Using math in physics: What’s the problem?

E. F. (Joe) Redish
University of Maryland

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This is an interactive talk!

If you have Chrome, Firefox, or Safari browser on a tablet, or laptop, send it to

http://app.tophat.com/e/132795

or scan the QR code at the right.

Then choose to log in as a **guest**
(button on the lower right of the screen);

Who are we?

At what level do you currently teach?

1. Pre-college
2. Community college
3. 4-year college
4. University

What population of students do you teach? (Select all that apply.)

1. High-school students
2. Mixed population of college students
3. Physics majors at the intro level
4. Physics majors at the advanced level
5. Engineering students at the intro level
6. Life science students at the intro level
7. Non-major specialized course

Do we have a problem?

I often find that my students have difficulty using math in physics.

A problem my life-science students have trouble with

A problem my life science/pre meds have trouble with

- What is 3½ divided by ¼?

A problem they don't have trouble with

- For the faculty-student get together you ordered 4 small pizzas divided into quarters. The faculty ate two slices.
  How many students can have a slice?

A problem my engineering students have trouble with

A problem my engineers have trouble with

- The Electric field is defined by

$$E = F/q$$

  where q is the charge on the test (probe) charge.
- If q is replaced by $q/3$ what happens to E?

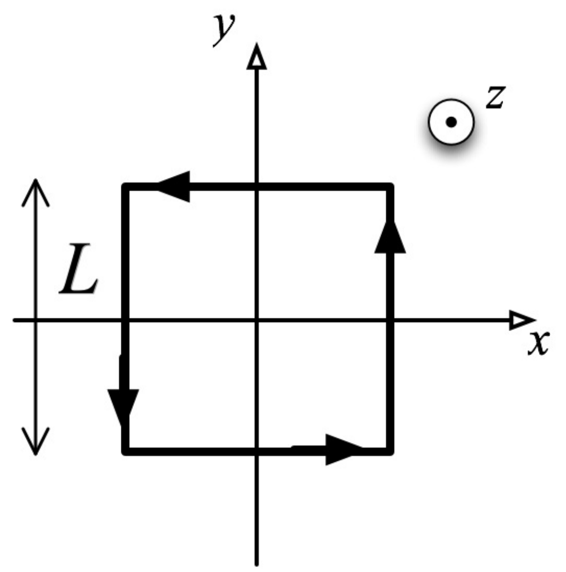
A problem my physics majors have trouble with

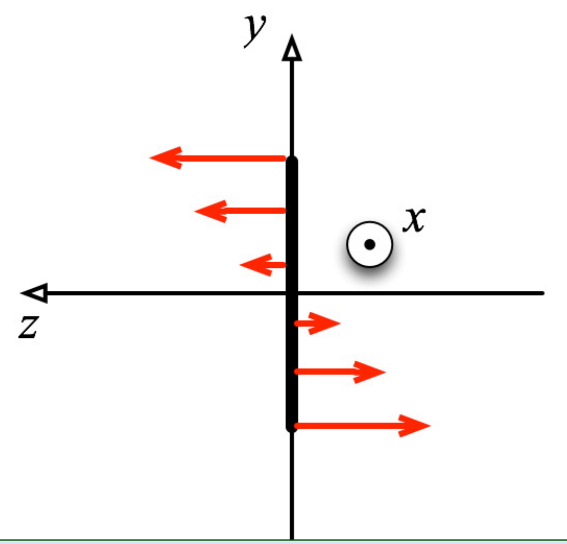
A problem my physics majors have trouble with*

- *A square loop of wire is centered on the origin and oriented as in the figure. There is a space-dependent magnetic field*

$$\vec{B} = B_0 y \hat{k}$$

- *If the wire carries a current, I, what is the net force on the wire?*





*** Griffiths, *Introduction to Electrodynamics* (Addison-Wesley, 1999).**

Two paths to a solution

- Student A
  - Huh! Looks pretty simple – like a physics 1 problem.
  - The sides cancel so I can just do

    $$\vec{F} = I\vec{L} \times \vec{B}$$

    on the top and bottom where B is constant.
  - Gonna get

    $$\vec{F} = IL^2 B_0 \hat{j}$$

- Student B
  - I'm pretty sure they want us to do the vector line integral around the loop.

    $$\vec{F} = \oint I\, d\vec{L} \times \vec{B}$$

  - It's pretty straightforward.
  - The sides do cancel, but I get the top and bottom do too, so the answer is zero.

From a video of two physics majors working together to solve a problem in a third-year E&M course.

What do you think happened next?

What do you think happened next?

1. Student A (it’s simple) immediately yields to student B (I did the integral).
2. Student B (I did the integral) immediately yields to student A (it’s simple).
3. They said, “Let’s figure out why these don’t agree,” and worked together to resolve the discrepancy.
4. They immediately gave up and called a TA over and asked what the answer was.

No argument!

