Fun with Fourier Transforms, or, How I Learned to Stop Worrying and Love Convolutions

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CSAAPT

Goal of Talk

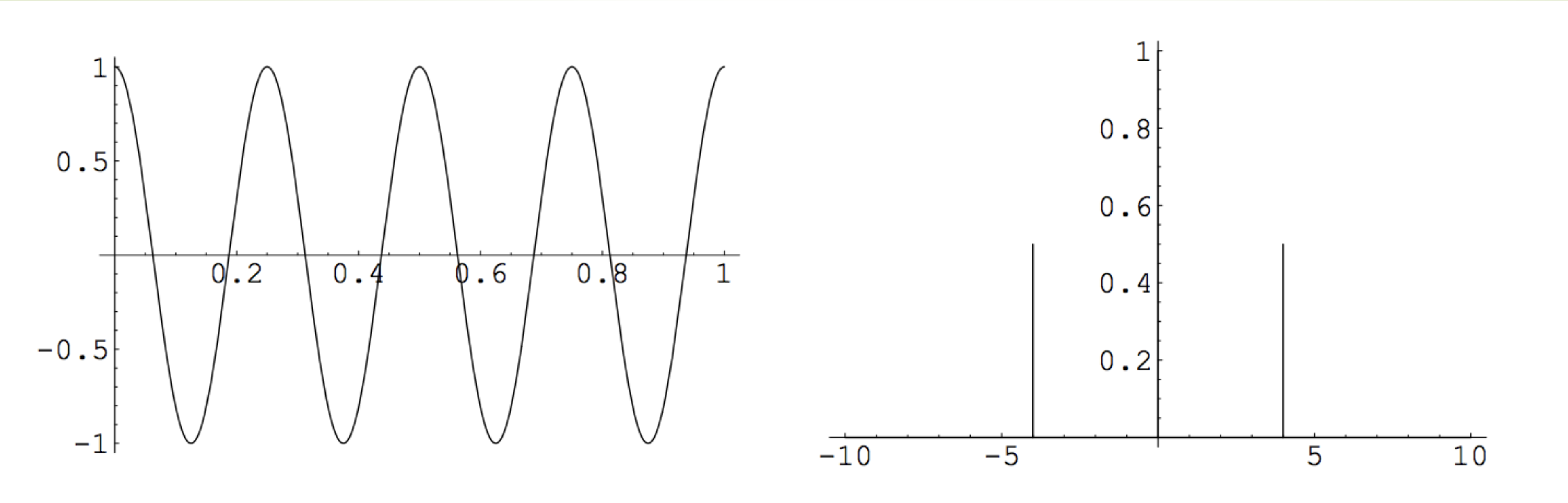
- Present Fourier Transforms in an accessible way without getting bogged down in the math.
- Give you some fun and interesting ways to motivate your students to love Fourier Transforms as much as I do!

What is a Fourier Transform?

- Decompose a function into a function of something else
  - Conjugate Variables t,f; x,p (Falls under Duality for the Mathematicians)
  - $$\hat{f}(\omega) = \int_{-\infty}^{\infty} \exp(-2\pi i \omega x) f(x)\, dx$$
    $$f(x) = \int_{-\infty}^{\infty} \exp(2\pi i \omega x) f(\omega)\, d\omega$$
- Time <
- Aperture Position <-> Angle (Diffraction, Telescope Design)
- Position <-> Momentum (Quantum Mechanics)

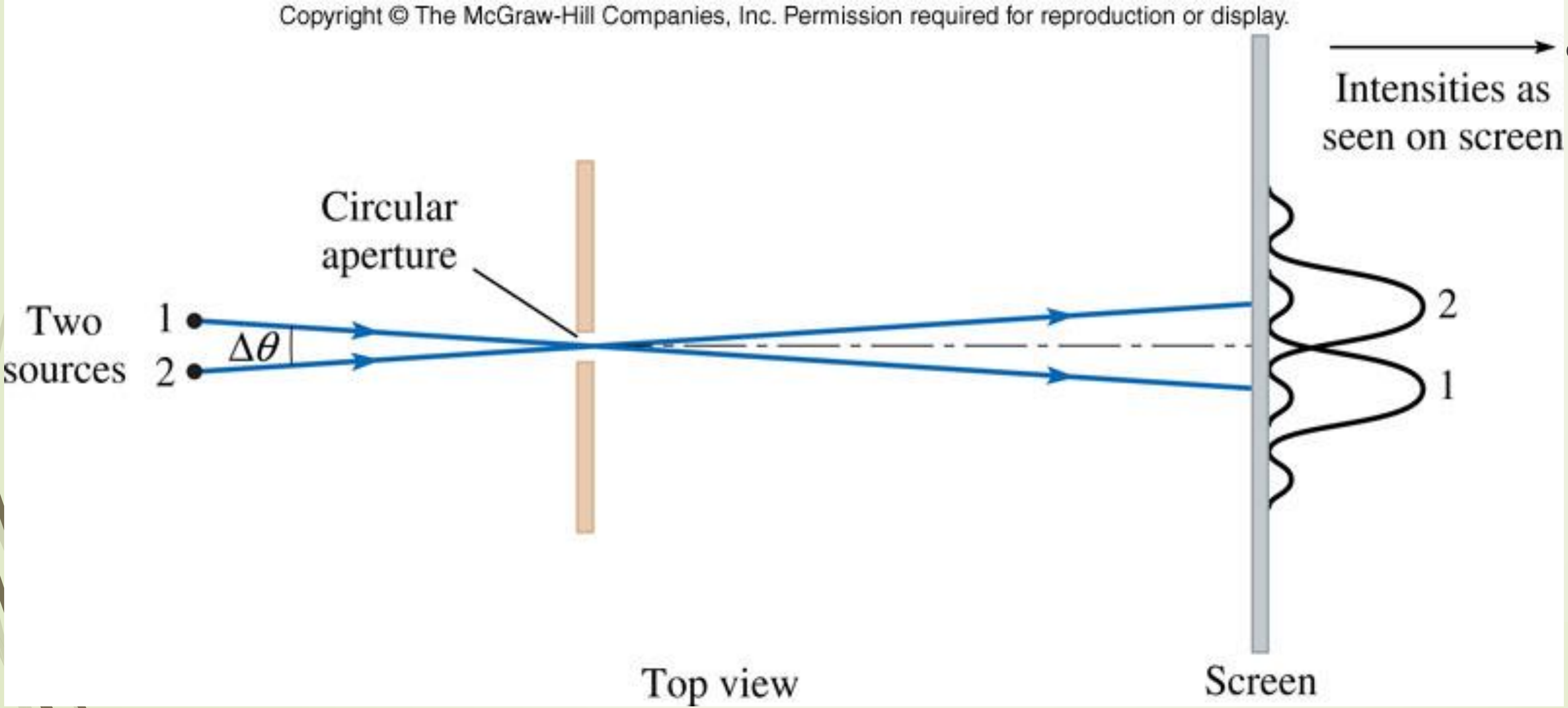
What is a Fourier Transform?

- Decompose a function into a function of something else
- Time <-> Frequency (Signal Analysis)
  - Left: Sound Wave. Right: Which key you press on a piano



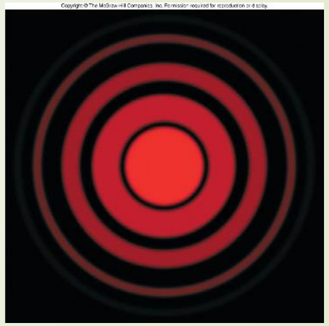
What ELSE is a Fourier Transform: Nature!

