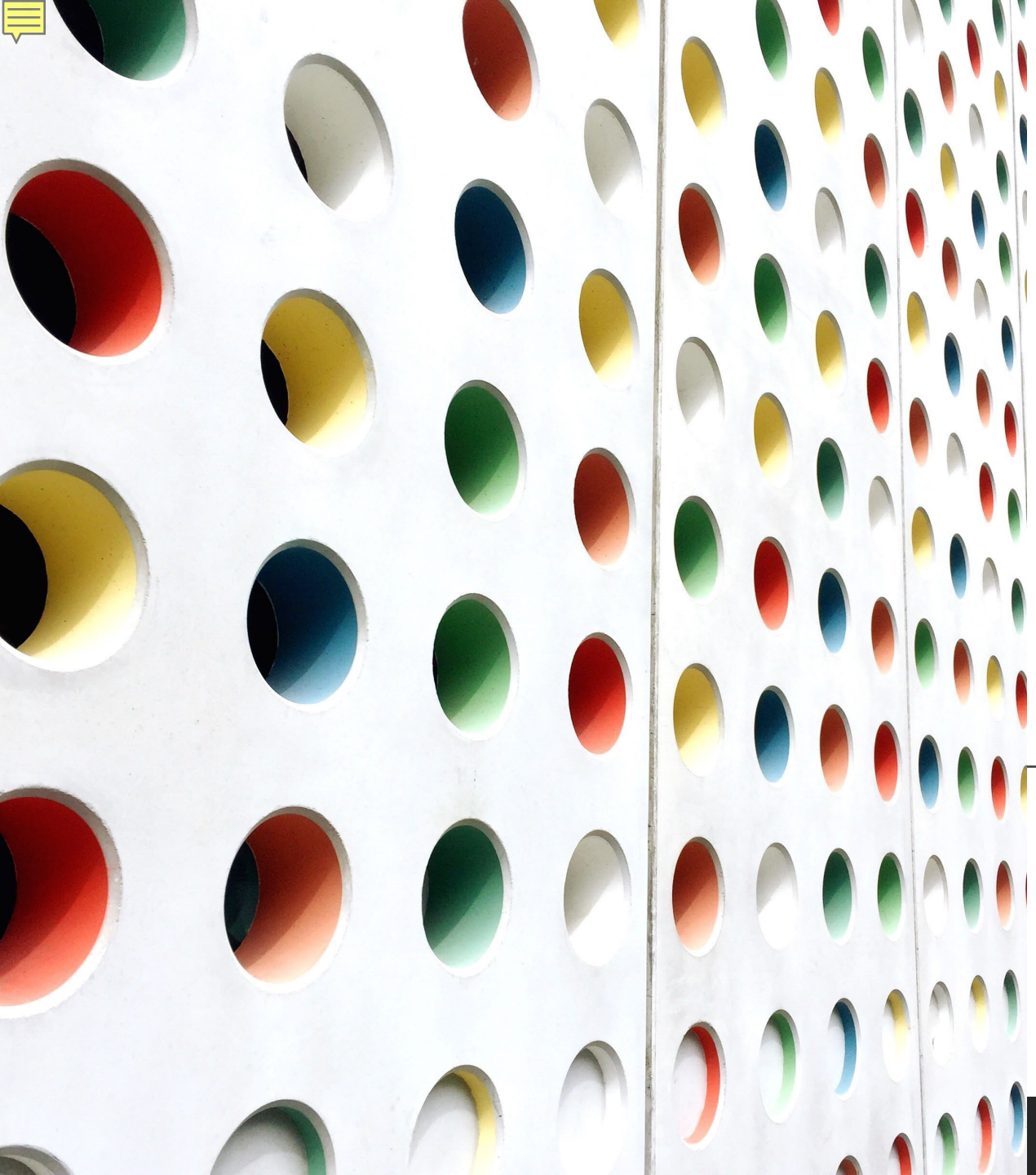


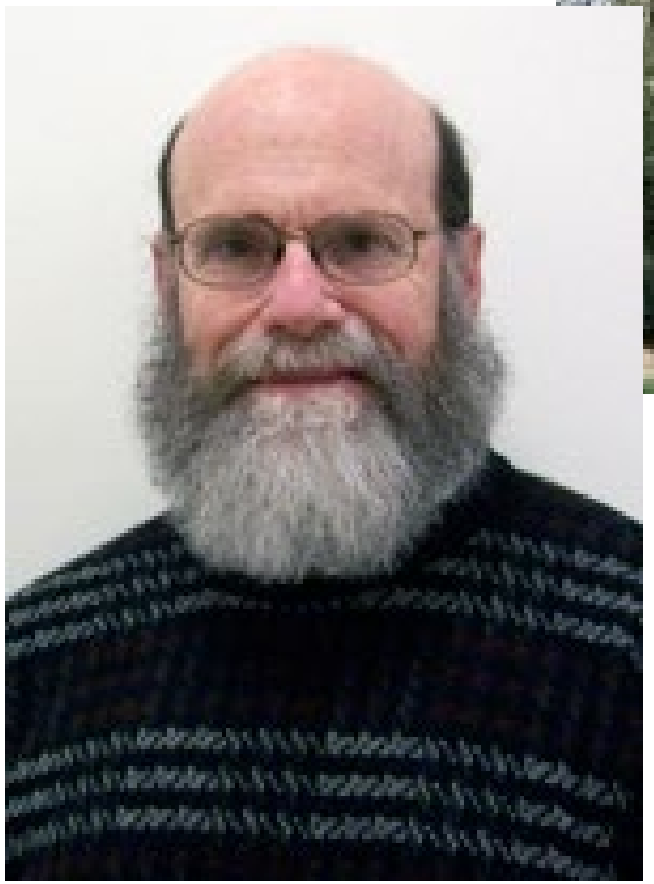
To Think Deeply of Simple Things| Reflections & Possible Results from an Ancient REU

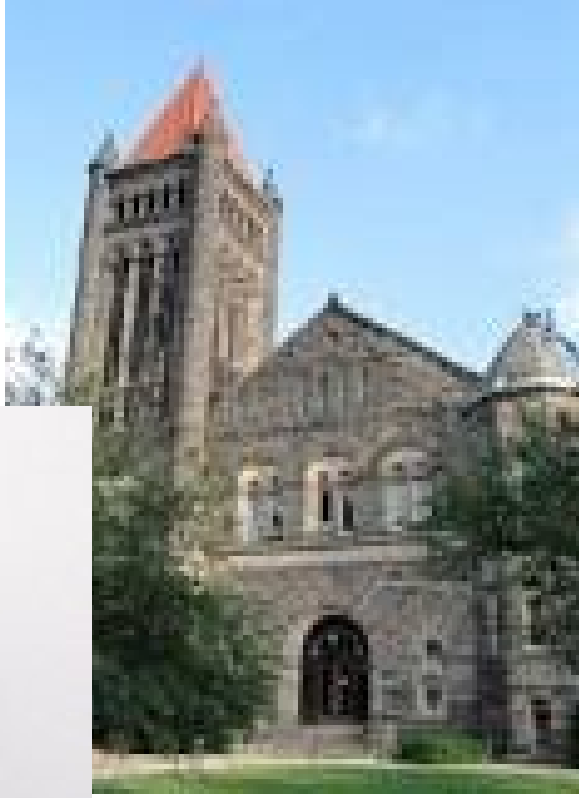
REBECCA ROSENBLATT

OCT 22, 2022

PLEASE NOTE THAT THE OPINIONS, FINDINGS, CONCLUSIONS AND RECOMMENDATIONS EXPRESSED HERE ARE MINE AND DO NOT NECESSARILY REFLECT THE VIEWS OF THE NATIONAL SCIENCE FOUNDATION (NSF)

