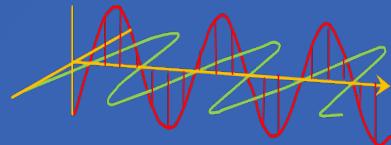
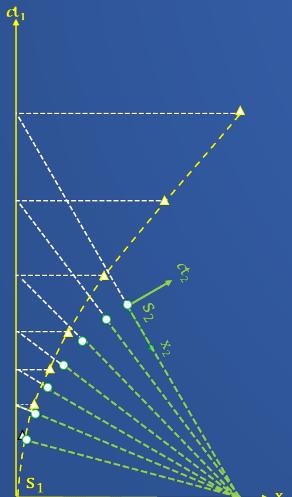


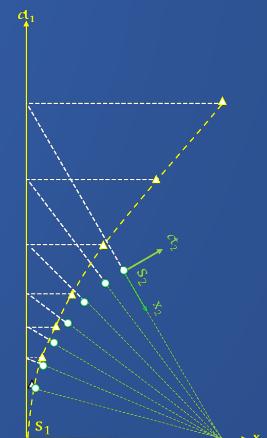
# Riding on a Light Beam

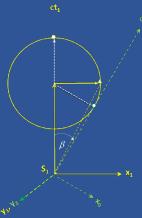


## Observing the Velocity Triangle

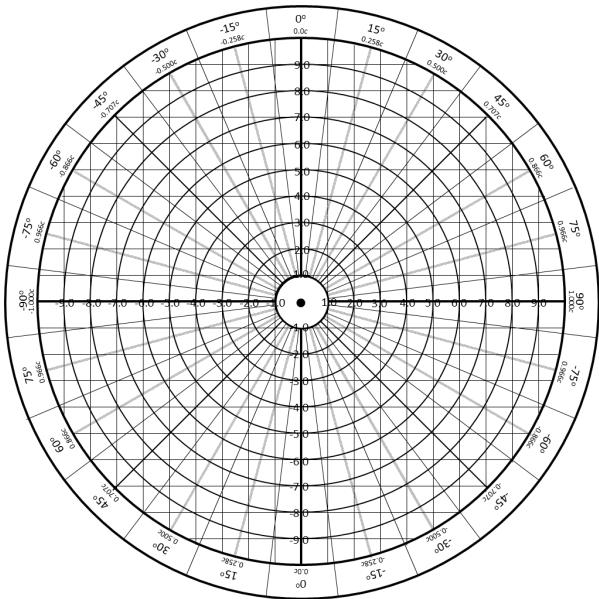
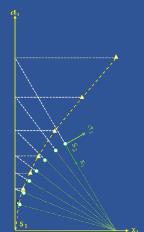


Lewis F. McIntyre  
CS AAPT Fall Session  
October 21, 2023  
[mcintyrel@verizon.net](mailto:mcintyrel@verizon.net)

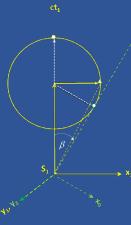




# INTRODUCTION



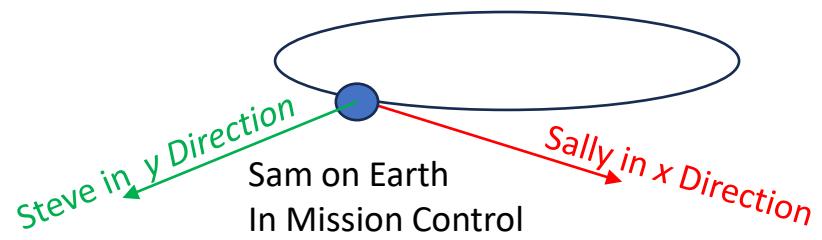
- VELOCITY TRIANGLE
  - Powerful Tool For Analysis
  - Does It Have Any Basis in Reality?



