## Using math in physics: What's the problem?

#### E. F. (Joe) Redish University of Maryland

AAPT Regional Meeting Chesapeake section 10/22/22

AAPT

#### This is an interactive talk!

If you have Chrome, Firefox, or Safari browser on a tablet, or laptop, send it to

#### http://app.tophat.com/e/132795

or scan the QR code at the right.

Then choose to log in as a **guest** (button on the lower right of the screen);







## Who are we?



#### At what level do you currently teach?

- 1. Pre-college
- 2. Community college
- 3. 4-year college
- 4. University



What population of students do you teach? (Select all that apply.)

- 1. High-school students
- 2. Mixed population of college students
- 3. Physics majors at the intro level
- 4. Physics majors at the advanced level
- 5. Engineering students at the intro level
- 6. Life science students at the intro level
- 7. Non-major specialized course

## Do we have a problem?

I often find that my students have difficulty using math in physics.

## A problem my life-science students have trouble with



## A problem my life science/pre meds have trouble with

#### • What is 3½ divided by ¼?

#### A problem they don't have trouble with

 For the faculty-student get together you ordered 4 small pizzas divided into quarters. The faculty ate two slices.
 How many students can have a slice?



## A problem my engineering students have trouble with



### A problem my engineers have trouble with

• The Electric field is defined by

E = F/q

where q is the charge on the test (probe) charge.

• If q is replaced by q/3 what happens to E?

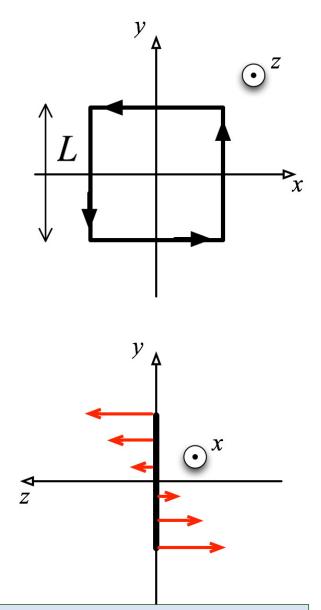
## A problem my physics majors have trouble with

A problem my physics majors have trouble with\*

• A square loop of wire is centered on the origin and oriented as in the figure. There is a spacedependent magnetic field

$$\vec{B} = B_0 y \hat{k}$$

• If the wire carries a current, I, what is the net force on the wire?



#### Two paths to a solution

- Student A
  - Huh! Looks pretty simple – like a physics 1 problem.
  - The sides cancel so I can just do

$$\vec{F} = I\vec{L} \times \vec{B}$$

on the top and bottom where B is constant.

- Gonna get
  - $\vec{F} = IL^2 B_0 \hat{j}$

#### Student B

 I'm pretty sure they want us to do the vector line integral around the loop.

$$\vec{F} = \oint I \, d\vec{L} \times \vec{B}$$

- It's pretty straightforward.
- The sides do cancel, but I get the top and bottom do too, so the answer is zero.

From a video of two physics majors working together to solve a problem in a third-year E&M course.



### What do you think happened next?

- Student A (it's simple) immediately yields to student B (I did the integral).
- 2. Student B (I did the integral) immediately yields to student A (it's simple).
- 3. They said, "Let's figure out why these don't agree," and worked together to resolve the discrepancy.
- 4. They immediately gave up and called a TA over and asked what the answer was.

#### No argument!

- Student A immediately folded his cards in response to student B's more mathematically sophisticated reason and agreed she must be right.
- Both students valued (complex) mathematical reasoning (where they could easily make a mistake) over a simple (and compelling) argument that blended math and physics reasoning.
- The students' expectations that the knowledge in the class was about learning to do complex math was supported by many class activities.

#### What's missing in all three examples

In each case, students fail to "make the blend"

 combining information
 about mathematical symbols
 with knowledge about
 what those symbols represent.

### Do we care?

Mathematical modeling is an essential part of learning to do and understand science.

- For physics majors, it's fairly obvious.
   Every advanced class is about the heavy use of math to describe physical systems.
- For engineers, it's fairly obvious. They use math to describe physical systems.
- For life science students and pre-meds, it's dicey. They rarely use math to describe physical systems in intro classes.



