Teaching Quantum Computing: An approach accessible to high school and beginning college students (and their educators)

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Big Ideas

- Address a diverse audience with little physics or advanced math background
- All you need is algebra and some trigonometry
- Use multiple representations: Dirac vectors, matrices, state space
- Importance of superposition and choice of basis states
- Start with polarized light as a prototypical quantum system
- Entanglement - another critical feature for QIS algorithms - ignored in a typical quantum mechanics course - approached via concrete examples
- Include information on quantum careers

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No need to shy away from the (appropriate) math

With simple matrices, algebra, and trig, students can see amazing quantum results:

- No-cloning theorem
- Bell States
- Bell's Theorem
- Deutsch Algorithm
- Simon Search Algorithm
- Shor Algorithm
- Quantum Cryptography
- Entanglement
- Entanglement Swapping
- Quantum State Teleportation

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What math is needed to introduce QIS and QC?

- Complex numbers not needed for many examples - introduce much later (Chapter 15 of 16)
- Need mostly only 2x2 matrices to represent gates (operations on states)
- Example: X-gate = NOT= $X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
- Matrix multiplication tools available online

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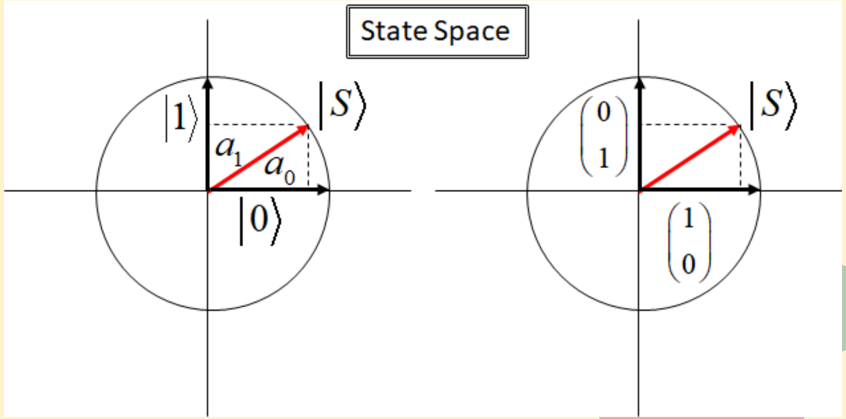
Dirac State Vectors, Matrices, State Space

Students need to learn the language of quantum: multiple representations of quantum states and operations on those states.

$$\mathrm{NOT}|1\rangle = |0\rangle$$
$$\mathrm{NOT}|0\rangle = |1\rangle$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$
$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$



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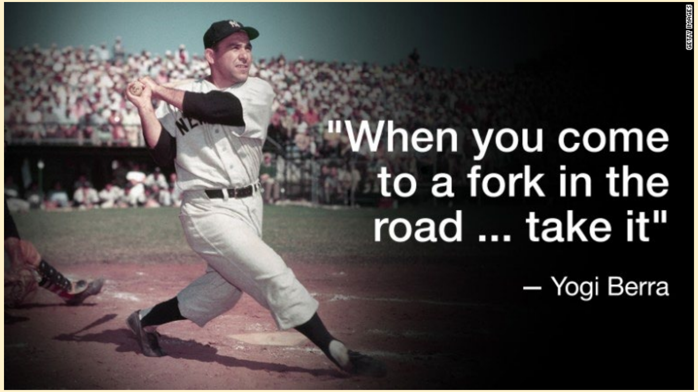
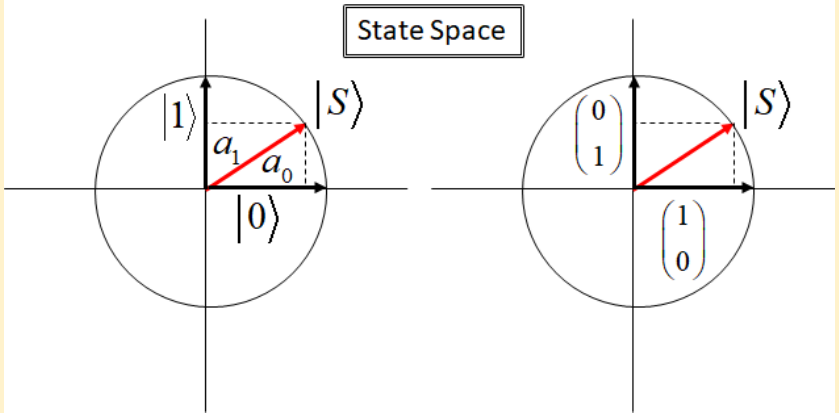