# OBSERVING THE VELOCITY TRIANGLE



- SALLY LEAVES EARTH AT NOON
  - Tangent to Earth's Orbit
  - Defined as  $x$  Direction
- STEVE LEAVES EARTH AT SAME TIME
  - Normal to Earth's Orbit
  - Defined as  $y$  Direction
- SAM REMAINS ON EARTH IN MISSION CONTROL
- WHERE WILL STEVE LOCATE SALLY AND SAM AT 1PM?



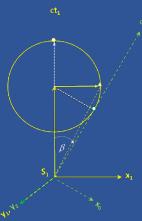


# THE VELOCITY TRIANGLE

Reference Frames  $S_1$ ,  $S_2$ , and  $S_3$



- THE VELOCITY TRIANGLE FOR  $S_1$  AND  $S_2$
- ORTHOGONAL OBSERVER  $S_3$
- SPHERICAL TRIGONOMETRIC SOLUTIONS FOR ORTHOGONAL VELOCITY ANGLES
  - Low Speed Approximations,  $v_{3I} \ll c$
  - High Speed Approximations,  $v_{3I} \rightarrow c$
- $S_3$ 's MEASUREMENTS ON  $x$ - $y$  PLANE
  - Vector Sum,  $v_{3I} \ll c$
  - Velocity Triangle as  $v_{3I} \rightarrow c$



# THE VELOCITY TRIANGLE

Sam and Sally



- VELOCITY TRIANGLE OAB:
  - $S_1$  in blue
  - $S_2$  in red
  - $\beta_{12} = \arcsin(v/c)$
- TRIGONOMETRIC LORENTZ TRANSFORM

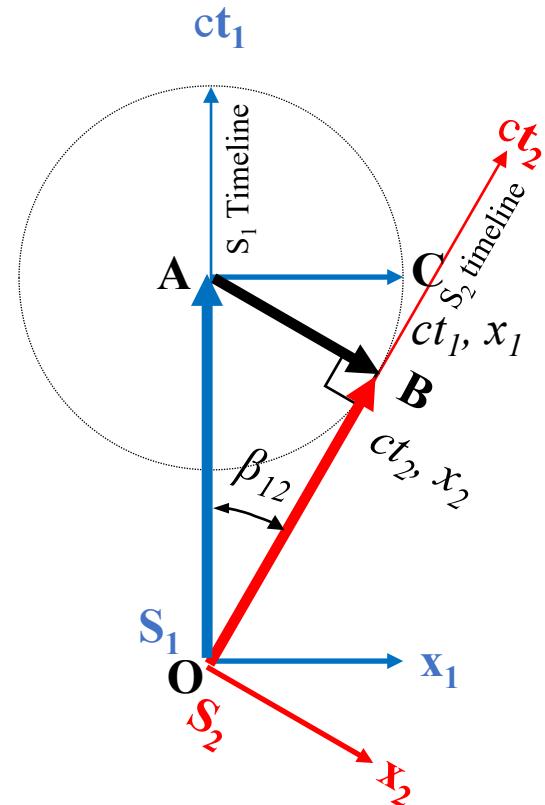
$$x_1 = \frac{x_2 + ct_2 \cdot \sin(\beta_{12})}{\cos(\beta_{12})}$$

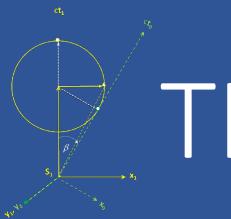
$$ct_1 = \frac{ct_2 + x_2 \cdot \sin(\beta_{12})}{\cos(\beta_{12})}$$

C IS  $S_1$ 's MEASUREMENT OF EVENT C ( $x_2=0$ )

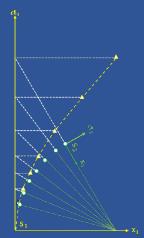
- C is Lorentz Transform of B
- $OA = ct_1 = ct_2 / \cos(\beta_{12})$
- $AC = x_1 = ct_2 \cdot \sin(\beta_{12}) / \cos(\beta_{12})$
- $v_{12}/c = x_1/ct_1 = \sin(\beta_{12})$