# Physics is the best place to learn to reason about the world mathematically! However...

- Many non-physics STEM majors succeed in their intro level courses by memorizing facts, algorithms, and heuristics.
- Most have never seen an example where using math improves their professional understanding.

It's not just IPLS students! Chemists and engineers too!

• We need to negotiate a change in student expectations

#### 10/23/21

F

The Math:

AAPT Chesapeake Section

#### 21

• My students prefer lots of plug-and-chug "practice problems"

- And they want to drop the units, only putting them in at the end. (Engineers too)
- and hate doing symbolic problems without numbers.
- Even though the math is trivial algebra? What's up with that? • My students are desparate to put numbers in right away
- My students often find that despite A's in calculus, they can't do the math I ask them to do.

rather than a few "thinking problems."

- As a result, they often resist thinking about the math.
- My students often see math as inappropriate in biology.

Some observations from ed research with life science students

Any other instructors out there seeing this as well?

#### So let's add to our learning goals

- Students will learn to use math effectively in science, including being able to:
  - **Calculate** the value of simple expressions and solve simple equations when the equations are given.
  - Work with equations in symbolic form including manipulating them and interpreting them for physical implications.
  - Generate appropriate symbolic equations describing physical situations for which no explicit numbers are given.
  - Reason about physical situations using symbolic mathematics both qualitiatively and quantitatively .

How often do we ask our IPLS students to do more than the first?

## What's going on?

#### Math in science is not the same as math in math

- Math in math classes tends to be about numbers (and the structure of abstract relationships).
   Math in science is not.
- Math in math classes tends to use a small number of symbols in constrained ways. Math in physics uses *lots* of symbols in different ways.
- The symbols in science classes often carry meaning that changes the way we interpret the quantity.
- In introductory math, equations are almost always about solving and calculating. In physics it's often about *explaining and making meaning*.

When we do a derivation of an equation we are **giving an explanation**. Intro physics students rarely understand that this is what we are doing.

#### Meta-misconceptions

- While some students may indeed lack mathematical skills, many (in my experience) can "do the math" when it's expressed as math.\* (At least after a brief review.)
- More serious problems (that are not easy to remedy by a review class or by math "drill-and-kill" activities) are students' inappropriate expectations about
  - the nature of the knowledge they are learning (*epistemological misconceptions*)
  - the nature of what mathematical symbols are being used to represent (*ontological misconceptions*)





- The critical difference in the way we use math in science is that our symbols do not just represent numbers.
- In physics, symbols represent a **mapping** of physical meaning into mathematical symbols that **blends** together
  - our conceptual knowledge of the physical content and
  - our structural knowledge of mathematical relationships and processes.
- Making this blend is a non-trivial mental process that is rarely taught explicitly. How can we teach it?

## What can we do about it?

#### Teaching the blend



- One way to begin to approach teaching the blend is to be more explicit about identifying and teaching the core types of reasoning we use in the blend.
- I have created a **math-in-science toolbelt** with specific analytic and problem solving strategies.
- I teach these tools ("epistemic games") explicitly .
- Every time I use them (in class or in the text) students see an icon that appears to remind them that we are using that tool.

## We can teach the blend through general purpose tools / strategies



Dimensional analysis		Reading the physics in a graph	
Estimation	S	Telling the story	
Functional dependence/ scaling		Diagrams	
Anchor equations	Ĵ	Special and extreme cases	
Toy models	Y K	Building equations from the physics	

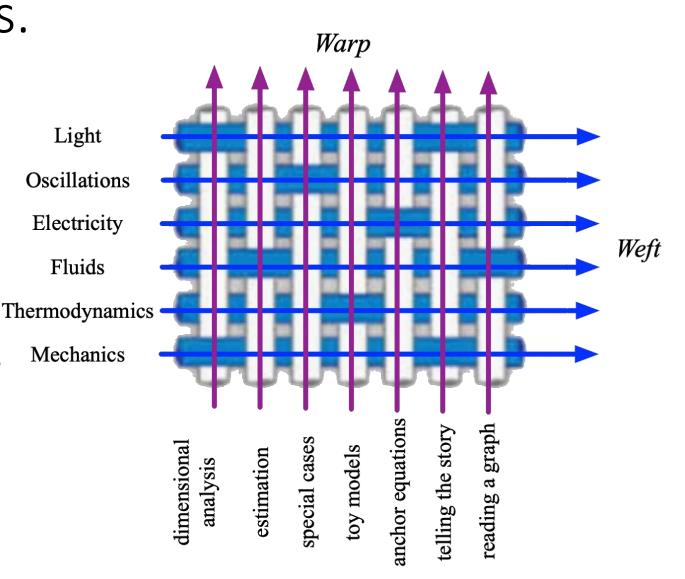
https://www.compadre.org/nexusph/course/Building\_your\_mathematical\_toolbelt