Summer 2005 REU at UIUC



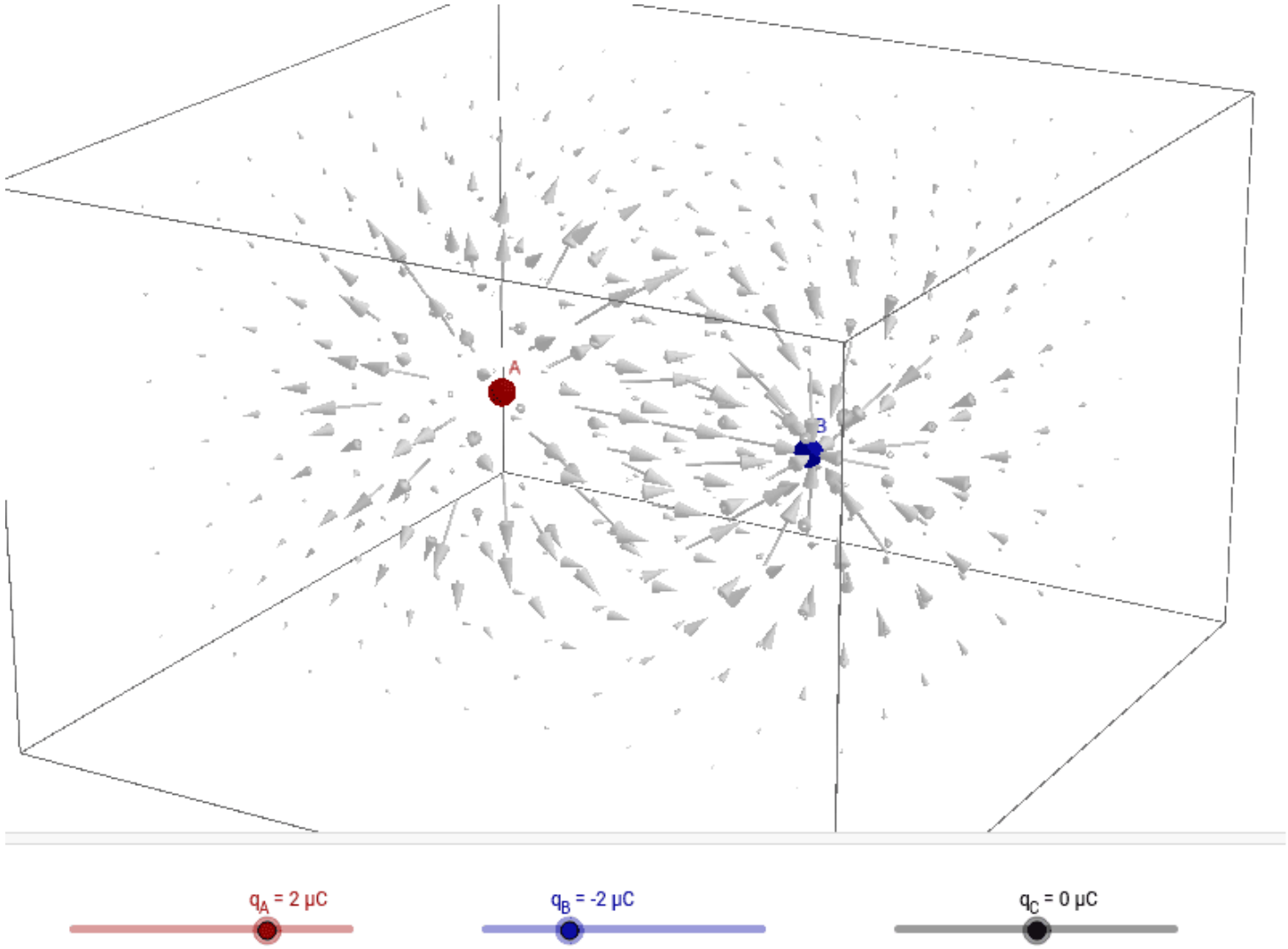


An Inverse Problem

IS THE PROCESS OF CALCULATING FROM A SET OF OBSERVATIONS THE CAUSAL FACTORS THAT PRODUCED THEM.