**IS IT REAL?  
OR JUST ANOTHER MATHEMATICAL CONSTRUCT?**

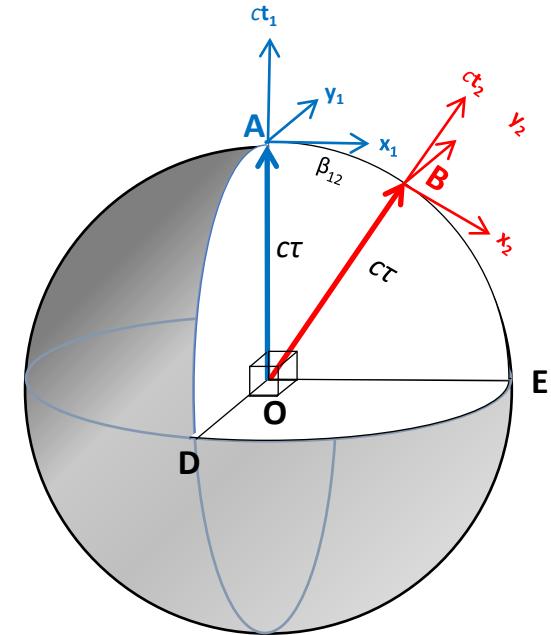




# THE ORTHOGONAL OBSERVER



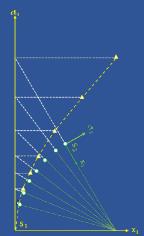
- ROTATE A BIT TO EXPOSE  $y$  AXIS
- SPHERE IS LOCUS OF IDENTICAL PROPER TIME  $c\tau$  FROM ORIGIN O





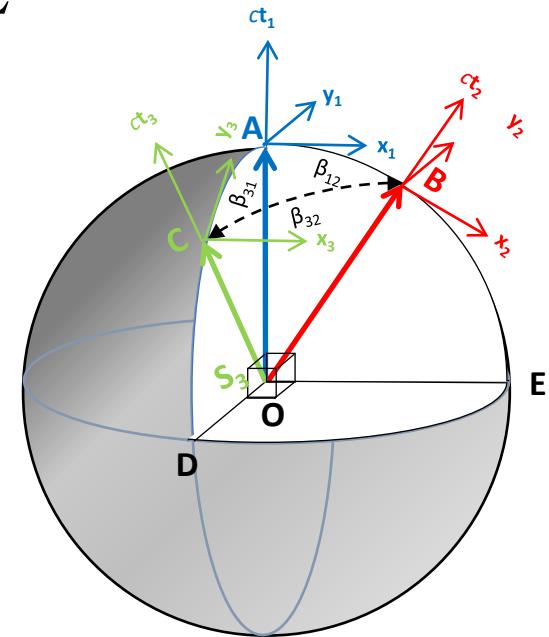
# THE ORTHOGONAL OBSERVER

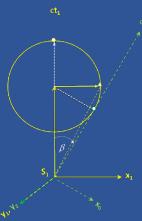
Velocity Angles



- ADD THIRD REFERENCE FRAME  $S_3$  IN GREEN

- Velocity Angle  $\beta_{31}$  with respect to  $S_1$  (Independent Variable)
- Velocity Angle  $\beta_{32}$  with respect to  $S_2$  (function of  $\beta_{31}$ )





# THE ORTHOGONAL OBSERVER

Velocity Angles



- $\beta_{12}$ ,  $\beta_{31}$  AND  $\beta_{32}$  FORM SPHERICAL TRIANGLE ABC ON SURFACE OF SPHERE

