPERIMENT USING THREE RANDOMLY SET ANALYZER ANGLES: $a=0°$, $b=120°$, $c=240°$



THE STERN-GERLACH EXPERIMENT USING THREE RANDOMLY SET ANALYZER ANGLES: a=0°, b=120°, c=240°

- According to EPR, the spins of the particles at the three angles, say: a=0°, b=120°, c=240° correspond to **actual elements of reality** ***if*** **they can be predicted with 100% certainty without disturbing the system.**

- If we measured the spin orientation of Particle 1at one of the 3 angles with its detector, what is the likelihood that a measurement at one of the other two angles in the second neighboring detector will agree with the results of the first detector?

BIG QUESTION

Do angles a, b, and **c correspond to simultaneous elements of reality?**

- For example, if we knew the spin at **a**=0°, what is the likelihood that the spin at **b**=120° or **c**=240° would be the same ?

Results for SG Detectors Randomly Set At Angles:

a=0°, b=120°, c=240°

If a and b either are both (u, u) or (d, d), then pair (a, b) = 1.
If a and b yielded (u, d) or (d, u) then pair (a, b) = 0.
Same for pairs (a, c) and (b, c).

Case #	a = 0°	b = 120°	c = 240°	(a, b)	(b, c)	(a, c)	Sum (a,b)+(b,c)+(a,c)	Average Sum ÷ 3
1	a^u	b^u	c^u	(u, u)	(u, u)	(u, u)	3	1 = 100%
2	a^u	b^u	c^d	(u, u)	0	0	1	0.333 = 33.3%
3	a^u	b^d	c^u	0	0	(u, u)	1	0.333 = 33.3%
4	a^u	b^d	c^d	0	(d, d)	0	1	0.333 = 33.3%
5	a^d	b^u	c^u	0	(u, u)	0	1	0.333 = 33.3%
6	a^d	b^u	c^d	0	0	(d, d)	1	0.333 = 33.3%
7	a^d	b^d	c^u	(d, d)	0	0	1	0.333 = 33.3%
8	a^d	b^d	c^d	(d, d)	(d, d)	(d, d)	3	1 = 100%

According to the "hidden variable" idea, the average likelihood of seeing a match for any pair must be at least **6/18 = 1/3 = 0.333**

$$P(a^u c^u) \le P(a^u b^u) + P(b^u c^u)$$

True!

Hidden Variable Theory vs. QT

- According QT, the average likelihood of seeing a match for any pair must be at least **0.250**, and for both (u,u) and (d,d) pairs it is **0.500**.
  - This is **less** than the "hidden variables" predicted value of at least **0.333** for both (u,u) and (d,d) pairs.

  - The value **0.250** is the "square" of the cosine of the angle (120°) between any two ***SG* detector angles**, (*A*, B), (B, *C*), or (*A*,*C*).

  The cos(120°) = –**0.500** and cos(120°)^2 = **0.250**.

$P(a^u c^u) \leq$ P(a[illegible]) + $P(b^u c^u)$ **Bell's Inequality is violated.**

Who is right?

- If the results obtained experimentally are not accordance with the **0.333** prediction (assumes hidden variables), and are closer to the QT prediction then our initial assumption above - that *A*, B and *C* exist simultaneously - must be **incorrect**.

Bell's Test Using Atomic Spins

- Simulated Stern-Gerlach apparatus (St. Andrews University, Scotland)
  - "**Entangled Spin ½ Particles Pairs and Hidden Variables**"
    - https://www.st-andrews.ac.uk/physics/quvis/simulations_html5/sims/quantum-versus-hv1/quantum-versus-hv1.html

- **Hidden Variable Idea** – The individual states of each particle coming from the sources form a "product" state:

$$|\psi\rangle = |\uparrow\rangle|\downarrow\rangle$$

- **QT Idea** – The particles coming from the source are in an "entangled" state:

$$|\psi\rangle = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

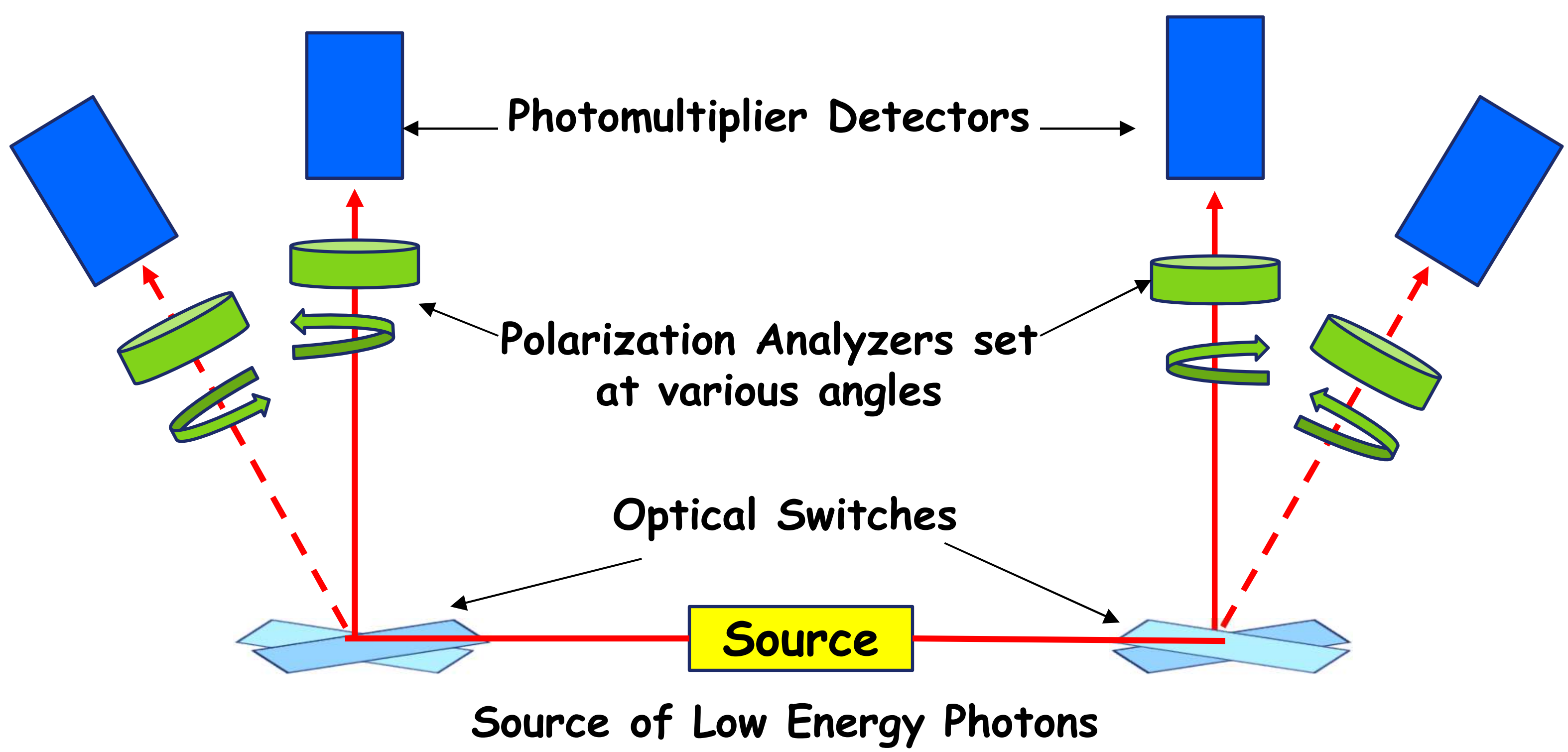
Timeline of Experiments to Test Bell's Theorem

1. Freedman and Clauser (1972)
2. Aspect et al. (1982)
3. Tittel et al. (1998)
4. Weihs et al. (1998): experiment under "strict Einstein locality" conditions
5. Pan et al. (2000) experiment on the GHZ state
6. Rowe et al. (2001): the first to close the detection loophole
7. Gröblacher et al. (2007) test of Leggett-type non-local realist theories
8. Salart et al. (2008): separation in a Bell Test
9. Ansmann et al. (2009): overcoming the detection loophole in solid state
10. Giustina et al. (2013), Larsson et al (2014): overcoming the detection loophole for photons
11. Christensen et al. (2013): overcoming the detection loophole for photons
12. Hensen et al., Giustina et al., Shalm et al. (2015): "loophole-free" Bell tests
13. Handsteiner et al. (2017): "Cosmic Bell Test" - Measurement Settings from Milky Way Stars
14. Rosenfeld et al. (2017): "Event-Ready" Bell test with entangled atoms and closed detection and locality loopholes

QT is supported in every case.

Alain Aspect's 1982 High Precision Experiment

Optical Switches are so far apart that any information (light) exchange between them cannot occur within the 40ns time period for the Switches to change polarizations.



Conclusions

- Experiments support the predictions of **QT**.
  - As a practical matter, a different version of Bell's Inequality - called the “CHSH Inequality”[1] is what was actually analyzed.
- These results means that our seemingly reasonable assumption (i.e. simultaneous **hidden variables** at **a**, **b** and **c** is invalid.
  - For classical composite systems each of the subsystems has well-defined properties.
  - In **QT** there exist states for which the wave function of the composite system is known, but the subsystems cannot be described in terms of individual wave functions and thus cannot be described separately.
    - Such states are not the product of individual wave functions, and , as such are not factorizable. They are entangled.
    - **S QT** explains: a particle’s attributes only exist within the context of an actual measurement.
  - We may agree with Bohr: **the Moon (Reality) is NOT there when we are not looking at it (so to speak).**

Some References

1. N. Gisin, *Quantum Chance –Nonlocality, Teleportation and Other Quantum Marvels* (Wiley, New York, 2004)

2. A. Aspect, J. Dalibard, G. Roger, *Experimental Test of Bell's Inequalities Using Time - Varying Analyzers*, Phys. Rev. Lett. 49 #25, 1804 (1982).

3. B. d'Espagnat, *The Quantum Theory and Reality*, Scientiific American, 158 (1979)

4. J. Blanton, *Does Bell's Inequality rule out local theories of quantum mechanics?*

5. J.S. Bell, *On the Einstein Podolsky Rosen Paradox*, Physics 1 #3,195 (1964).

6. A. Einstein, B. Podolsky, N. Rosen: *Can a Quantum-Mechanical Description of Physical Reality be Considered Complete?* Phys. Rev. 41, 777 (1935).

7. N. David Mermin, *Is the Moon There When Nobody Looks? Reality and the Quantum Theory*, Physics Today, 38 (April 1985)

8. L. Maccone, *A simple proof of Bell's inequality*, Am. J. of Phys.81, 11, 854 (2013):

9. Video: Bell's Theorem: The Quantum Venn Diagram Paradox: https://www.youtube.com/watch?v=zcqZHYo7ONs

10. Video: Quantum Entanglement & Spooky Action at a Distance https://www.youtube.com/watch?v=ZuvK-od647c