- Student A immediately folded his cards
  in response to student B’s more mathematically
  sophisticated reason and agreed she must be right.
- Both students valued (complex) mathematical reasoning
  (where they could easily make a mistake)
  over a simple (and compelling) argument
  that blended math and physics reasoning.
- The students’ expectations that the knowledge
  in the class was about learning to do complex math
  was supported by many class activities.

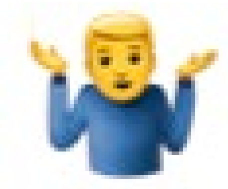
What’s missing in all three examples

- In each case, students fail to “**make the blend**” — combining information about mathematical symbols with knowledge about what those symbols represent.

Do we care?

Mathematical modeling is an essential part of learning to do and understand science.

- For physics majors, it's fairly obvious. Every advanced class is about the heavy use of math to describe physical systems.
- For engineers, it's fairly obvious. They use math to describe physical systems.
- For life science students and pre-meds, it's dicey. They rarely use math to describe physical systems in intro classes.

Physics is the best place to learn to reason about the world mathematically! 

However…

- Many non-physics STEM majors succeed in their intro level courses by memorizing facts, algorithms, and heuristics.
- Most have never seen an example where using math improves their professional understanding.
- We need to negotiate a change in student expectations

It's not just IPLS students! Chemists and engineers too!

The Math:
Some observations from ed research with life science students

- My students often see math as inappropriate in biology.
  - As a result, they often resist thinking about the math.
- My students often find that despite A’s in calculus, they can’t do the math I ask them to do.
  - Even though the math is trivial algebra? What’s up with that?
- My students are desparate to put numbers in right away and hate doing symbolic problems without numbers.
  - And they want to drop the units, only putting them in at the end. (Engineers too)
- My students prefer lots of plug-and-chug “practice problems” rather than a few “thinking problems.”

Any other instructors out there seeing this as well?

So let’s add to our learning goals

- Students will learn to use math effectively in science, including being able to:
  - **Calculate** the value of simple expressions and solve simple equations when the equations are given.
  - **Work with equations in symbolic form** including manipulating them and interpreting them for physical implications.
  - **Generate appropriate symbolic equations** describing physical situations for which no explicit numbers are given.
  - **Reason about physical situations using symbolic mathematics** both qualitiatively and quantitatively .
  - ….

How often do we ask our IPLS students to do more than the first?

What’s going on?

Math in science is not the same as math in math

- Math in math classes tends to be about numbers (and the structure of abstract relationships). Math in science is not.
- Math in math classes tends to use a small number of symbols in constrained ways. Math in physics uses *lots* of symbols in different ways.
- The symbols in science classes often carry meaning that changes the way we interpret the quantity.
- In introductory math, equations are almost always about solving and calculating. In physics it's often about ***explaining* and *making meaning***.

When we do a derivation of an equation we are ***giving an explanation.*** *Intro physics students rarely understand that this is what we are doing.*

Meta-misconceptions

- While some students may indeed lack mathematical skills, many (in my experience) can “do the math” when it’s expressed *as* math.* (At least after a brief review.)
- More serious problems (that are not easy to remedy by a review class or by math ”drill-and-kill” activities) are students’ **inappropriate expectations** about
  - the nature of the knowledge they are learning (***epistemological misconceptions***)
  - the nature of what mathematical symbols are being used to represent (***ontological misconceptions***)

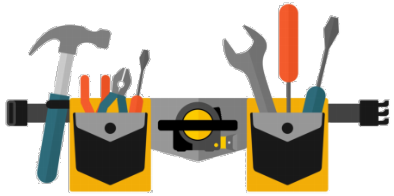
* Torigoe & Glasdding, AJP 79 (2011) 133-140
  Meltzer, King, & Jones, many AAPT talks (2016-2020)

The key

- The critical difference in the way we use math in science is that our symbols do not just represent numbers.
- In physics, symbols represent a **mapping** of physical meaning into mathematical symbols that **blends** together
  - our conceptual knowledge of the physical content and
  - our structural knowledge of mathematical relationships and processes.
- Making this blend is a non-trivial mental process that is rarely taught explicitly.
  How can we teach it?

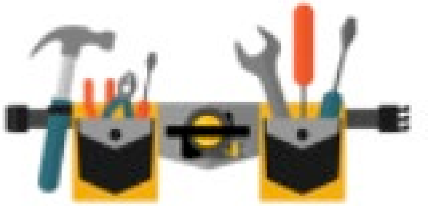
What can we do about it?

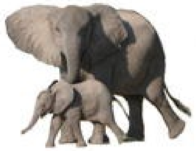
Teaching the blend



- One way to begin to approach teaching the blend is to be more explicit about identifying and teaching the core types of reasoning we use in the blend.
- I have created a **math-in-science toolbelt** with specific analytic and problem solving strategies.
- I teach these tools ("epistemic games") explicitly .
- Every time I use them (in class or in the text) students see an icon that appears to remind them that we are using that tool.