- Aperture Shape/Position <-> Angle (Diffraction: Telescope Design)
  - θ = λ /D; θ (resolution angle) = λ (wavelength)/D (diameter) =



asses)

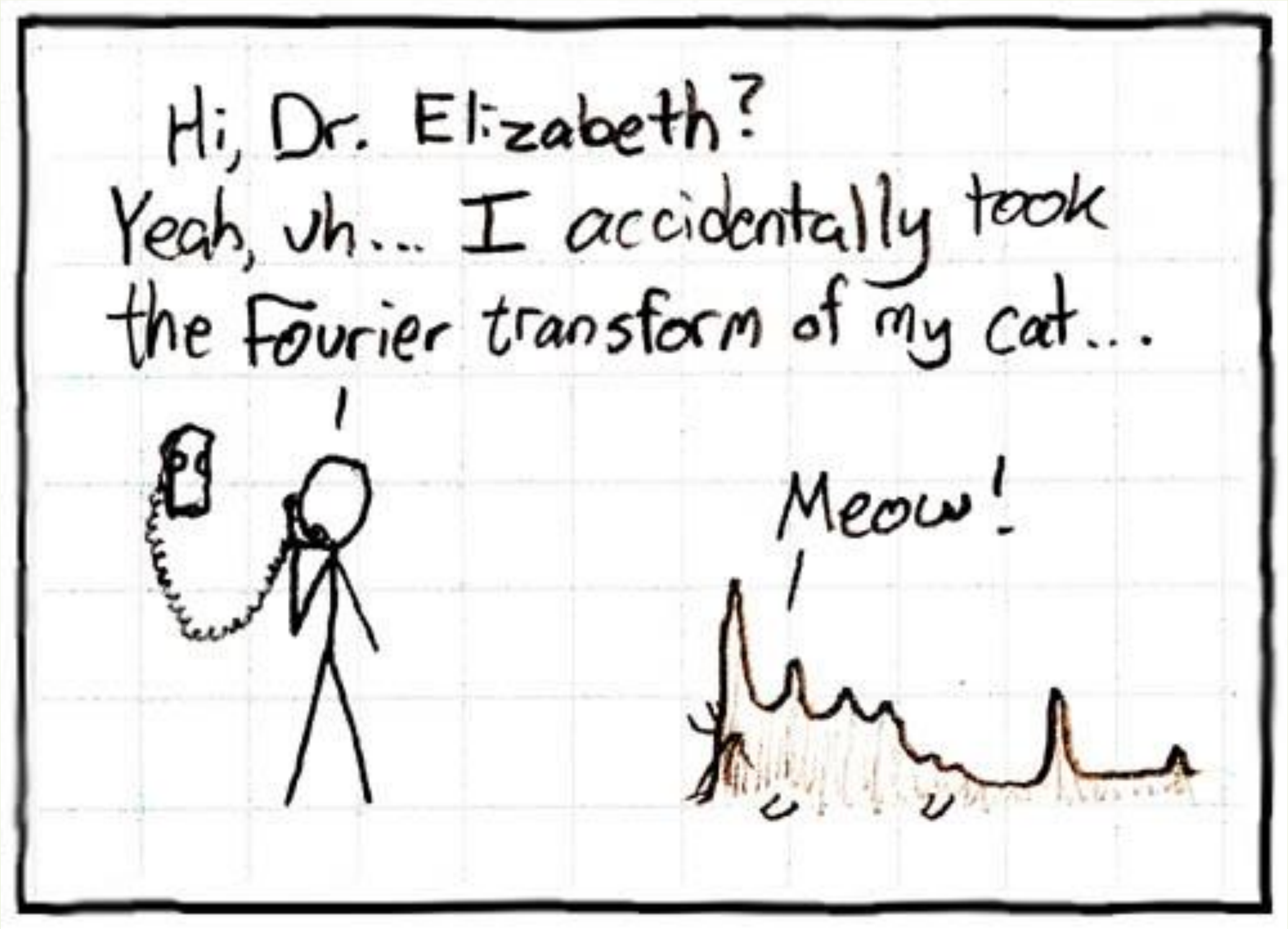
Laser Through a Small Hole:



= F.T. of a Small Circle

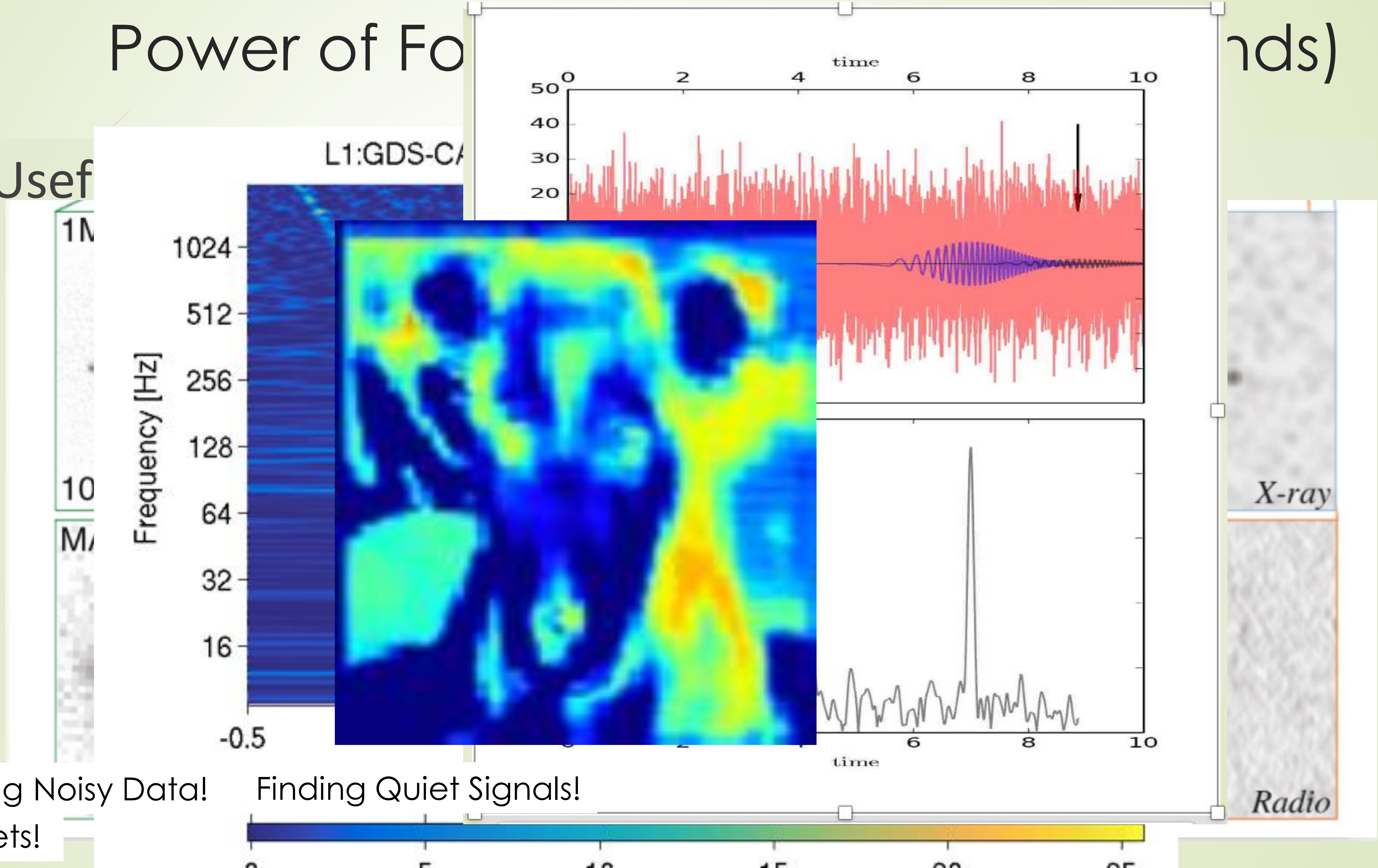
With great power, comes great responsibility…

https://xkcd.com/26/



Power of Fo nds)

- Usef



Astronomy!