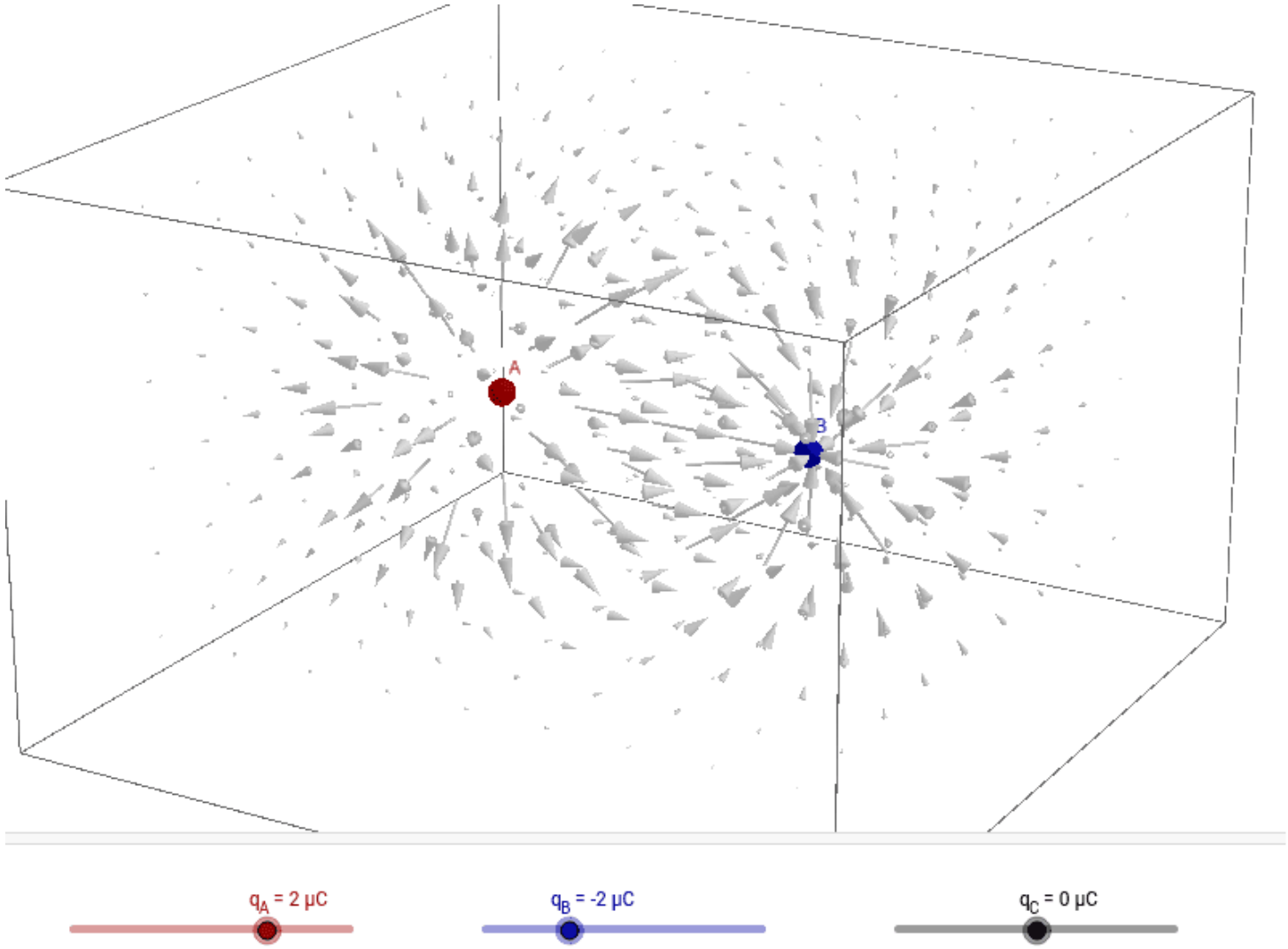
Electric field

Where and how many measurements do I need to uniquely determine the placement of the point charges that created it?