State Superposition: a critical quantum feature. Important to introduce early and to emphasize over and over.



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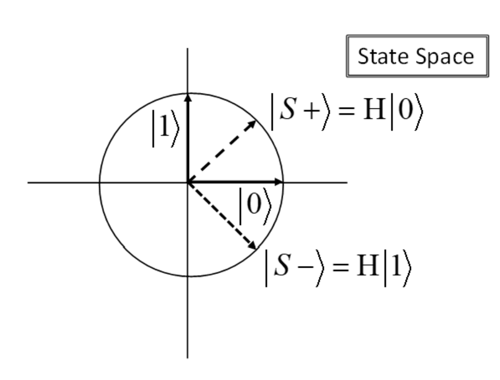
Hadamard Gate produces superposition

$$H \Rightarrow \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

Matrix representation

$$H|0\rangle = \frac{1}{\sqrt{2}}\{|0\rangle + |1\rangle\}$$

Dirac vector representation



Hadamard Gate collapses superposition

$$H[H|0\rangle] \Rightarrow \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\left[\frac{1}{\sqrt{2}}\left\{\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix}\right\}\right]$$

$$= \frac{1}{2}\left\{\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right\}$$

$$= \frac{1}{2}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \Rightarrow |0\rangle$$

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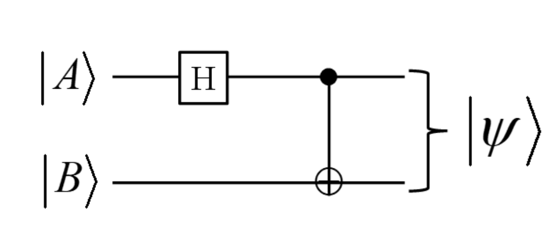
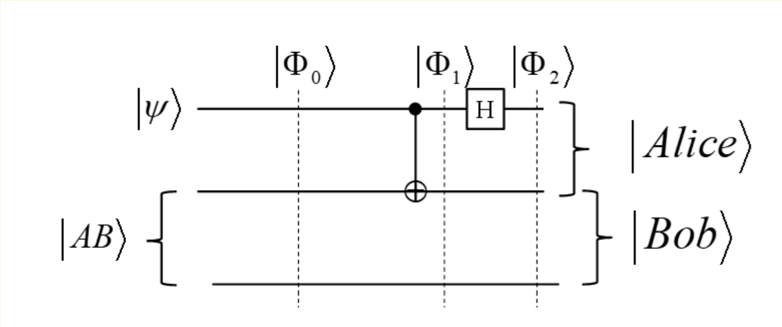
Importance of superposition

Find the Hadamard Gate!