Understanding Noisy Data!

Finding Quiet Signals!

Identifying Pets!

Convolution: One of the best uses of Fourier Transforms, but: Who uses that?

- Audio Synthesis
  - http://iub.edu/~emusic/etext/synthesis/chapter4_convolution.shtml
  - Dog (click) * Concert Hall (x3)
  - Dog in a concert hall!

    =

  - Deconvolution: Take the dog out of the concert hall https://www.dsprelated.com/freebooks/mdft/Convolution_Theorem.html#table:ffttable

Phenomenal, Cosmic Power

- Convolution: Complicated!!

$$(f * g)(x) = \int_{-\infty}^{\infty} f(x-y)g(y)$$

- Convolution Theorem: Makes the Complicat

$$\widehat{(f * g)}(\omega) = \hat{f}(\omega)$$

- Convolution is a crazy integral; Multiplicatio
- Even Better: Deconvolution = Division

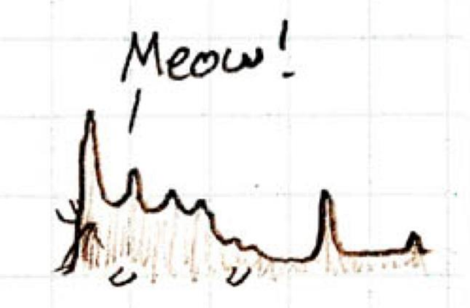
Table 7.1: Direct versus FFT convolution times in milliseconds (convolution length = 2^M) using Matlab 5.2 on an 800 MHz Athlon Windows PC.

M	Direct	FFT	Ratio
1	0.07	0.08	0.91
2	0.08	0.08	0.92
3	0.08	0.08	0.94
4	0.09	0.10	0.97
5	0.12	0.12	0.96
6	0.18	0.12	1.44
7	0.39	0.15	2.67
8	1.10	0.21	5.10
9	3.83	0.31	12.26
10	15.80	0.47	33.72
11	50.39	1.09	46.07
12	177.75	2.53	70.22
13	709.75	5.62	126.18
14	4510.25	17.50	257.73
15	19050.00	72.50	262.76
16	[illegible]	[illegible]	[illegible]

Cats vs. Dogs: Image Recognition

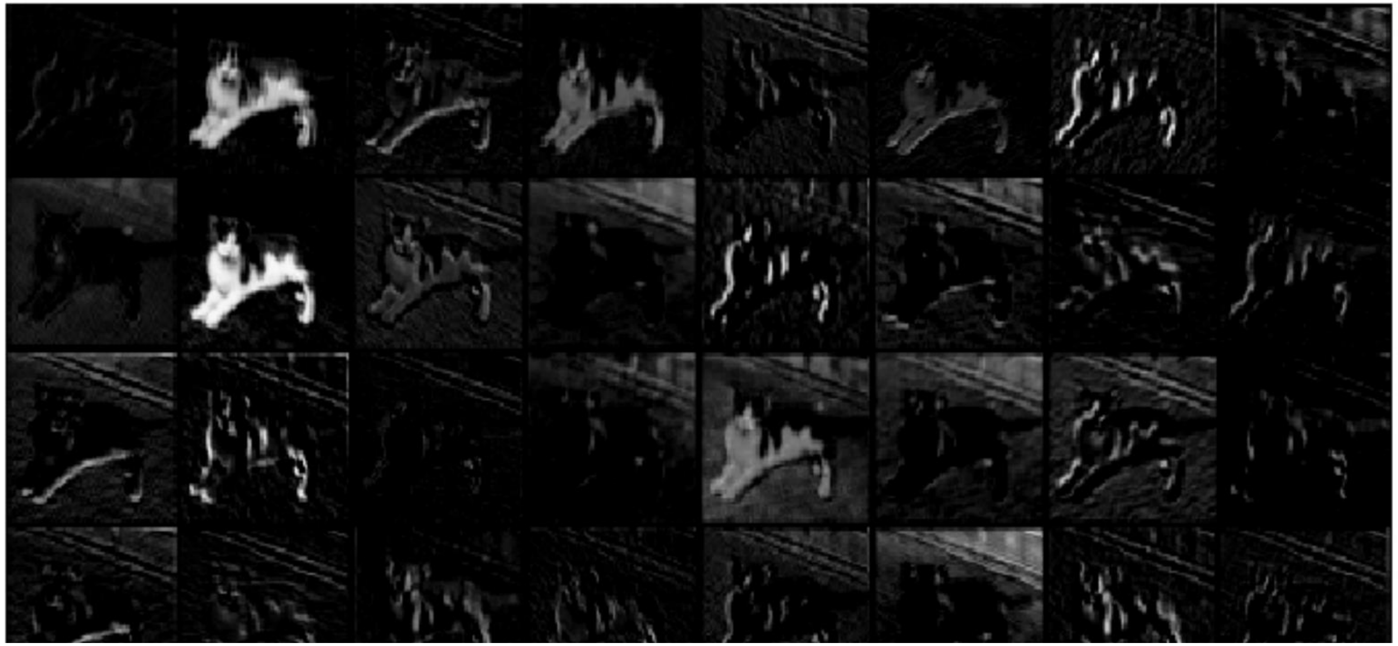
- Convolutional Neural Networks
- Instead of using all the pixel values in an image
- Convolve a Filter With the Image
  - Get another, different image
  - (The Neural Network picks the best filters)

