Teaching Quantum Computing: An approach accessible to high school and beginning college students (and their educators)

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Big Ideas

- Address a **diverse audience** with little physics or advanced math background
- All you need is **algebra and some trigonometry**
- Use **multiple representations**: Dirac vectors, matrices, state space
- Importance of **superposition** and choice of basis states
- Start with **polarized light** as a prototypical quantum system
- **Entanglement** - another critical feature for QIS algorithms - ignored in a typical quantum mechanics course - approached via **concrete examples**
- Include information on **quantum careers**
No need to shy away from the (appropriate) math

With simple matrices, algebra, and trig, students can see amazing quantum results:

- No-cloning theorem
- Bell States
- Bell’s Theorem
- Deutsch Algorithm
- Simon Search Algorithm
- Shor Algorithm
- Quantum Cryptography
- Entanglement
- Entanglement Swapping
- Quantum State Teleportation
What math is needed to introduce QIS and QC?

- Complex numbers not needed for many examples - introduce much later (Chapter 15 of 16)
- Need mostly only 2x2 matrices to represent gates (operations on states)
- Example: X-gate = NOT = \[
\begin{pmatrix}
0 & 1 \\
1 & 0
\end{pmatrix}
\]
- Matrix multiplication tools available online
Dirac State Vectors, Matrices, State Space

Students need to learn the language of quantum: multiple representations of quantum states and operations on those states.

\[
\begin{pmatrix}
0 & 1 \\
1 & 0 \\
\end{pmatrix}
\begin{pmatrix}
1 \\
0 \\
\end{pmatrix} =
\begin{pmatrix}
0 \\
1 \\
\end{pmatrix}
\]

\[
\begin{pmatrix}
0 & 1 \\
1 & 0 \\
\end{pmatrix}
\begin{pmatrix}
0 \\
1 \\
\end{pmatrix} =
\begin{pmatrix}
1 \\
0 \\
\end{pmatrix}
\]
**State Superposition**: a critical quantum feature. Important to introduce early and to emphasize over and over.
Hadamard Gate produces superposition

Matrix representation

\[
H \mapsto \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}
\]

\[
H|0\rangle = \frac{1}{\sqrt{2}} \left(|0\rangle + |1\rangle\right)
\]

Dirac vector representation

Hadamard Gate collapses superposition

\[
H[H|0\rangle] = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \frac{1}{\sqrt{2}} \left(|0\rangle + |1\rangle\right)
\]

\[
= \frac{1}{2} \left[\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} |0\rangle + \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} |1\rangle\right]
\]

\[
= \frac{1}{2} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \Rightarrow |0\rangle
\]
Importance of superposition

Find the Hadamard Gate!

\[ |A\rangle \rightarrow \text{H} \rightarrow |\psi\rangle \]

\[ |B\rangle \rightarrow \text{H} \rightarrow |\psi\rangle \]

Bell Circuit

Quantum State Teleportation

Deutsch Algorithm

\[ |\psi\rangle \rightarrow |\phi_0\rangle \rightarrow |\phi_1\rangle \rightarrow |\phi_2\rangle \]

|\psi\rangle \rightarrow |\phi_0\rangle \rightarrow |\phi_1\rangle \rightarrow |\phi_2\rangle \]

Quantum State Teleportation

Deutsch Algorithm
Use polarized light as a quantum paradigm

Polarized light can be described as a state of superposition between 2 orthogonal basis states

Again: multiple representations

\[ |S\rangle = a_h |hlp\rangle + a_v |vlp\rangle \]

\[ |S\rangle = a_h \begin{pmatrix} 1 \\ 0 \end{pmatrix} + a_v \begin{pmatrix} 0 \\ 1 \end{pmatrix} \]
Measurements with different basis states give more information about a quantum state. Therefore, we need to know how to describe a change of basis states.
Three Polarizer Problem

After A, light is in **superposition** of C basis states and some will be transmitted thru C.

After C, light is in **superposition** of B basis states and some will be transmitted thru B.

\[
|\text{hlp}\rangle_A = \frac{1}{\sqrt{2}} \left( |\text{hlp}\rangle_C + |\text{vlp}\rangle_C \right)
\]

\[
|\text{hlp}\rangle_C = \frac{1}{\sqrt{2}} \left( |\text{hlp}\rangle_B + |\text{vlp}\rangle_B \right)
\]
Entanglement: Another critical quantum state feature.

Shows up in many quantum cryptography and quantum computing algorithms.

Need two (or more) qubits, say, A and B (spin-$\frac{1}{2}$ systems)

$$|S\rangle = |AB\rangle = \frac{1}{\sqrt{2}} |\uparrow_A\rangle |\uparrow_B\rangle + \frac{1}{\sqrt{2}} |\downarrow_A\rangle |\downarrow_B\rangle \neq |A\rangle |B\rangle$$

- Entanglement shows up as **correlations among the expectation values** for the measurement results on the two qubits individually.
- Introduce with **specific examples** first.
Why we MUST teach quantum

- Make **contemporary physics** accessible to a wider range of people
- **Workforce development:**
  - Quantum Information Sciences (cryptography, etc.)
  - Quantum Computing
  - Quantum Sensing
- Recruit a **larger number of and more diverse** students to physics
- Emphasize the **diversity of careers** that contribute to QIS and QC
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