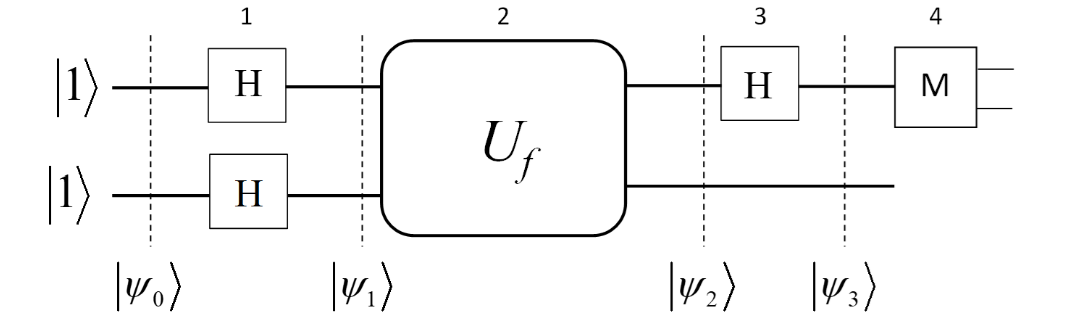


Bell Circuit



Quantum State Teleportation

Deutsch Algorithm



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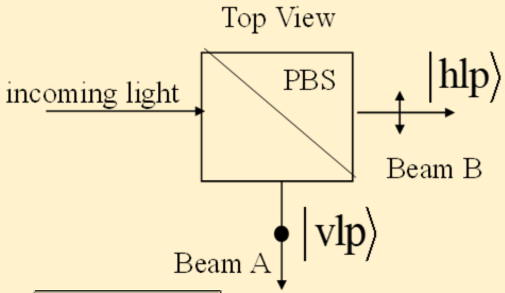
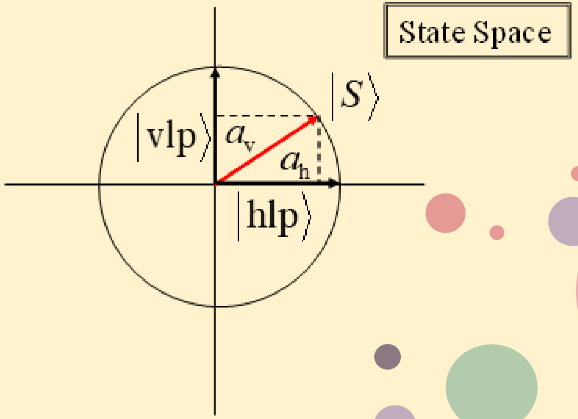
Use polarized light as a quantum paradigm

Polarized light can be described as a state of superposition between 2 orthogonal basis states



Again: multiple representations

$$|S\rangle = a_h |\text{hlp}\rangle + a_v |\text{vlp}\rangle$$



$$|S\rangle = a_h \begin{pmatrix} 1 \\ 0 \end{pmatrix} + a_v \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

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Change of basis states

- Measurements with different basis states give more information about a quantum state.
- Therefore, we need to know how to describe a change of basis states

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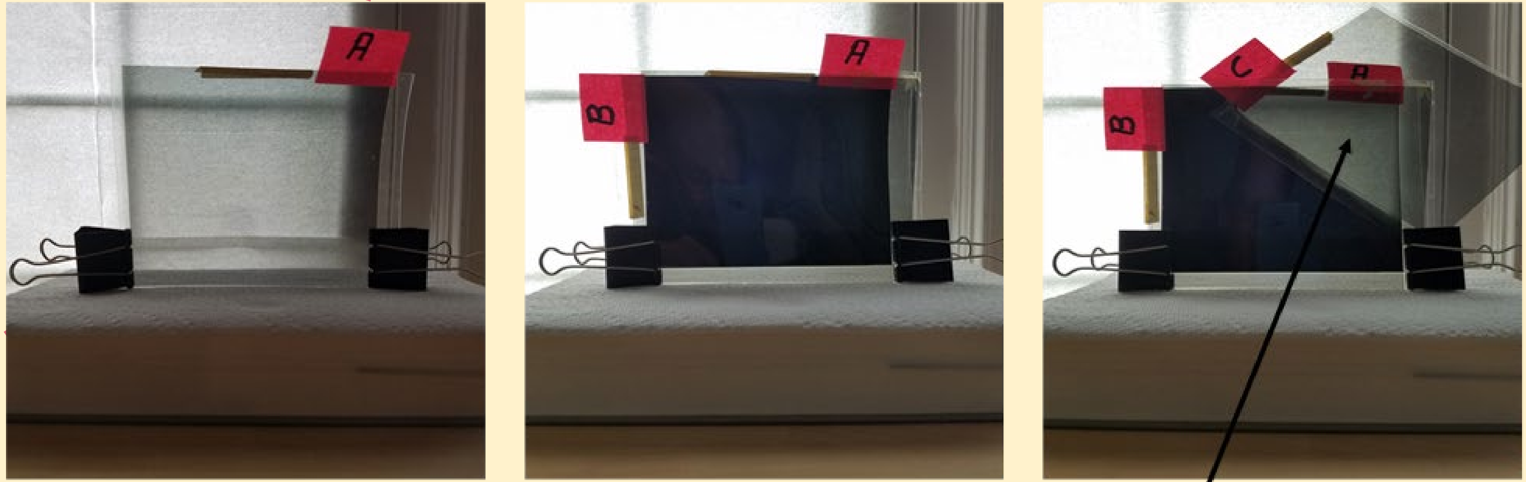
Three Polarizer Problem