We can teach the blend through general purpose tools / strategies

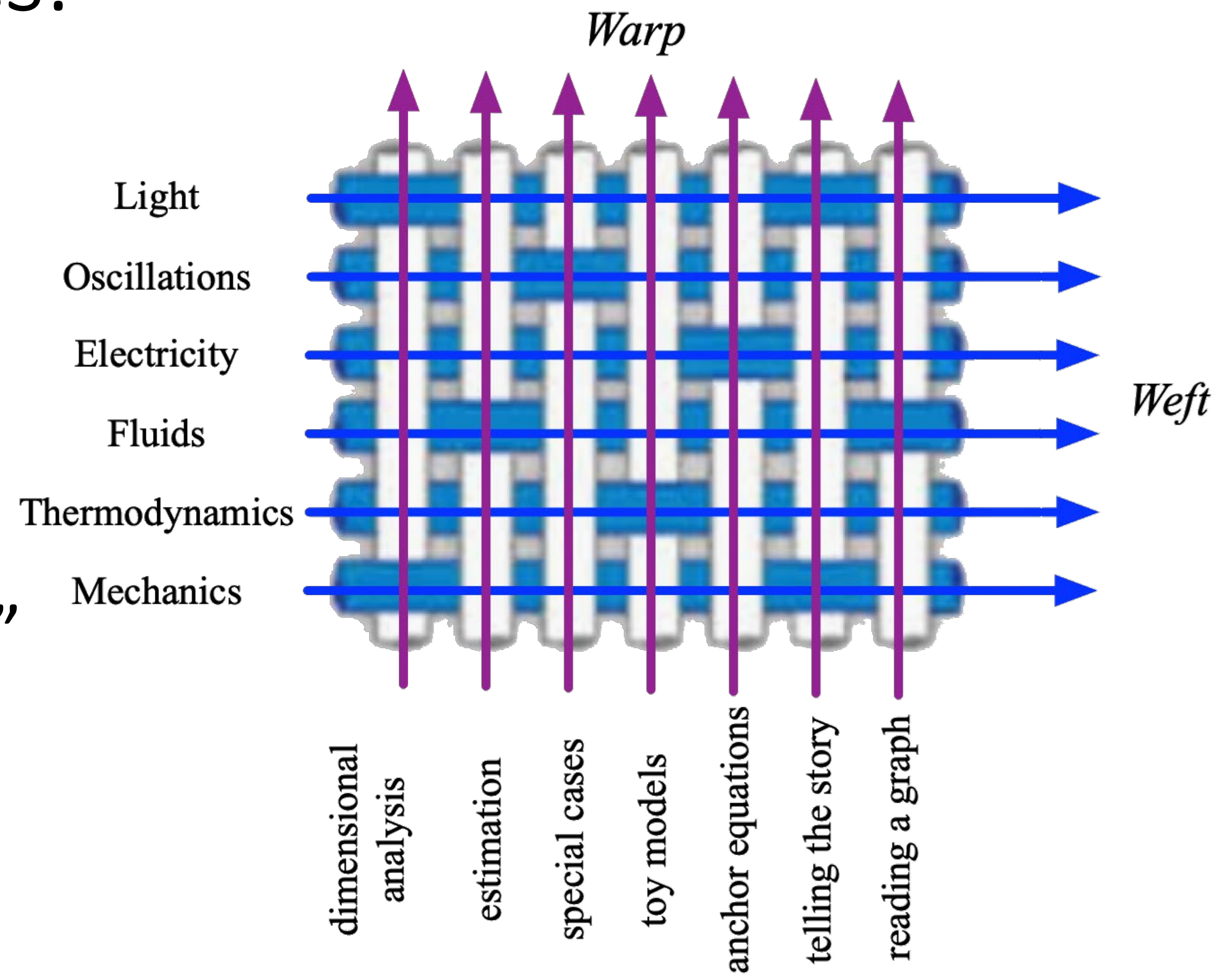


Dimensional analysis		Reading the physics in a graph	
Estimation		Telling the story	
Functional dependence/ scaling		Diagrams	
Anchor equations		Special and extreme cases	
Toy models		Building equations from the physics	

https://www.compadre.org/nexusph/course/Building_your_mathematical_toolbelt

Skills apply to all topics.