$\tilde{}$ =

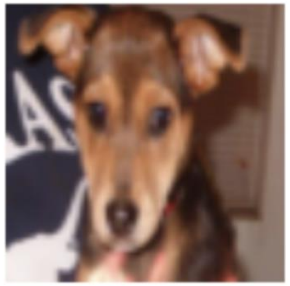


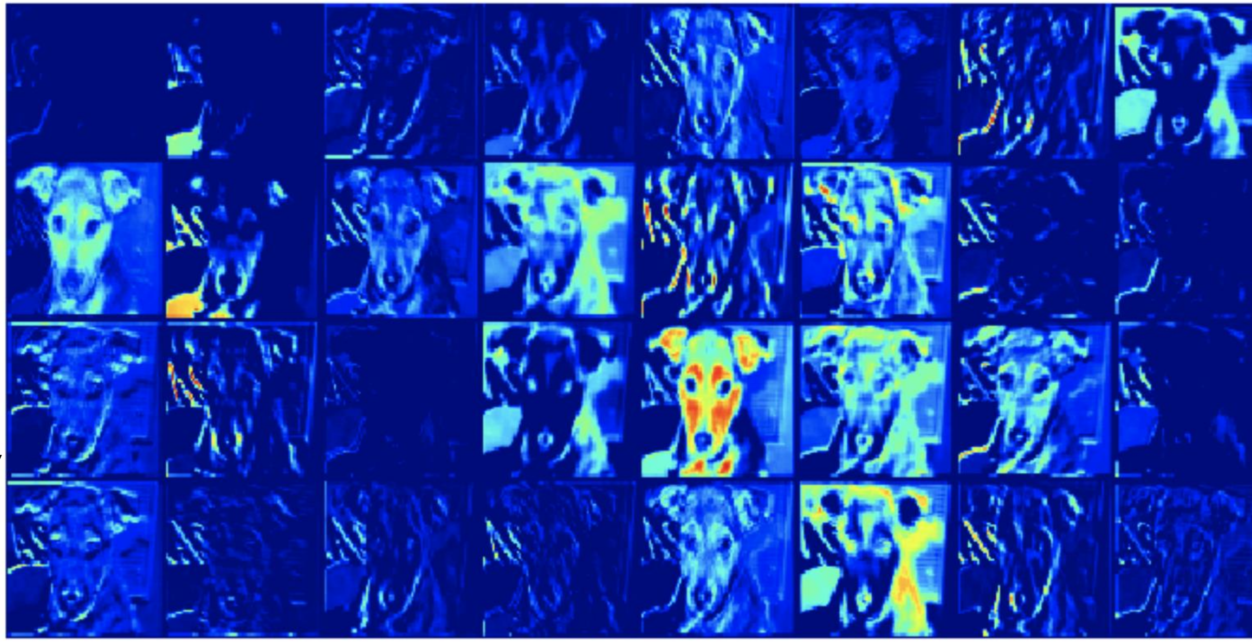
- http://www.subsubroutine.com/sub-subroutine/2016/9/30/cats-and-dogs-and-convolutional-neural-networks
  - Search for "What has it learned" (I've put this on the next slide)

Cats vs. Dogs



- Each "new" image is a "feature" input to the NN
- NN Can now distinguish between a cat and a dog!



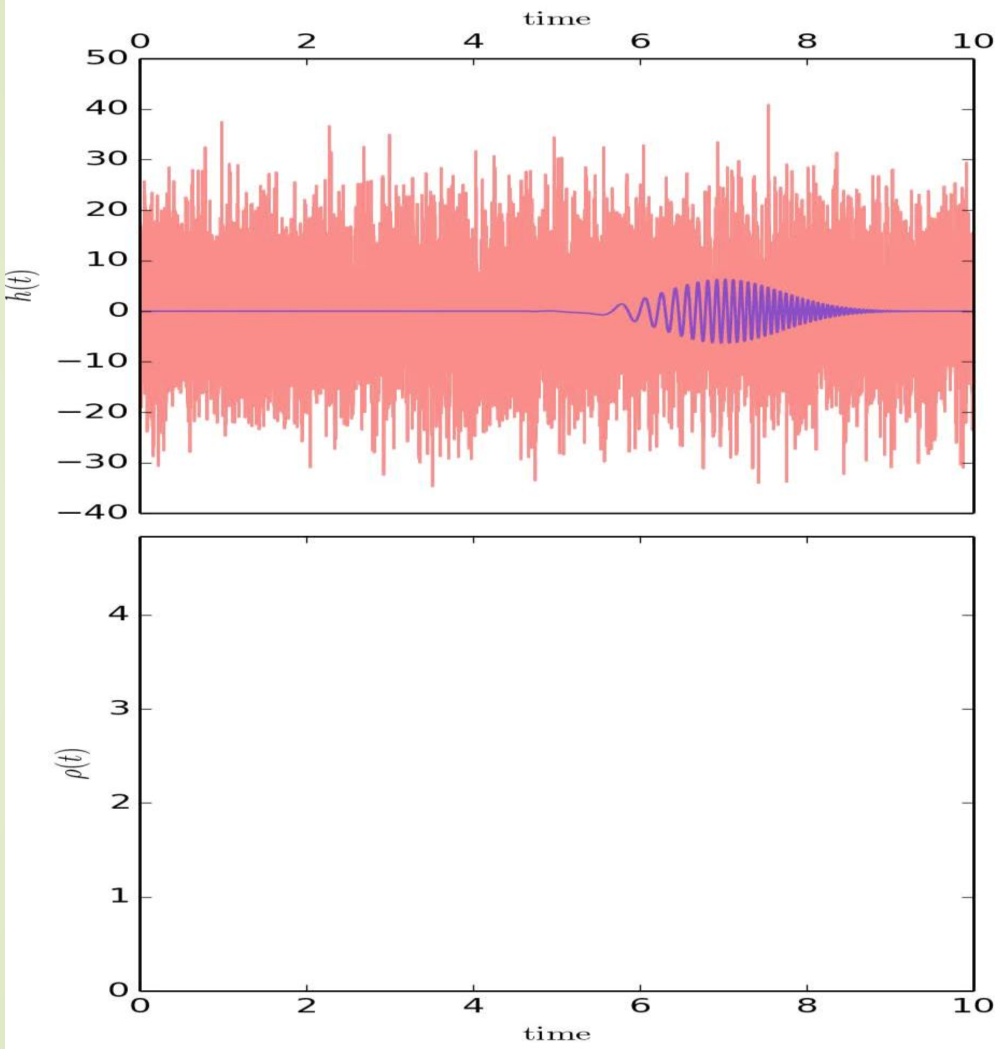


(Dogs are obviously cooler)

Finding a weak (gravitational wave) signal in noise

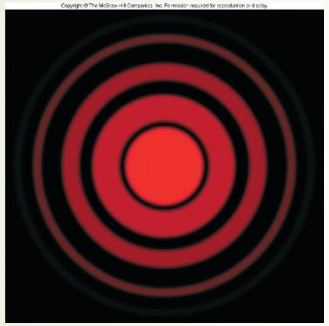
$$(s|h)(t) = 4\mathrm{Re}\int_{f_{\mathrm{low}}}^{f_{\mathrm{high}}} \frac{\tilde{s}(f)\tilde{h}^*(f)}{S_n(f)} e^{2\pi i f t}\,\mathrm{d}f,$$

- "Matched filtering" lets us find a weak signal submerged in noise.
- If you know the signal waveform:
  - Multiply the waveform by the data, for all possible times when the signal might have arrived.
  - = Take the Convolution of Data with the Waveform
- When there's a match, you see it!
- S+N -> Project onto S;
  - <N,S> << <S,S>

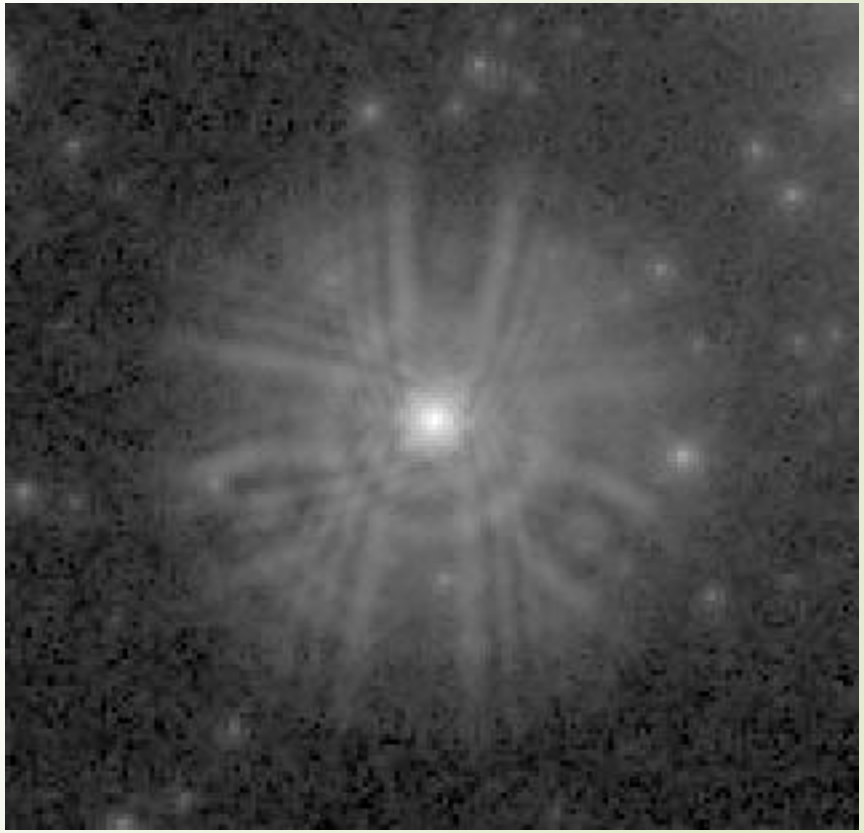


Beam Pattern of a Telescope

- Telescopes create an image of a point source that doesn't look like a point
- Perfect Circular Aperature