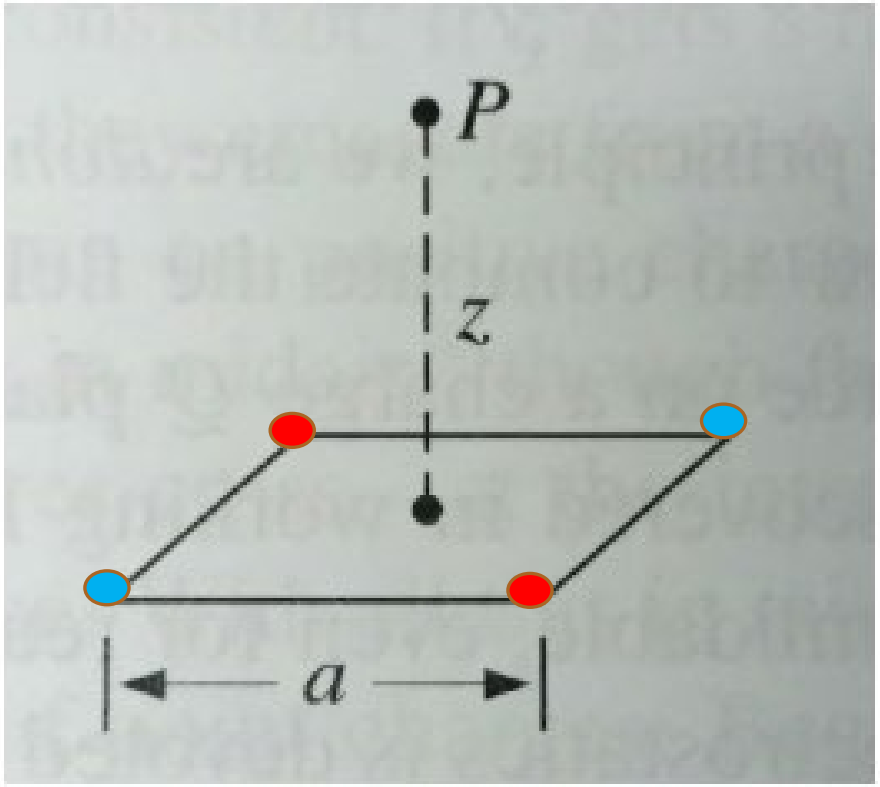


Electric field

What is the zero set of an electric field from a finite set of point charges?



I can know the field at each point P along a line in z



Question: Taking an unknown finite electrical field, what method of taking a limited set of measurements will guarantee that if the measurements are all zero, then the field itself is zero everywhere?

Yanushauskas used the fact that the field in this case is the gradient of a harmonic function to show that the zero set consists of a **locally finite set of points and analytic curves**.

- A. I. Yanushauskas, On the zeros of the gradient of a harmonic function, Dokl. Akad. Nauk SSSR 158 (1964) 547-549.

Result in 1D: Measure field in line of points

If points are along a line, the field along that line can only have finite number of zeros.

$$X(x,y) = X(x,0) = \sum_{j=1}^{M} \frac{a_j(x-x_j)}{((x-x_j)^2)^{3/2}} = \sum_{j=1}^{M} \frac{a_j(x-x_j)}{|x-x_j|} \frac{1}{(x-x_j)^2}\,.$$

Let $I_0 := (-\infty, x_1)$, $I_M := (x_M, \infty)$, and $I_j := (x_j, x_{j+1})$, $j = 1, 2, \ldots, M-1$. The function $h(x) := \dfrac{x-x_j}{|x-x_j|}$ is identically -1 or identically 1 on any of these intervals I_j for $j = 0, 1, \ldots, M$. Hence, $X(x,y) = X(x,0)$ is a rational function on any of these I_j and can only have a finite number of zeros on I_j. This means that $F(x,0)$ can be zero only finitely many times too. □

Result in 2D: Charges on Line & Field in Plane

Analytic Attack..... Dead End

Y component is not zero unless net charge is zero

X component is not zero unless there is Q*x symmetry

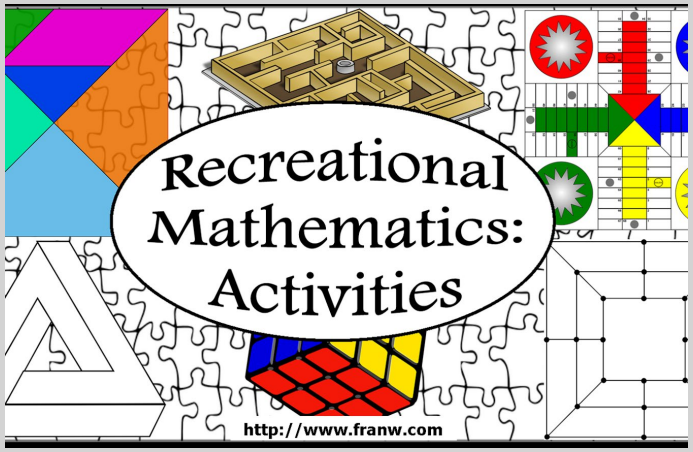
But ultimately this approach was not working

Result in 2D: Charges on Line & Field in Plane

Proposition 3.5. In the Special Case the zero set of F = (X, Y) contains at most $9M^2 4^M$ points. (Where M is the number of charges).

Proposition 3.6. (B´ezout's Theorem) Given any two coprime polynomials f, g ∈ R[x, y] of degrees d1 and d2, respectively, Z(f) ∩ Z(g) contains at most d1d2 points.

https://arxiv.org/abs/2106.04706 https://arxiv.org/abs/2208.12857



Reflections

What is research?

What makes a good research problem?

What is the value of NSF funding?

Thank you! rrosenbl@nsf.gov

rosenblatt.rebecca@gmail.com