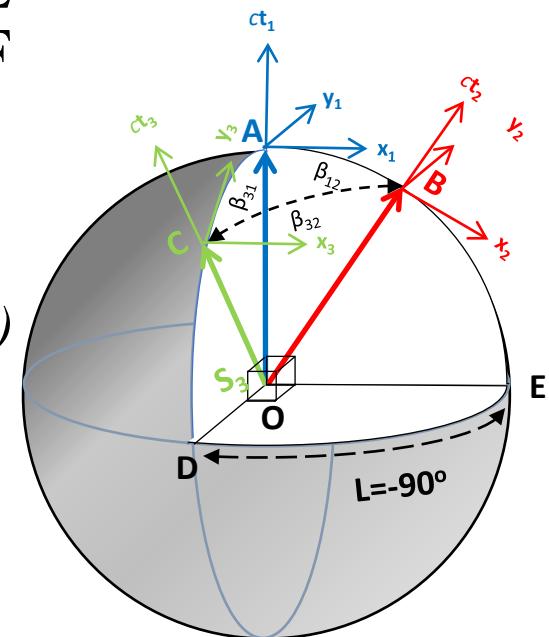
- Great Circle Ranges:

$$\cos(\beta_{32}) = \cos(\beta_{12}) \cdot \cos(\beta_{31}) - \cos(L) \cdot \sin(\beta_{12}) \cdot \cos(\beta_{31})$$

Which Reduces to

$$\sin^2(\beta_{32}) = \sin^2(\beta_{12}) + \sin^2(\beta_{31}) - \sin^2(\beta_{12}) \cdot \sin^2(\beta_{31})$$

using  $\cos^2(\beta) = 1 - \sin^2(\beta)$





# THE ORTHOGONAL OBSERVER

Mapping  $S_1$  and  $S_2$  onto  $x_3$ - $y_3$  Plane



- $S_3$  OBSERVES

- $S_1$  Receding Only in  $y_3$  Direction:

$$s_{31} = y_{31} = ct_3 \cdot \sin(\beta_{31})$$

- $S_2$  Receding in  $x_3$  and  $y_3$  Directions

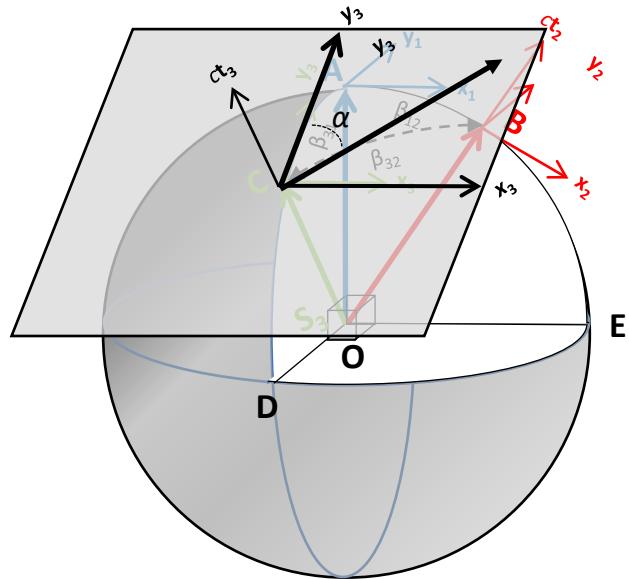
$$s_{32} = ct_3 \cdot \sin(\beta_{32})$$

$$x_{32} = s_{32} \cdot \sin(\alpha)$$

$$y_{32} = s_{32} \cdot \cos(\alpha)$$

Where  $\alpha$  is the Great Circle Bearing  
from  $S_3$  to  $S_2$

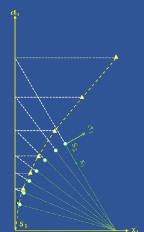
$$\cos(\alpha) = [\cos(\beta_{12}) - \cos(\beta_{31}) \cdot \cos(\beta_{32})] / [\sin(\beta_{31}) \cdot \sin(\beta_{32})]$$





# THE ORTHOGONAL OBSERVER

$$V_{31} \ll C$$



VELOCITY ANGLE  $\beta_{32}$ :