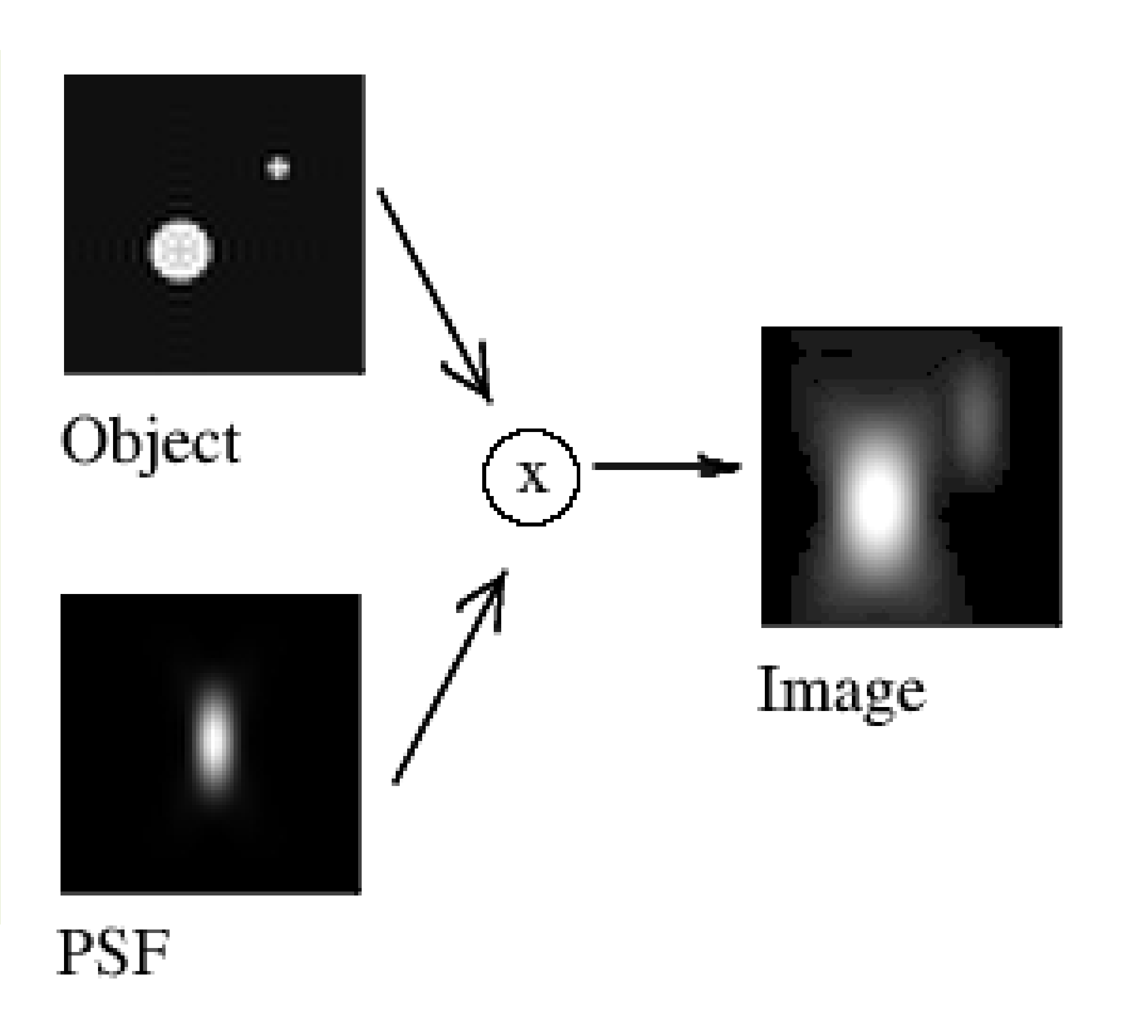


- Extreme Example:
  - Hubble with Flawed Optics ->



Called the Point-Spread Function = PSF

- What we see is the convolution of PSF with the real light from objects
- Must Deconvolve to get original back!
- Hey! In Fourier Land, this is just division!