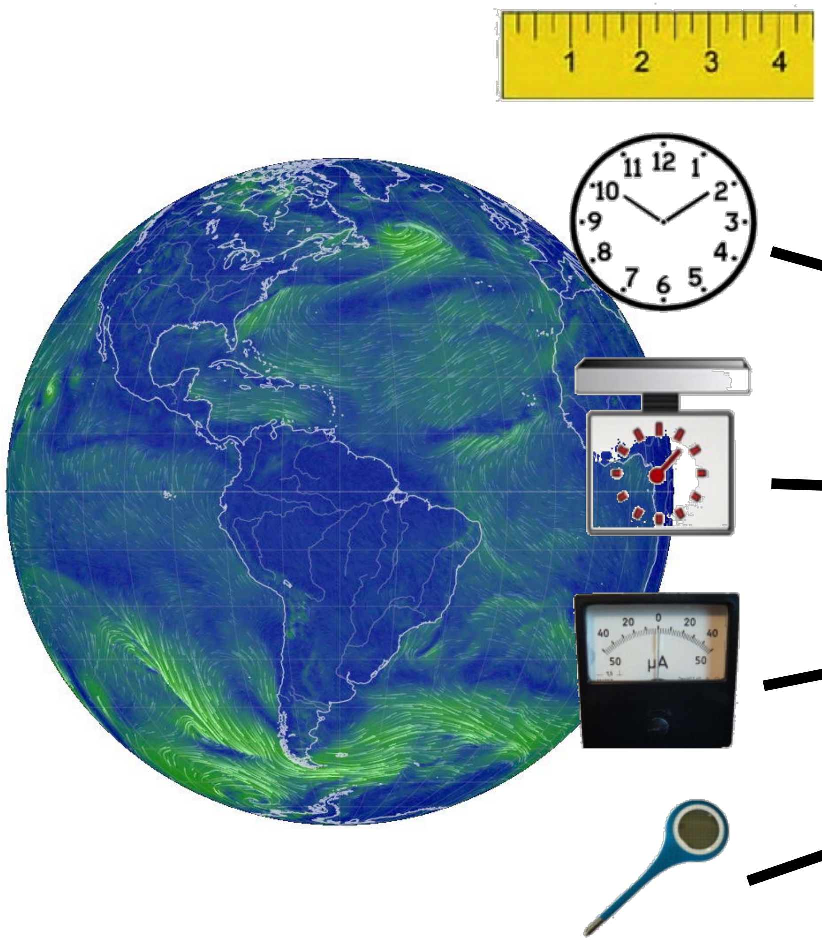
- Adding a focus on developing math skills doesn't add new topics.
- It modifies the way we teach topics we already include.
- It does require us to be "meta" — to be explicit about tools since students don't easily generalize.



Redish, Using math in physics: Overview, *The Physics Teacher*, **59** (2021) 314

The ontological tools:
What kinds of things are our symbols?

These are discussed in detail in my papers in the 9/21 & 10/21 TPT.



L, r

Δt

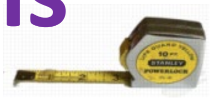
m

Q

T

Not numbers!
Measurements!
Units matter!

Dimensional analysis



Estimation

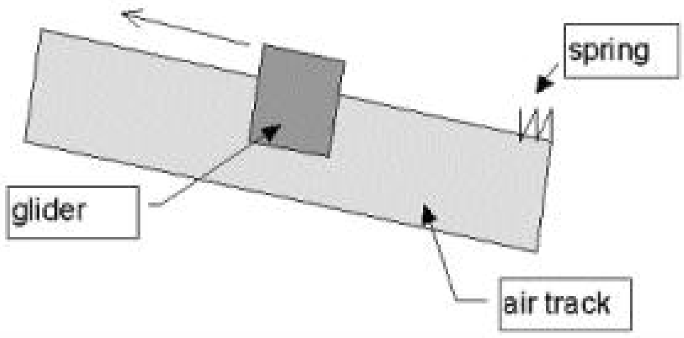
Without the skills of dimensional analysis and estimation, the numbers in our problems have no physical meaning!

Reading the physics in a graph

- One of the best ways to help students build the blend is through interpreting graphs.
- Graphs are a powerful tool for making sense of what’s happening physically, but many students see a graph as an answer — something the teacher assigned them to do.
- Learning how to interpret graphs both physically and mathematically is something they need to learn.

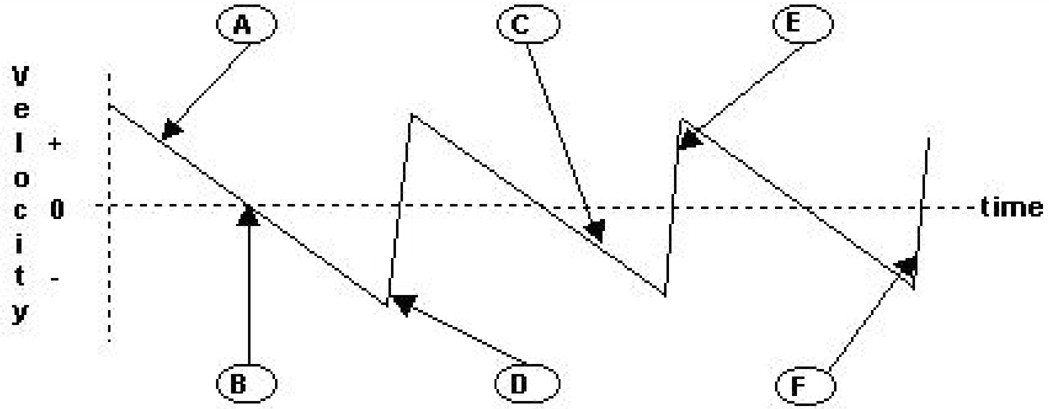
A tilted airtrack has a spring at one end. A cart on it is pressed against the spring and released. The cart bounces up and down.