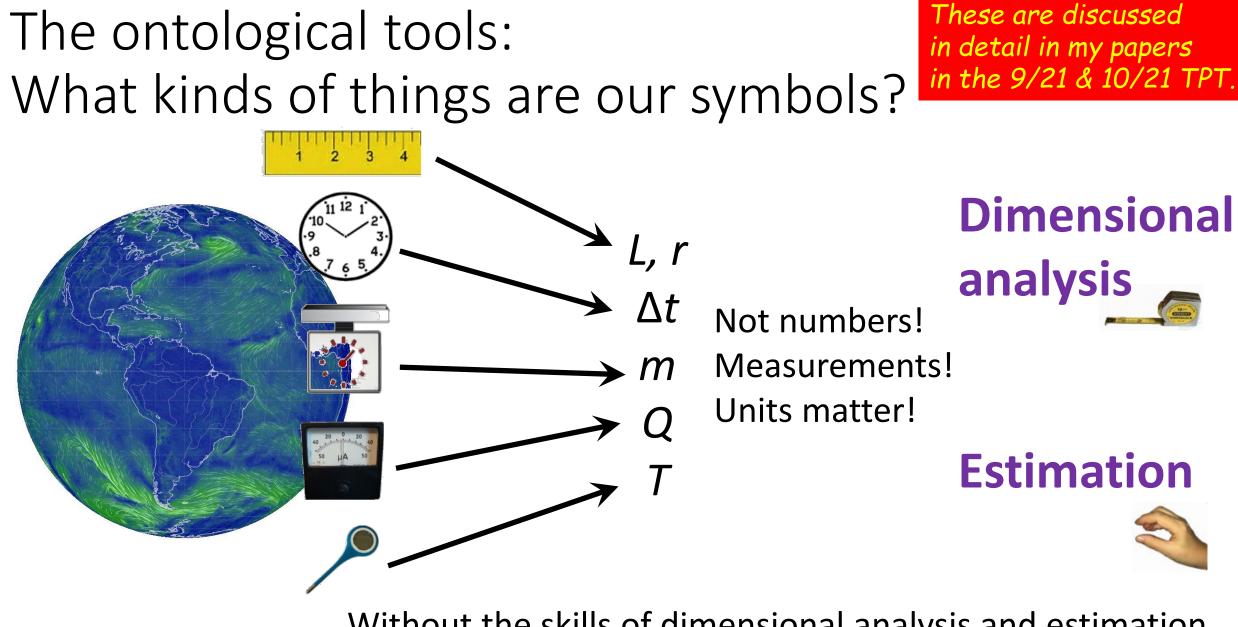
### Skills apply to all topics.

- Adding a focus

   Adding a focus
   On developing math skills
   doesn't add new topics.
- It modifies the way we teach topics we already include.
- It does require us to be "meta"

   to be explicit about tools
   since students don't easily
   generalize.





Without the skills of dimensional analysis and estimation, the numbers in our problems have no physical meaning!

### Reading the physics in a graph



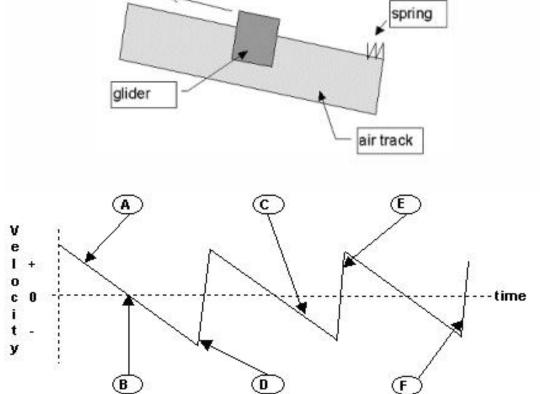
- One of the best ways to help students build the blend is through interpreting graphs.
- Graphs are a powerful tool for making sense of what's happening physically, but many students see a graph as an answer something the teacher assigned them to do.
- Learning how to interpret graphs both physically and mathematically is something they need to learn.