Triangular region in which
C overlaps with both A and B

After A, light is in superposition of C basis states and some will be transmitted thru C

$$|\mathrm{hlp}\rangle_A = \frac{1}{\sqrt{2}}\left(|\mathrm{hlp}\rangle_C + |\mathrm{vlp}\rangle_C\right)$$

After C, light is in superposition of B basis states and some will be transmitted thru B.

$$|\mathrm{hlp}\rangle_C = \frac{1}{\sqrt{2}}\left[|\mathrm{hlp}\rangle_B + |\mathrm{vlp}\rangle_B\right]$$

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Entanglement: Another critical quantum state feature.

Shows up in many quantum cryptography and quantum computing algorithms.

Need two (or more) qubits, say, A and B (spin-½ systems)

$$|S\rangle = |AB\rangle = \frac{1}{\sqrt{2}}|\uparrow_A\rangle|\uparrow_B\rangle + \frac{1}{\sqrt{2}}|\downarrow_A\rangle|\downarrow_B\rangle \neq |A\rangle|B\rangle$$

- Entanglement shows up as **correlations among the expectation values** for the measurement results on the two qubits individually.
- Introduce with **specific examples** first.

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Why we MUST teach quantum

- Make contemporary physics accessible to a wider range of people
- Workforce development:
  - Quantum Information Sciences (cryptography, etc.)
  - Quantum Computing
  - Quantum Sensing
- Recruit a larger number of and more diverse students to physics
- Emphasize the diversity of careers that contribute to QIS and QC

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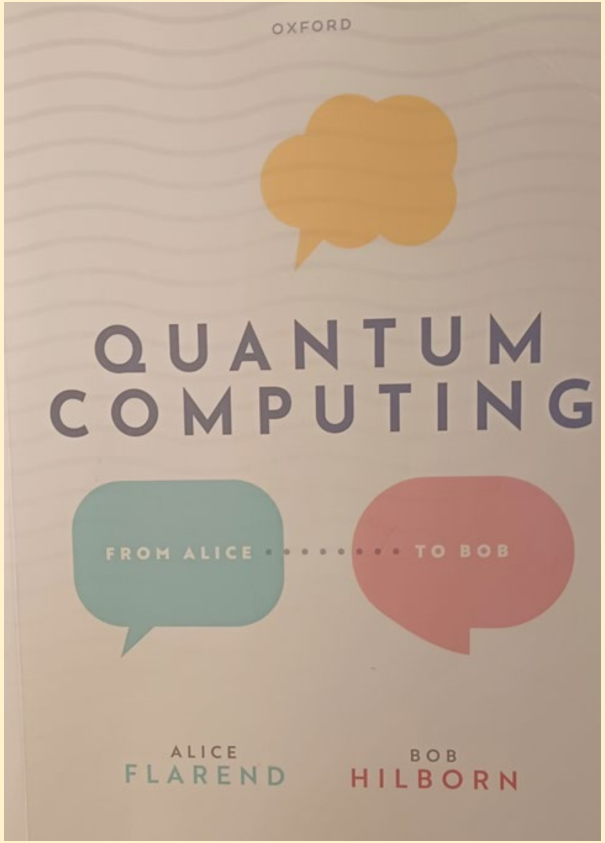

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