The graph represents the velocity of the cart as a function of time starting at the moment of release. Positive is to the left of the diagram.

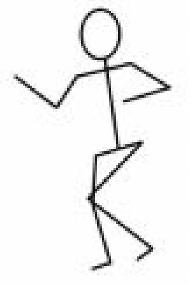


Which of the letters on the graph can identify an instant of time at which the cart is

1. instantaneously not moving
2. in contact with the spring
3. moving down the track
4. at its highest point on the track
5. has an acceleration of zero.

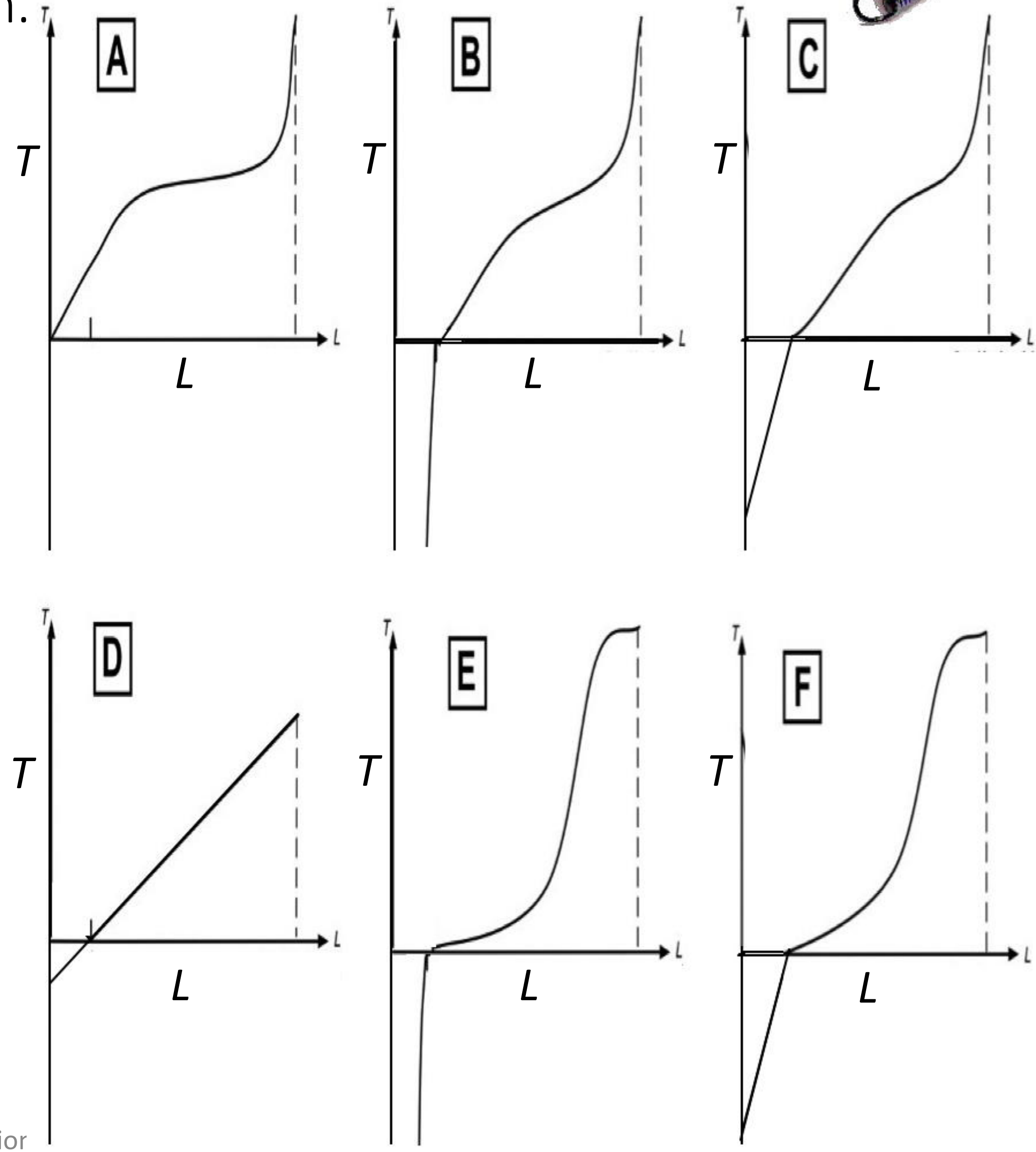


Toy models



- One of the most powerful tools we use in physics in learning to make sense of the world with math are ***toy models***.
- We consider the simplest possible example that shows a phenomenon and beat it into submission — until we understand it completely.
- These are extremely valuable starting points for building more complex models of realistic situations.
- The problem is that biologists (and engineers) tend to want to focus on realistic situations. This leads them to see toy models as bogus and irrelevant.