http://exoplanet.as.arizona.edu/~lclose/a302/lecture9/Lecture_9.html

Fixing a Hubble Image

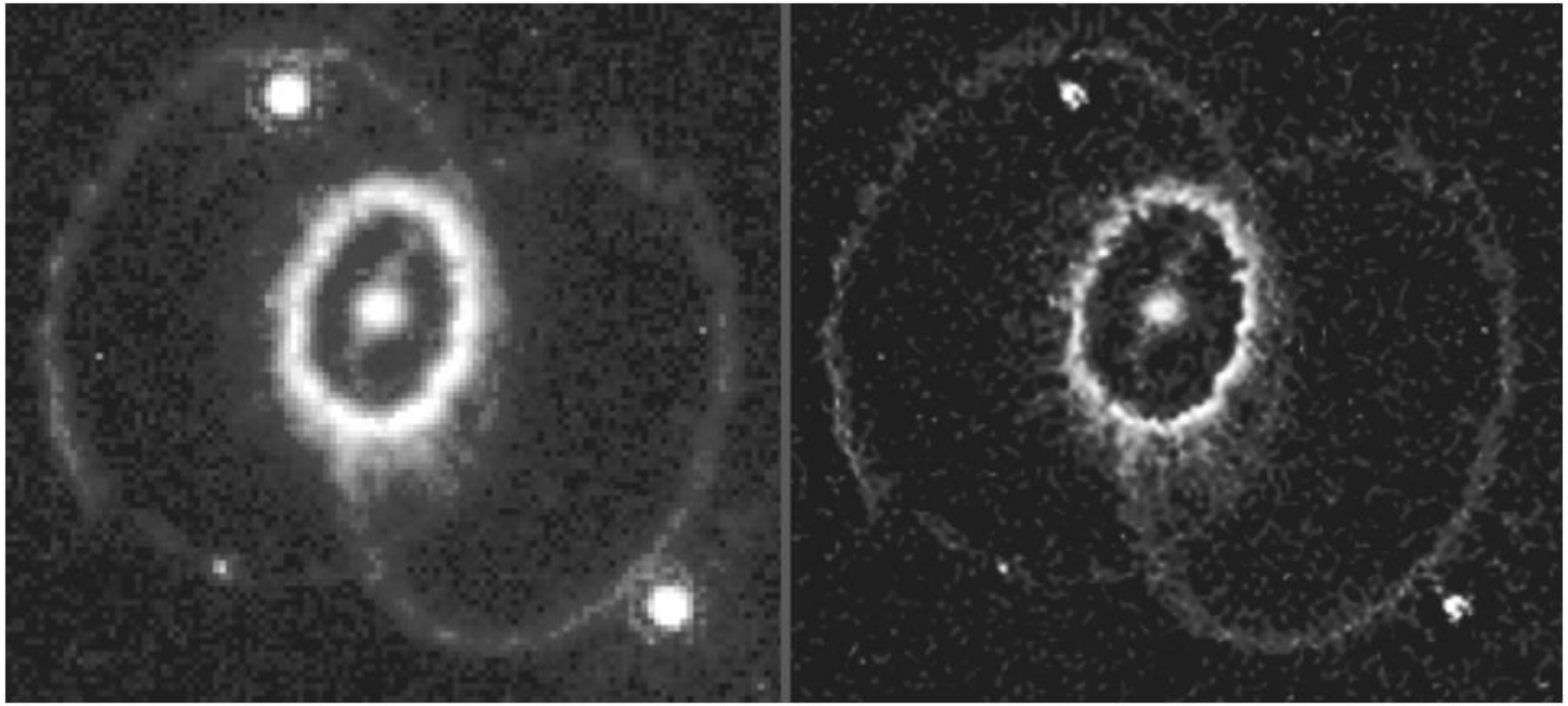


Figure 7. (Left) Image of the remnants of Supernova 1987a observed with the WFPC2 PC in filter F656N. (Right) Result of deconvolving the image with a Tiny Tim model PSF. The nebular rings are better separated from the background stars and the bright ring around the supernova remnant is better defined.

Krist et.al. https://ui.adsabs.harvard.edu/link_gateway/2011SPIE.8127E..0JK/doi:10.1117/12.892762

Fixing the instrument helps too!

Both Hubble images were "raw," they were not processed using computer image reconstruction techniques that improved aberrated images made before the servicing mission.

https://en.wikipedia.org/wiki/File:Hubble_Images_of_M100_Before_and_After_Mirror_Repair_-_GPN-2002-000064.jpg

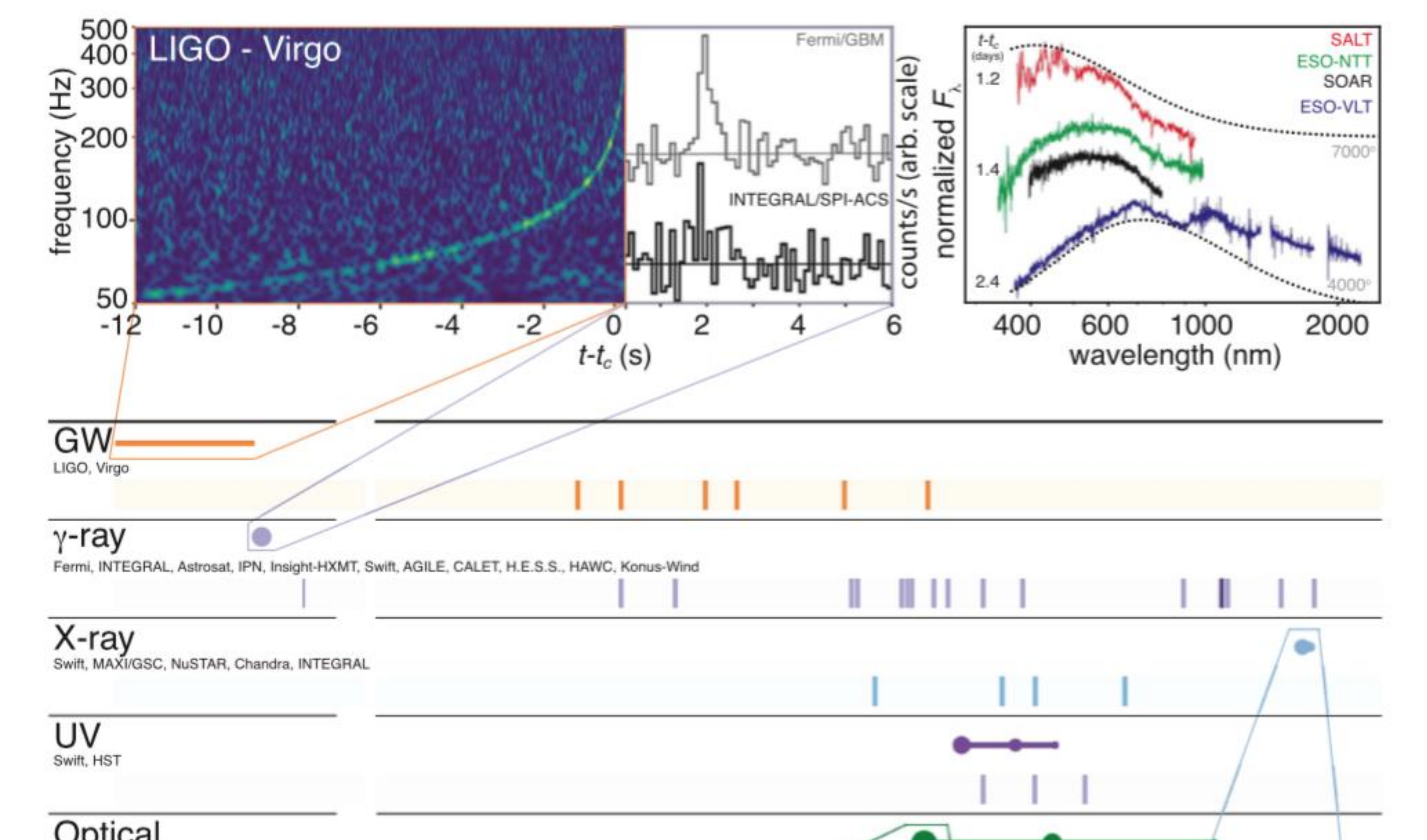
Thank you!!
Extra Time? Extra Slides!

- ryan.fisher@cnu.edu
- If you have any students interested in gravitational waves, please share my email with them!
- I love giving talks about all things gravitational-waves, astronomy and physics.

Enables: Multi-Messenger Detection of a GRB/BNS

- Gamma-Ray-Burst
- GW Detection
- X-Ray
- Optical
- UV
- Etc.!

B.P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration), Astrophys. J. Lett. 848, L12 (2017).



Mul

- Gamma-Ray-Burst
- GW Detection
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- Etc.!