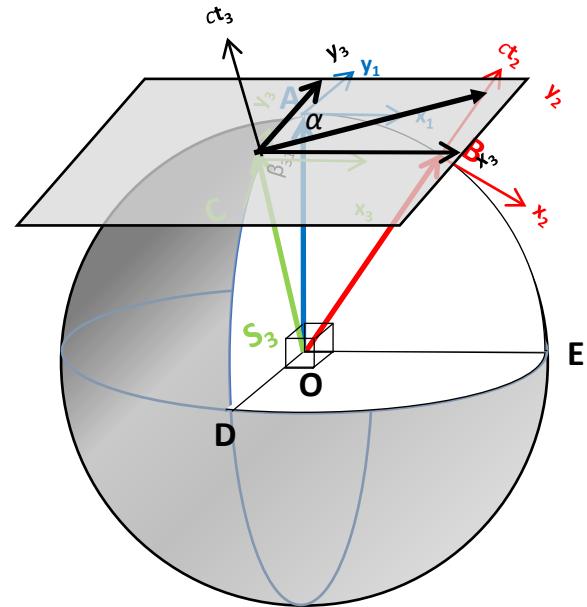
$$\begin{aligned}\sin^2(\beta_{32}) &= \sin^2(\beta_{12}) + \sin^2(\beta_{31}) - \sin^2(\beta_{12}) \cdot \sin^2(\beta_{31}) \\ &\approx \sin^2(\beta_{12}) + \sin^2(\beta_{31})\end{aligned}$$

BEARING ANGLE  $\alpha$ :

$$\beta_{32} \rightarrow \beta_{12}$$

$$\cos(\beta_{31}) \rightarrow 1$$

$$\begin{aligned}\cos(\alpha) &= [\cos(\beta_{12}) - \cos(\beta_{31}) \cdot \cos(\beta_{32})] / [\sin(\beta_{31}) \cdot \sin(\beta_{32})] \approx 0 \\ \text{so } \alpha &\rightarrow 90^\circ\end{aligned}$$





# THE ORTHOGONAL OBSERVER

$$V_{31} \rightarrow C$$



VELOCITY ANGLE  $\beta_{32}$ :

$$\sin(\beta_{31}) \rightarrow 1$$

$$\begin{aligned} \sin^2(\beta_{32}) &= \sin^2(\beta_{12}) + \sin^2(\beta_{31}) - \sin^2(\beta_{12}) \cdot \sin^2(\beta_{31}) \\ &\rightarrow \sin^2(\beta_{31}) \end{aligned}$$

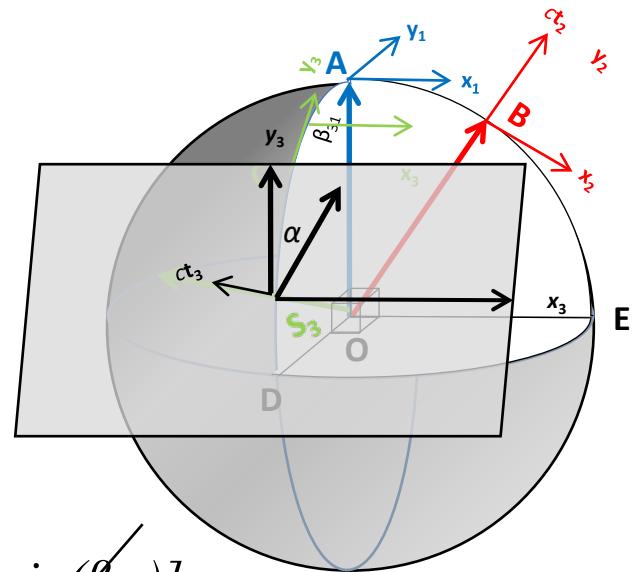
BEARING ANGLE  $\alpha$ :

$$\cos(\beta_{31}) \rightarrow 0$$

$$\sin(\beta_{31}) \rightarrow \sin(\beta_{32}) \rightarrow 1$$

$$\text{SO}$$

$$\begin{aligned} \cos(\alpha) &= [\cos(\beta_{12}) - \cos(\beta_{31}) \cdot \cos(\beta_{32})] / [\sin(\beta_{31}) \cdot \sin(\beta_{32})] \\ &\rightarrow \cos(\beta_{12}) \\ \alpha &\rightarrow \beta_{12} \end{aligned}$$