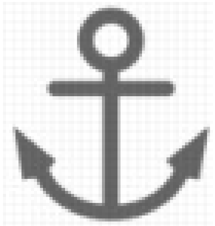
Real springs only follow the Hooke's law model for small displacements around their rest length. Which graph might represent *T* (tension – pos for stretch, neg for squeeze) vs *L* (length) for a real spring?



A real spring behaves as follows:

- stretching from its rest length, it obeys Hooke's law for a small stretch.
- As you stretch it further, it gets stiffer as the coils begin to bend.
- Eventually it straightens out into a long straight wire which is very hard to stretch.
- If you keep pulling harder, the wire suddenly stretches easily and breaks.
- If you try to compress it, the coils get pushed together and you can squeeze very hard without getting much compression.

Anchor equations



- Anchor equations provide stable starting points for thinking about whole blocks of physics content.
- They are the **central principles** that provide a foothold — a starting point for organizing our understanding of an entire topic.
  - Coding for conceptual knowledge
  - A good starting point for unpacking other relevant knowledge
  - A good starting point for solving problems

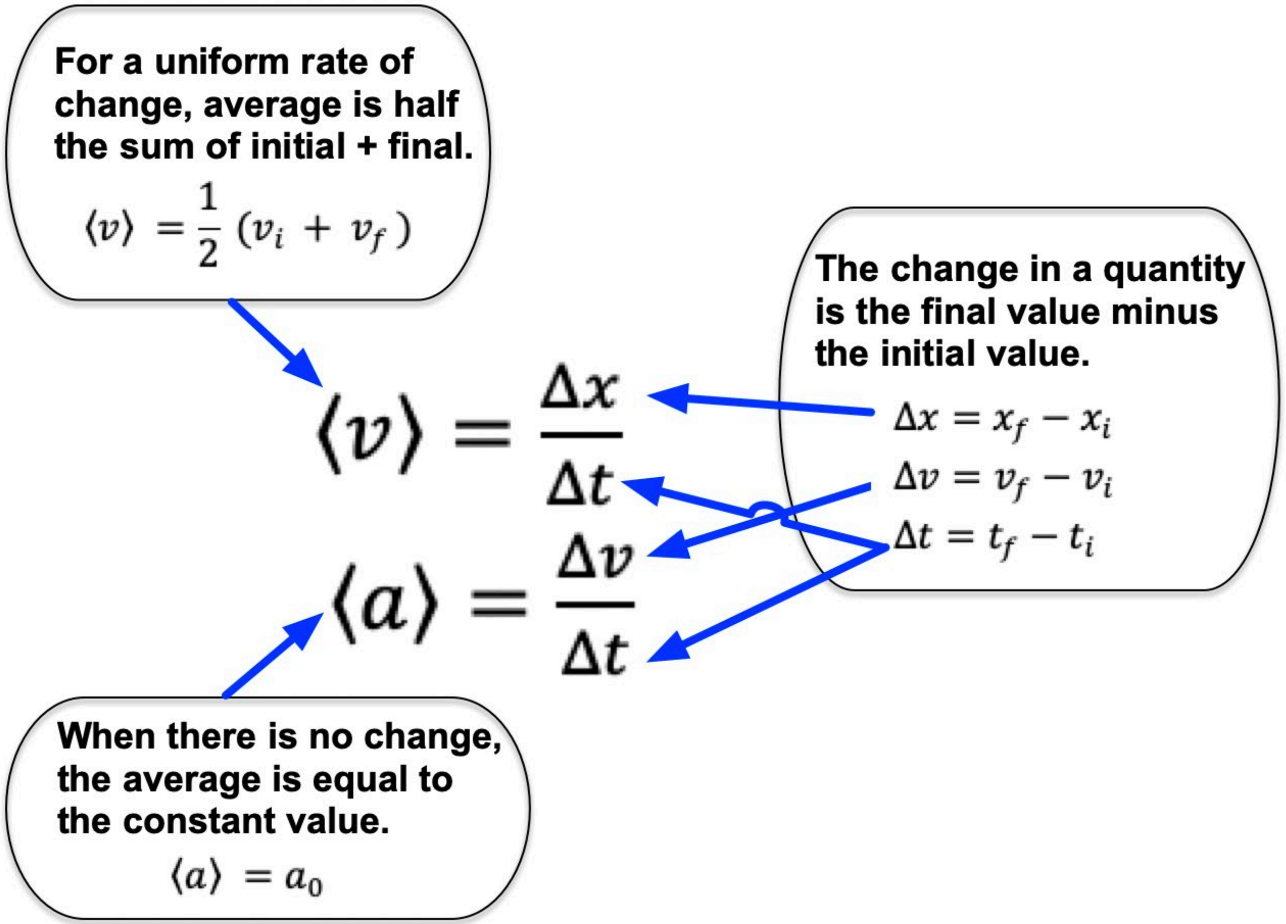
An example: Kinematics concepts (blended)