A tilted airtrack has a spring at one end. A cart on it is pressed against the spring and released. The cart bounces up and down.

The graph represents the velocity of the cart as a function of time starting at the moment of release. Positive is to the left of the diagram.

Which of the letters on the graph can identify an instant of time at which the cart is

- 1. instantaneously not moving
- 2. in contact with the spring
- 3. moving down the track
- 4. at its highest point on the track
- 5. has an acceleration of zero.





#### Toy models

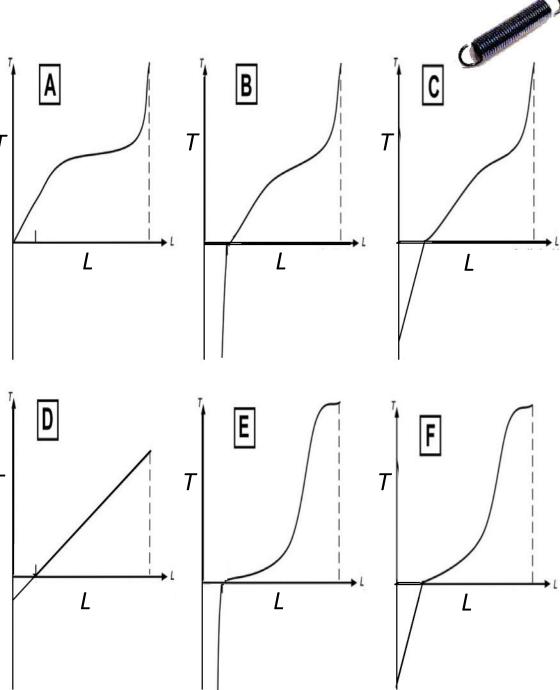
- One of the most powerful tools we use in physics in learning to make sense of the world with math are *toy models*.
- We consider the simplest possible example that shows a phenomenon and beat it into submission — until we understand it completely.
- These are extremely valuable starting points for building more complex models of realistic situations.
- The problem is that biologists (and engineers) tend to want to focus on realistic situations. This leads them to see toy models as bogus and irrelevant.



Real springs only follow the Hooke's law model for small displacements around their rest length. Which graph might represent T (tension – pos for stretch, neg for squeeze) vs L (length) for a real spring?

A real spring behaves as follows:

- stretching from its rest length, it obeys Hooke's law for a small stretch.
- As you stretch it further, it gets stiffer as the coils begin to bend.
- Eventually it straightens out into a long straight wire which is very hard to stretch.
- If you keep pulling harder, the wire suddenly stretches easily and breaks.
- If you try to compress it, the coils get pushed together and you can squeeze very hard without getting much compression. 10/23/21



**AAPT Chesapeake Sectior** 

#### Anchor equations



- Anchor equations provide stable starting points for thinking about whole blocks of physics content.
- They are the central principles that provide a foothold

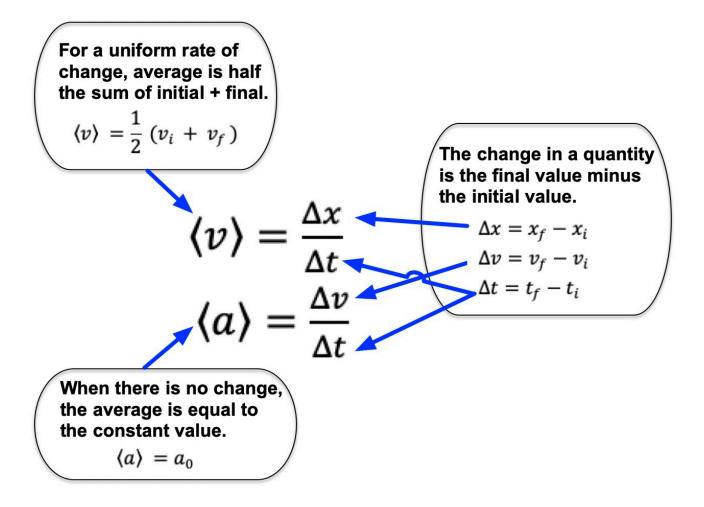
   a starting point for organizing our understanding
   of an entire topic.
  - Coding for conceptual knowledge
  - A good starting point for unpacking other relevant knowledge
  - A good starting point for solving problems