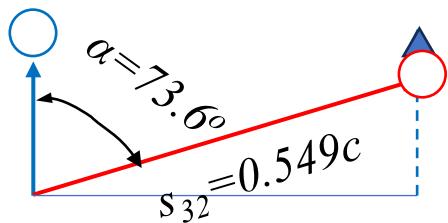


# $S_3 \ x_3 - y_3$ MEASUREMENTS

$v_{31} \ll c$

$$y_{31}=0.17$$

$$x_{31}=0.0$$



$$x_1 = x_3 = 0.5$$

$$y_{32}=0.15$$

$$x_{31}=1.0$$

**SALLY WRT SAM**

$$v_{12}=0.5c$$

$$\beta_{12}=30^\circ$$

$$TIME=1.0$$

**STEVE WRT SAM**

$$v_{31}=0.17c$$

$$\beta_{31}=10^\circ$$

**STEVE WRT SALLY**

$$v_{32}=0.549c$$

$$\beta_{32}=31.47^\circ$$



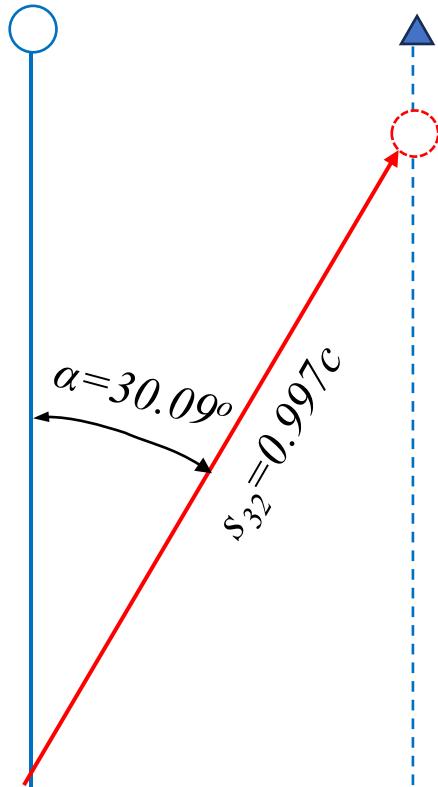


# $S_3$ x-y MEASUREMENTS

$$v_{31} \rightarrow c$$



$$\begin{aligned}y_{31} &= 0.996 \\x_{31} &= 0.0\end{aligned}$$



$$x_I = x_3 = 0.5$$

**SALLY WRT SAM**

$$v_{12} = 0.5c$$

$$\beta_{12} = 30^\circ$$

$$TIME = 1.0$$

$$\begin{aligned}y_{32} &= 1.725 \\x_{31} &= 1.0\end{aligned}$$