- The **average velocity** is given by the change in position (How far did you move?) divided by the time interval (How long did it take to do it?).
- The **average acceleration** is given by the change in velocity (How much did it change?) divided by the time interval (How long did it take to do it?).

$$< v > = \frac{\Delta x}{\Delta t} \qquad v = \frac{dx}{dt}$$

$$< a > = \frac{\Delta v}{\Delta t} \qquad a = \frac{dv}{dt}$$

Unpacking the anchor equations



Using these equations, problems are solved by:

1. Identifying relevant quantities in the physical problem (mapping physics to symbol)
2. Calling on relevant math concepts and matching them to the identified physical values
3. Identifying knowns and unknowns and manipulating to get solvable equations
4. Solving the problem

Packing Concepts into Equations: Equations as a conceptual organizer

When we just write "F=ma" our students often miss the rich set of conceptual associations hidden in the equations and mis-use them.

Force is what you have to pay attention to when considering motion

What matters is the sum of the forces

on the object being considered

These stand for 3 equations that are independently true for each direction.

$$\vec{a}_A = \vec{F}^{net}_A / m_A$$

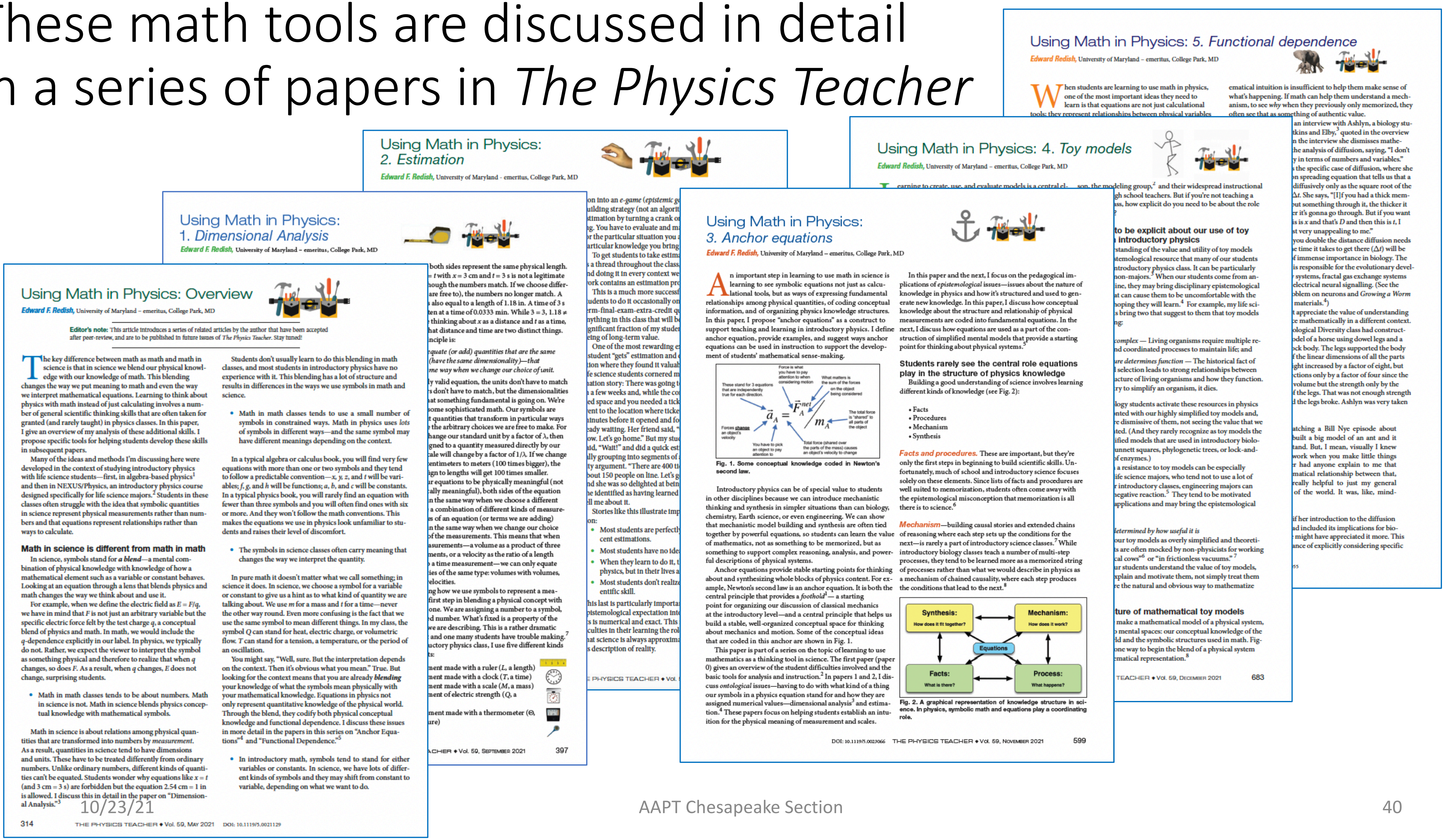
The total force is "shared" to all parts of the object