## An example: Kinematics concepts (blended)

- The **average velocity** is given by the change in position (How far did you move?) divided by the time interval (How long did it take to do it?).
- The **average acceleration** is given by the change in velocity (How much did it change?) divided by the time interval (How long did it take to do it?).

$$< v > = \frac{\Delta x}{\Delta t}$$
  $v = \frac{dx}{dt}$   
 $< a > = \frac{\Delta v}{\Delta t}$   $a = \frac{dv}{dt}$ 

## Unpacking the anchor equations



Using these equations, problems are solved by:

- Identifying relevant quantities in the physical problem (mapping physics to symbol)
- Calling on relevant math concepts and matching them to the identified physical values
- Identifying knowns and unknowns and manipulating to get solvable equations
- 4. Solving the problem

Packing Concepts into Equations: Equations as a conceptual organizer

> hidden in the equations Force is what and mis-use them. you have to pay attention to when What matters is considering motion the sum of the forces These stand for 3 equations on the object that are independently being considered true for each direction. The total force is "shared" to all parts of the object Forces change an object's velocity Total force (shared over You have to pick the parts of the mass) causes an object to pay an object's velocity to change attention to

When we just write "F=ma"

our students often miss the rich

set of conceptual associations

#### These math tools are discussed in detail in a series of papers in *The Physics Teacher*

Using Math in Physics:

nciple is:

elocities.

Edward F. Redish, University of Maryland - emeritus, College Park, MD

2. Estimation

#### Edward Redish, University of Maryland - emeritus, College Park, MD Then students are learning to use math in physics,

Using Math in Physics: 4. Toy models

In this paper and the next, I focus on the pedagogical im-

plications of epistemological issues-issues about the nature of

knowledge in physics and how it's structured and used to gen-

erate new knowledge. In this paper, I discuss how conceptual

measurements are coded into fundamental equations. In the

struction of simplified mental models that provide a starting

Students rarely see the central role equations

Building a good understanding of science involves learning

play in the structure of physics knowledge

point for thinking about physical systems.

different kinds of knowledge (see Fig. 2):

knowledge about the structure and relationship of physical

Edward Redish, Untversity of Maryland - emeritus, College Park, MD

one of the most important ideas they need to learn is that equations are not just calculational tools: they represent relationships between physical variables

ematical intuition is insufficient to help them make sense of what's happening. If math can help them understand a mechanism, to see why when they previously only memorized, they often see that as something of authentic value.

an interview with Ashlyn, a biology stutkins and Elby,<sup>3</sup> quoted in the overview

he analysis of diffusion, saying, "I don't

the specific case of diffusion, where she

n spreading equation that tells us that a

diffusively only as the square root of the

∆t. She says, "[I]f you had a thick mem-

out something through it, the thicker it

er it's gonna go through. But if you want

responsible for the evolutionary devel-

ological Diversity class had construct-

odel of a horse using dowel legs and a

ock body. The legs supported the body

f the linear dimensions of all the parts

abt increased by a factor of eight but

ctions only by a factor of four since the

f the legs. That was not enough strength

d the legs broke. Ashlyn was very taken

atching a Bill Nye episode about

built a big model of an ant and it

tand. But, I mean, visually I knew

work when you make little things

r had anyone explain to me that

matical relationship between that.

really helpful to just my general

of the world. It was, like, mind-

if her introduction to the diffusion

d included its implications for bio-

might have appreciated it more. This

ance of explicitly considering specific

olume but the strength only by the

is is x and that's D and then this is t. I

st very unappealing to me."

n the interview she dismisses mathe-

in terms of numbers and variables"



earning to create, use, and evaluate models is a central el- son, the modeling group,<sup>2</sup> and their widespread instructional gh school teachers. But if you're not teaching a ss, how explicit do you need to be about the role

Using Math in Physics: 5. Functional dependence

#### to be explicit about our use of toy introductory physics

you double the distance diffusion needs e time it takes to get there  $(\Delta t)$  will be immense importance in biology. The systems, fractal gas exchange systems electrical neural signalling. (See the oblem on neurons and Growing a Worm materials.4) appreciate the value of understanding mathematically in a different context.

complex — Living organisms require multiple rend coordinated processes to maintain life; and

logy students activate these resources in physics nted with our highly simplified toy models and, e dismissive of them, not seeing the value that we ted. (And they rarely recognize as toy models the ified models that are used in introductory bioloinnett squares, phylogenetic trees, or lock-andenzymes.)