**STEVE WRT SAM**

$$v_{31} = 0.996c$$

$$\beta_{31} = 85.67^\circ$$

**STEVE WRT SALLY**

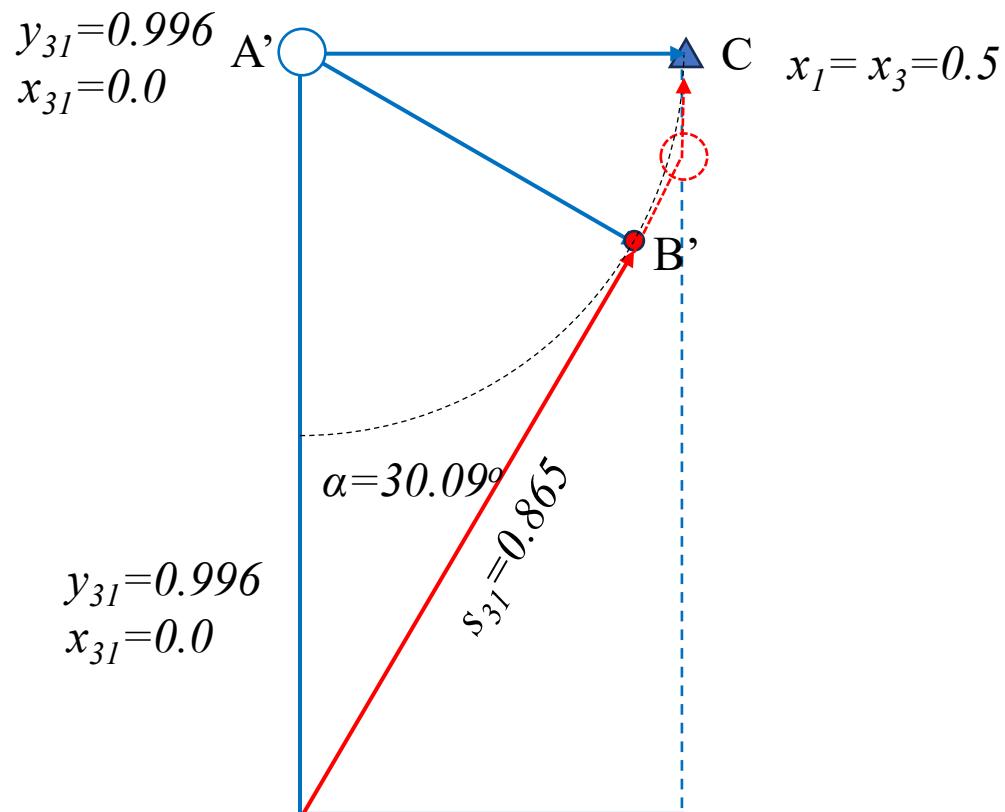
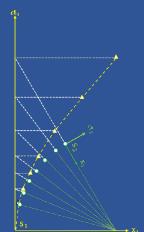
$$v_{32} = 0.997c$$

$$\beta_{32} = 85.67^\circ$$



# $S_3$ x-y MEASUREMENTS

$v_{31} \rightarrow C$



***SALLY WRT SAM***

$$v_{12}=0.5c$$

$$\beta_{12}=30^\circ$$

$$TIME=1.0$$

***STEVE WRT SAM***

$$v_{31}=0.996c$$

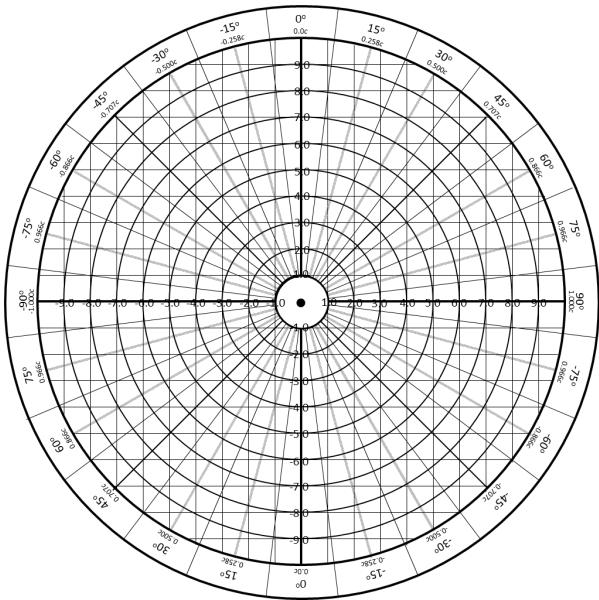
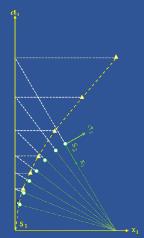
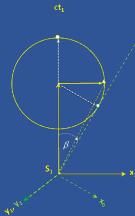
$$\beta_{31}=85.67^\circ$$

***STEVE WRT SALLY***

$$v_{32}=0.997c$$

$$\beta_{32}=85.67^\circ$$

# SUMMARY



- VELOCITY TRIANGLE

- It is Observable
- Reflects Underlying Physical Reality of Relativity