Forces **change** an object's velocity

You have to pick an object to pay attention to

Total force (shared over the parts of the mass) causes an object's velocity to change

These math tools are discussed in detail in a series of papers in *The Physics Teacher*

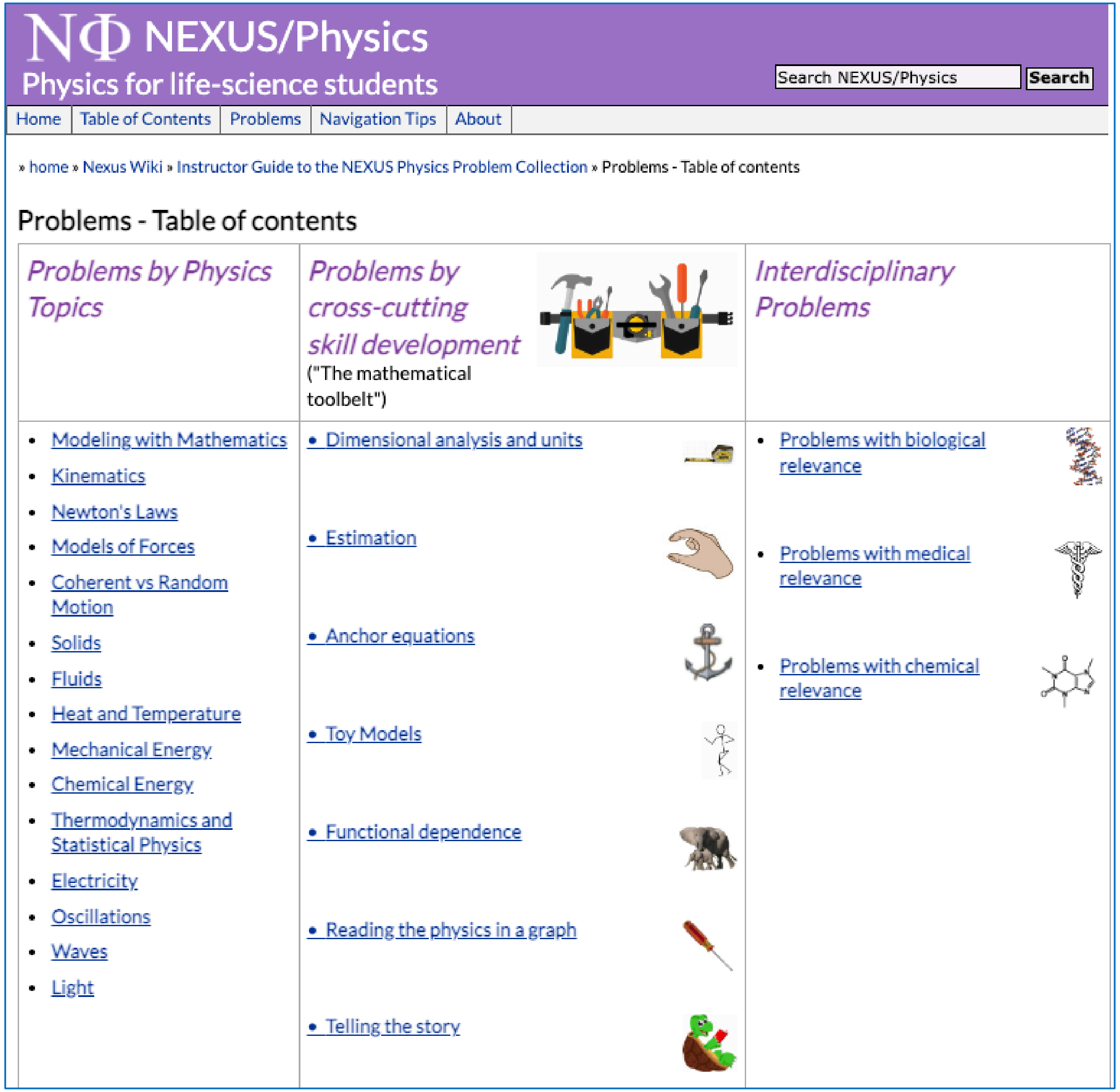


Problem collection

- A collection of problems designed to help students develop math-in-science skills is at the NEXUS/Physics website.

  (Solutions by request or in autograded ExpertTA.)

- http://compadre.org/nexusph/



Question for discussion

- The ideas for this talk were largely developed based on 20 years of research with algebra-based physics and life-science (and pre-med) students.
- Do these ideas ring true for engineers and physics majors?