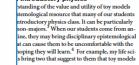
introductory classes, engineering majors can regative reaction.5 They tend to be motivated applications and may bring the epistemological

our toy models as overly simplified and theoretis are often mocked by non-physicists for working cal cows"6 or "in frictionless vacuums." r students understand the value of toy models, xplain and motivate them, not simply treat them e the natural and obvious way to mathematize

#### ture of mathematical tov models

make a mathematical model of a physical system, mental spaces: our conceptual knowledge of the ld and the symbolic structures used in math. Figone way to begin the blend of a physical system matical representation.8

EACHER + Vol. 59, DECEMBER 2021 683



re determines function - The historical fact of selection leads to strong relationships between ture of living organisms and how they function. ry to simplify an organism, it dies.

a resistance to toy models can be especially

ife science majors, who tend not to use a lot of

#### rmined by how useful it is

Using Math in Physics: Overview

Using Math in Physics:

Edward F. Redish, University of Maryland - emeritus, College Park, MD

1. Dimensional Analysis

Edward F. Redish, University of Maryland - emeritus, College Park, MD

Editor's note: This article introduces a series of related articles by the author that have been accepte after peer-review, and are to be published in future issues of The Physics Teacher. Stay tuned!

he key difference between math as math and math in science is that in science we blend our physical knowledge with our knowledge of math. This blending changes the way we put meaning to math and even the way we interpret mathematical equations. Learning to think about physics with math instead of just calculating involves a number of general scientific thinking skills that are often taken for granted (and rarely taught) in physics classes. In this paper, I give an overview of my analysis of these additional skills. I propose specific tools for helping students develop these skills in subsequent papers.

Many of the ideas and methods I'm discussing here were developed in the context of studying introductory physics with life science students-first, in algebra-based physics1 and then in NEXUS/Physics, an introductory physics course designed specifically for life science majors,<sup>2</sup> Students in these classes often struggle with the idea that symbolic quantities in science represent physical measurements rather than numbers and that equations represent relationships rather than ways to calculate

#### Math in science is different from math in math

In science, symbols stand for a hlend-a mental combination of physical knowledge with knowledge of how a mathematical element such as a variable or constant behaves Looking at an equation through a lens that blends physics and math changes the way we think about and use it.

For example, when we define the electric field as E = F/q, we have in mind that F is not just an arbitrary variable but the specific electric force felt by the test charge q, a conceptual blend of physics and math. In math, we would include the a-dependence explicitly in our label. In physics, we typically do not. Rather, we expect the viewer to interpret the symbol as something physical and therefore to realize that when q changes, so does F. As a result, when q changes, E does not change, surprising students.

· Math in math classes tends to be about numbers. Math in science is not. Math in science blends physics conceptual knowledge with mathematical symbols.

Math in science is about relations among physical quantities that are transformed into numbers by measurement As a result, quantities in science tend to have dimensions and units. These have to be treated differently from ordinary numbers. Unlike ordinary numbers, different kinds of quantities can't be equated. Students wonder why equations like x = t(and 3 cm = 3 s) are forbidden but the equation 2.54 cm = 1 in is allowed. I discuss this in detail in the paper on "Dimensional Analysis. 10/23/21

314

Students don't usually learn to do this blending in math classes, and most students in introductory physics have no experience with it. This blending has a lot of structure and results in differences in the ways we use symbols in math and science.

 Math in math classes tends to use a small number of symbols in constrained ways. Math in physics uses lots of symbols in different ways-and the same symbol may have different meanings depending on the context.

In a typical algebra or calculus book, you will find very few equations with more than one or two symbols and they tend to follow a predictable convention—x, w, z, and t will be variables; f, g, and h will be functions; a, b, and c will be constants. In a typical physics book, you will rarely find an equation with fewer than three symbols and you will often find ones with six or more. And they won't follow the math conventions. This makes the equations we use in physics look unfamiliar to students and raises their level of discomfort.