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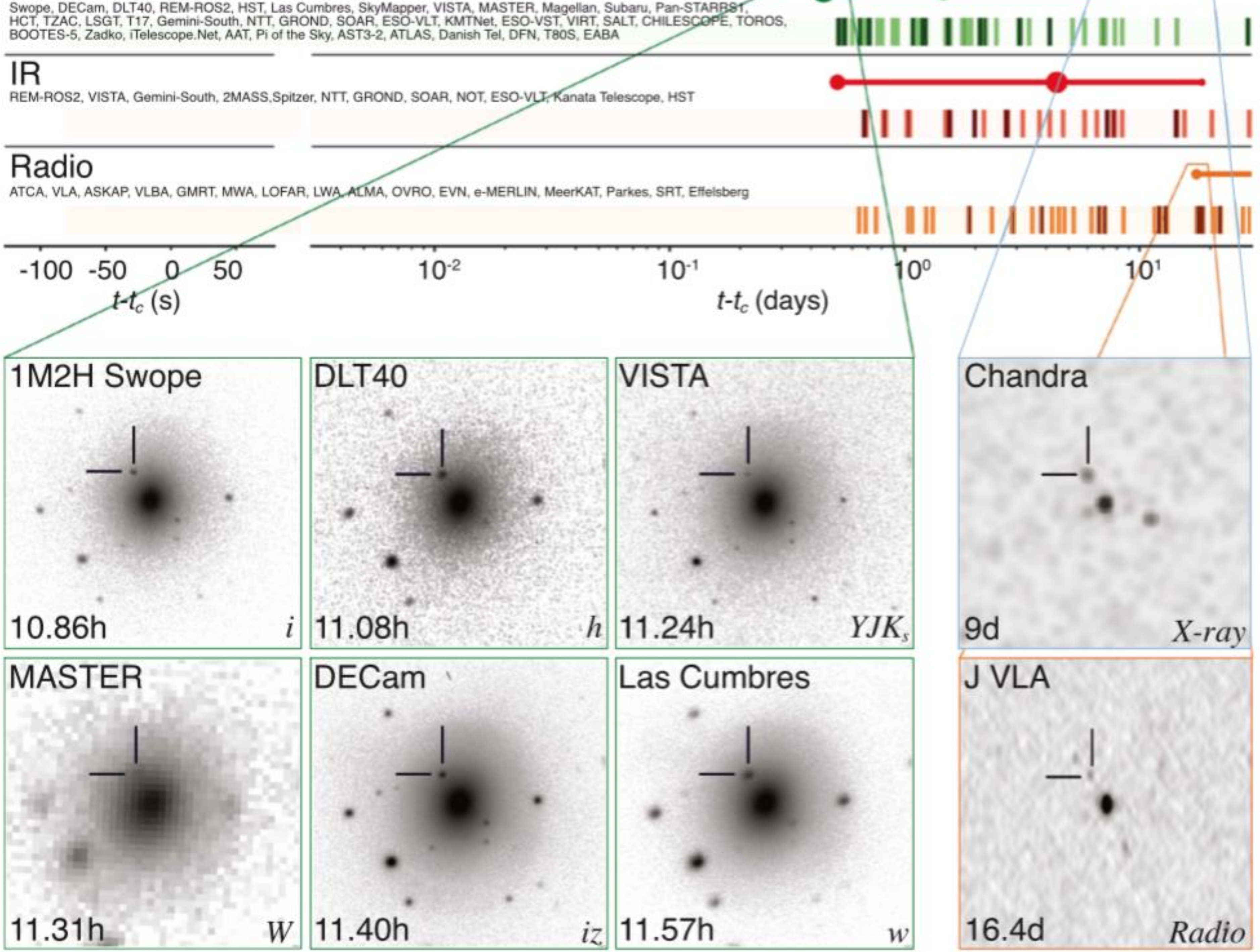
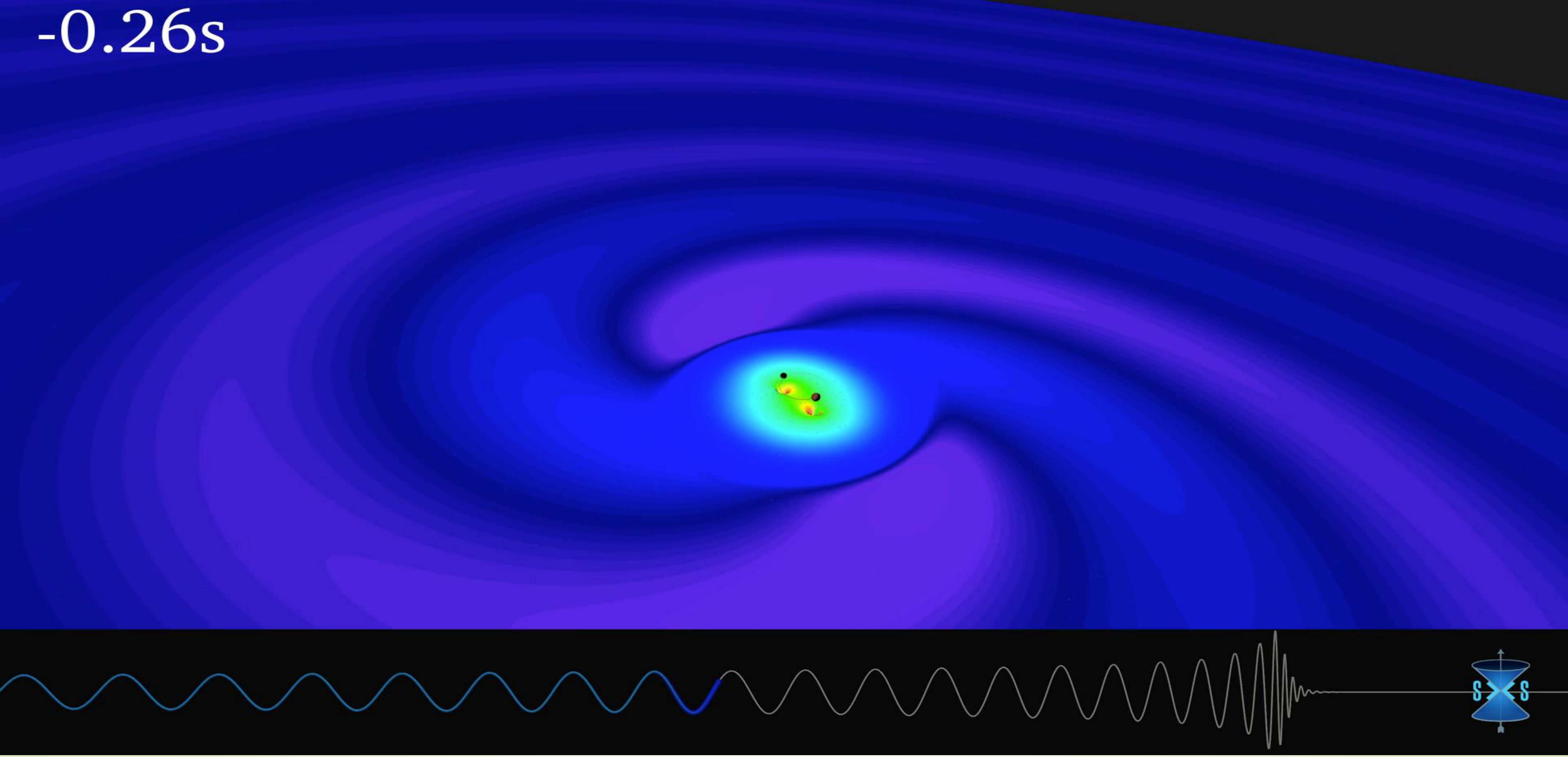