> The symbols in science classes often carry meaning that changes the way we interpret the quantity.

In pure math it doesn't matter what we call something; in science it does. In science, we choose a symbol for a variable or constant to give us a hint as to what kind of quantity we are talking about. We use m for a mass and t for a time-never the other way round. Even more confusing is the fact that we use the same symbol to mean different things. In my class, the symbol Q can stand for heat, electric charge, or volumetric flow. T can stand for a tension, a temperature, or the period of an oscillation

You might say, "Well, sure. But the interpretation depends on the context. Then it's obvious what you mean." True. But looking for the context means that you are already blending your knowledge of what the symbols mean physically with your mathematical knowledge. Equations in physics not only represent quantitative knowledge of the physical world. Through the blend, they codify both physical conceptual knowledge and functional dependence. I discuss these issues in more detail in the papers in this series on "Anchor Equations"<sup>4</sup> and "Functional Dependence."

· In introductory math, symbols tend to stand for either variables or constants. In science, we have lots of different kinds of symbols and they may shift from constant to variable, depending on what we want to do.



(have the same dimensionality) that

n the same way when we change our choice

rements-a volume as a product of three

of the measurements. This means that when

ments, or a velocity as the ratio of a length

a time measurement-we can only equate

ies of the same type: volumes with volumes,

ng how we use symbols to represent a mea-

first step in blending a physical concept with

nent of electric strength (Q, a

To get students to take estim both sides represent the same physical length. a thread throughout the class doing it in every context we twith x = 3 cm and t = 3 s is not a legitimate hough the numbers match. If we choose differk contains an estimation pr This is a much more successf are free to), the numbers no longer match. A dents to do it occasionally o also equal to a length of 1.18 in. A time of 3 s ten at a time of 0.0333 min. While 3 = 3, 1.18 ≠ rm-final-exam-extra-credit o thing in this class that will be thinking about x as a distance and t as a time, nificant fraction of my stude hat distance and time are two distinct things. ing of long-term value.

on into an e-game (epistemic g

ullding strategy (not an algori

timation by turning a crank o

g. You have to evaluate and m

r the particular situation you

rticular knowledge you bring

One of the most rewarding e quate (or add) quantities that are the same udent "gets" estimation and tion where they found it valuat me way when we change our choice of unit. science students cornered n valid equation, the units don't have to match ation story: There was going t don't have to match, but the dimensionalities a few weeks and while the co at something fundamental is going on. We're d space and you needed a tick ome sophisticated math. Our symbols are nt to the location where ticke quantities that transform in particular ways nutes before it opened and fo the arbitrary choices we are free to make. For ady waiting. Her friend said, ange our standard unit by a factor of  $\lambda$ , then ow. Let's go home," But my stu gned to a quantity measured directly by our I. "Watt!" and did a quick est ale will change by a factor of  $1/\lambda$ . If we change illy grouping into segments of timeters to meters (100 times bigger), the argument. "There are 400 th ign to lengths will get 100 times smaller. out 150 people on line. Let's p r equations to be physically meaningful (not nd she was so delighted at bei ally meaningful), both sides of the equation e identified as having learned n the same way when we choose a different me about it. a combination of different kinds of measure-Stories like this illustrate imp s of an equation (or terms we are adding)

> Most students are perfectly cent estimations. Most students have no the When they learn to do it, t

physics, but in their lives Most students don't realiz entific skill.

is last is particularly importa one. We are assigning a number to a symbol, temological expectation int number. What's fixed is a property of the s numerical and exact. This ve are describing. This is a rather dramatic ilties in their learning the ro and one many students have trouble making. at science is always approxim uctory physics class, I use five different kinds lescription of reality.





CHER + Vol. 59, SEPTEMBER 2021 397

AAPT Chesapeake Section

THE PHYSICS TEACHER . Vol. 59, May 2021 DOI: 10.1119/5.0021129



#### Synthesis Fig. 1. Some conceptual

Introductory physics can be of special value to students in other disciplines because we can introduce mechanistic thinking and synthesis in simpler situations than can biology, chemistry, Earth science, or even engineering. We can show that mechanistic model building and synthesis are often tied together by powerful equations, so students can learn the value of reasoning where each step sets up the conditions for the of mathematics, not as something to be memorized, but as something to support complex reasoning, analysis, and power-introductory biology classes teach a number of multi-step

Using Math in Physics:

Edward F. Redish, University of Maryland - emeritus, College Park, MD

n important step in learning to use math in science is

learning to see symbolic equations not just as calcu-

Lational tools, but as ways of expressing fundamental

relationships among physical quantities, of coding conceptual

information, and of organizing physics knowledge structures.

anchor equation, provide examples, and suggest ways anchor

equations can be used in instruction to support the develop-

ment of students' mathematical sense-making.

In this paper, I propose "anchor equations" as a construct to

3. Anchor equations

ful descriptions of physical systems. Anchor equations provide stable starting points for thinking of processes rather than what we would describe in physics as about and synthesizing whole blocks of physics content. For ex- a mechanism of chained causality, where each step produces ample, Newton's second law is an anchor equation. It is both the the conditions that lead to the next. central principle that provides a foothold<sup>1</sup> - a starting

at the introductory level-and a central principle that helps us build a stable, well-organized conceptual space for thinking about mechanics and motion. Some of the conceptual ideas that are coded in this anchor are shown in Fig. 1.

mathematics as a thinking tool in science. The first paper (paper (1) gives an overview of the student difficulties involved and the basic tools for analysis and instruction.<sup>2</sup> In papers 1 and 2, I discuss ontological issues-having to do with what kind of a thing our symbols in a physics equation stand for and how they are assigned numerical values-dimensional analysis<sup>3</sup> and estima-

Synthesis How does it fit togethe

Facts and procedures. These are important, but they're only the first steps in beginning to build scientific skills. Unfortunately, much of school and introductory science focuses solely on these elements. Since lists of facts and procedures are well suited to memorization, students often come away with the epistemological misconception that memorization is all there is to science 6 Mechanism-building causal stories and extended chains

· Facts

• Procedures

Mechanism

support teaching and learning in introductory physics. I define next, I discuss how equations are used as a part of the con-

next-is rarely a part of introductory science classes.<sup>7</sup> While

processes, they tend to be learned more as a memorized string

point for organizing our discussion of classical mechanics

This paper is part of a series on the topic of learning to use tion.4 These papers focus on helping students establish an intu-

DOI: 10.1119/5.0023056 THE PHYSICS TEACHER • Vol. 59. NOVEMBER 2021

Facts

What is there? What happens? Fig. 2. A graphical representation of knowledge structure in sci-In physics, symbolic math and equations play a role. ition for the physical meaning of measurement and scales

599

Mechanism

How does it work?

Process:

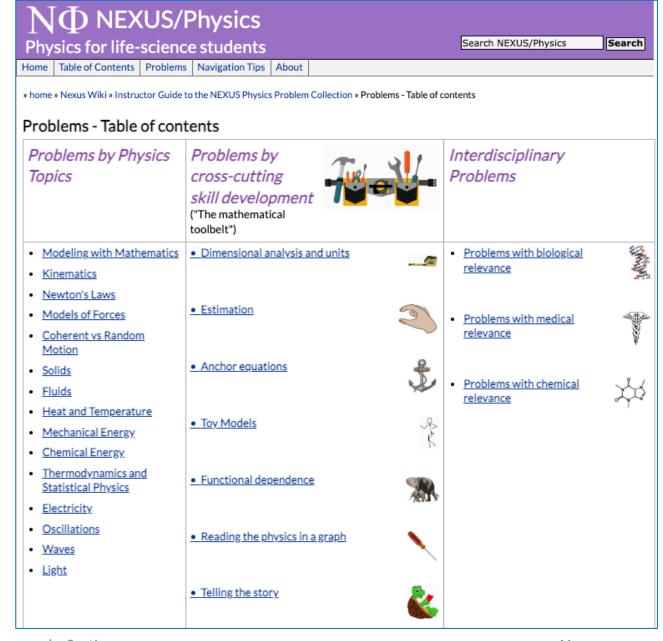
### Problem collection

 A collection of problems designed to help students develop math-in-science skills is at the NEXUS/Physics website.

(Solutions by request or in autograded ExpertTA.)

• <u>http://compadre.org/nexusph/</u>





#### Question for discussion

- The ideas for this talk were largely developed based on 20 years of research with algebra-based physics and life-science (and pre-med) students.
- Do these ideas ring true for engineers and physics majors?