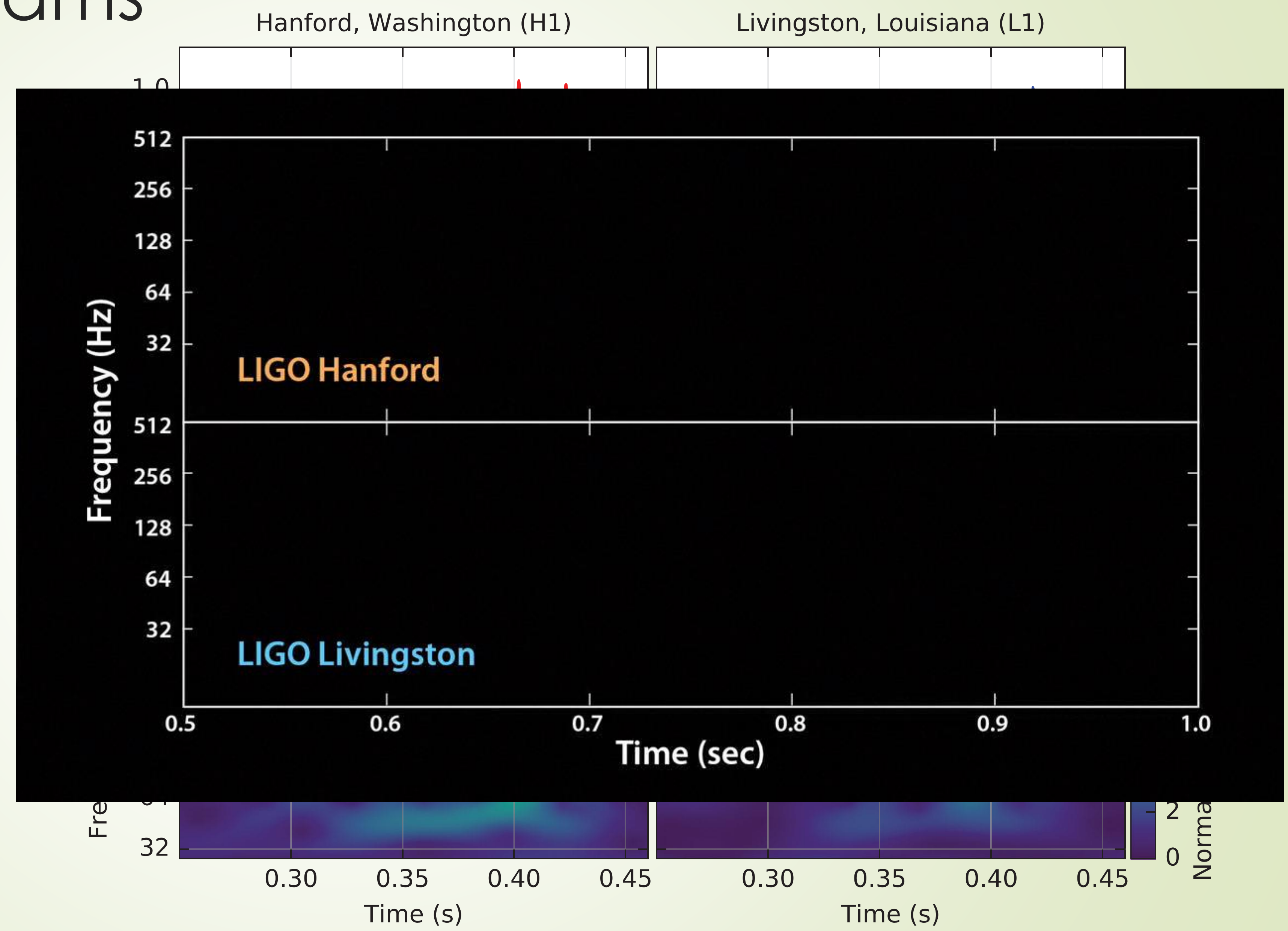


Our Gravitational Wave: Simulation

Credit: Harald Pfeiffer, for SXS Collaboration; Simulations performed on Compute Canada HPC systems

Spectrograms

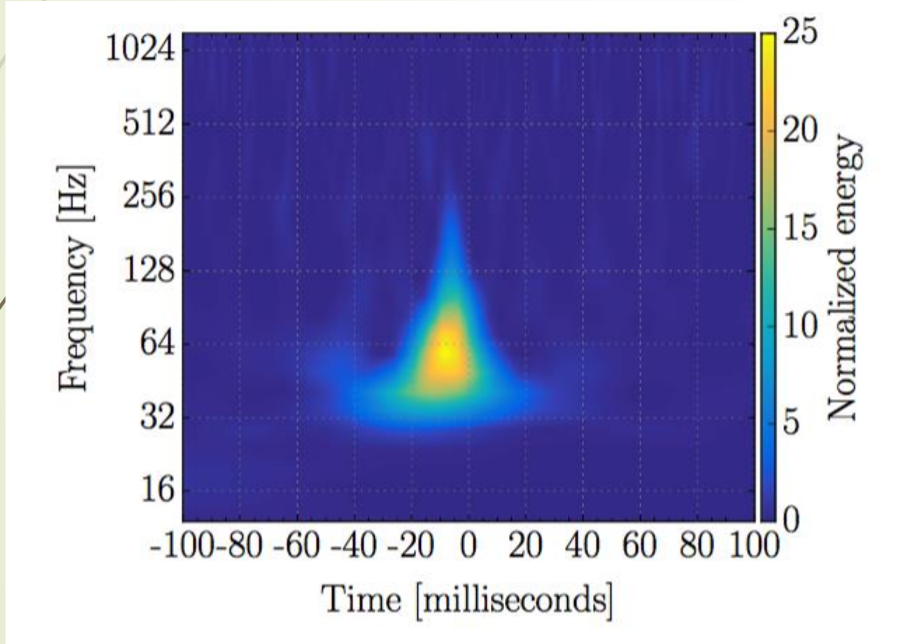
Fourier Transform of a Gravitational Wave!

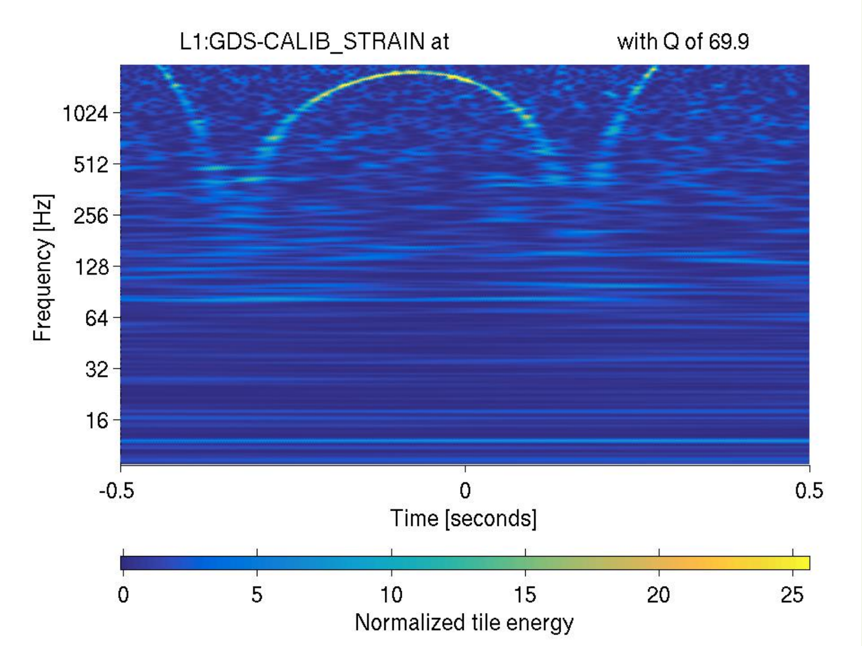


PRL 116, 061102 (2016)

Reminder: The Piano keys are now arranged vertically, and the time is horizontal

Detector Characterization: Looking for Noise With Lots (and lots) of Fourier Transforms!





Abbott, et al. **Class.Quant.Grav. 33 (2016) no.13, 134001**

Gravity Spy!

- Goal: Identify Different Types of Noise Artifacts in LIGO Data
- Attempt 1: Convolutional Neural Network
  - FAIL
- Attempt 2: Use Humans to Train a Convolutional Neural Network
  - https://www.zooniverse.org/projects/zooniverse/gravity-spy/classify
  - SUCCESS! : We now have a CNN identifying different classes of noise in our instrument, and we are using other algorithms to identify what caused each class
- (I'm on "Level 2", (I sold my level 10 account for $100 on ebay))