Using the Sound from a Tuning Fork to Demonstrate Heisenberg Uncertainty Principle

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Heisenberg Uncertainty Principle

One cannot know precisely two complementary observables simultaneously.

$$\Delta a \Delta b \geq \frac{h}{4\pi}$$

Complementary Observables

$$s(a) = \int s(b) e^{-iab} db$$

Time and angular frequency are complementary observables:

$$s(t) = \int s(\omega) e^{-i\omega t} d\omega$$

Process

Record the sound of a 341.3-Hz tuning fork for 1.024s @ 1kHz: $\hat{s}(t)$

Multiply the signal with a window function of various sizes: $s(t) = W(t)\hat{s}(t)$

Rectangular window

Gaussian window

Get the frequency spectrum of the resulting signal: $s(\omega) = \frac{1}{T}\int s(t)e^{i\omega t}dt$

Discrete Fourier Transform

Signal duration: T = 1.024s

$$s(\omega) = \frac{1}{T}\int s(t)e^{i\omega t}dt$$

$$t_n = \frac{nT}{1024} \qquad n = 0, 1,2 \ \ldots, 1023$$

$$\omega_m = 2\pi f_m = \left(\frac{2\pi}{T}\right)m \qquad m = 0, 1,2 \ \ldots, 512$$

$$s(f_m) = \frac{1}{1024}\sum_{n=0}^{1023} s(t_n)\, e^{\frac{i(2\pi)nm}{1024}}$$

Tasks

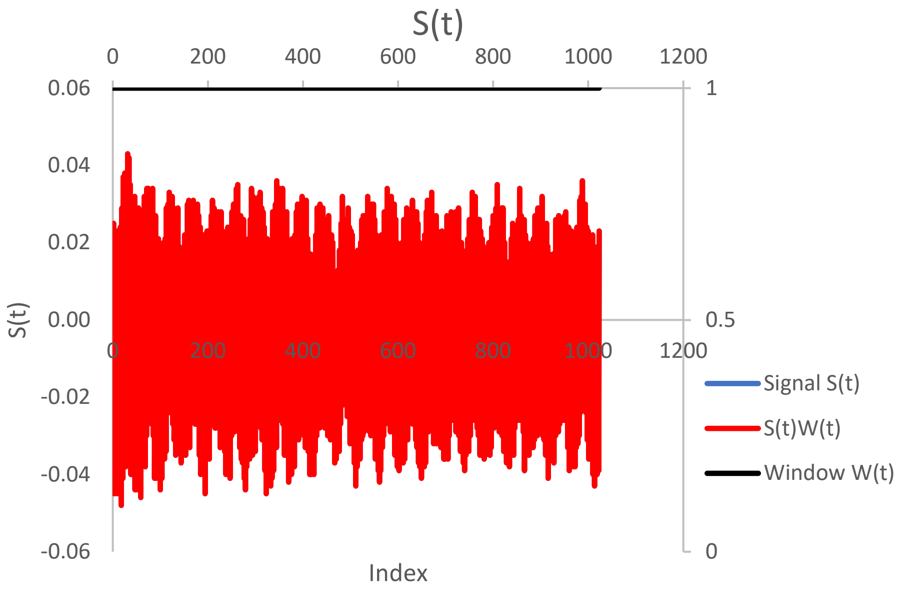
Students don’t know about Fourier analysis, so will be provided with the software.

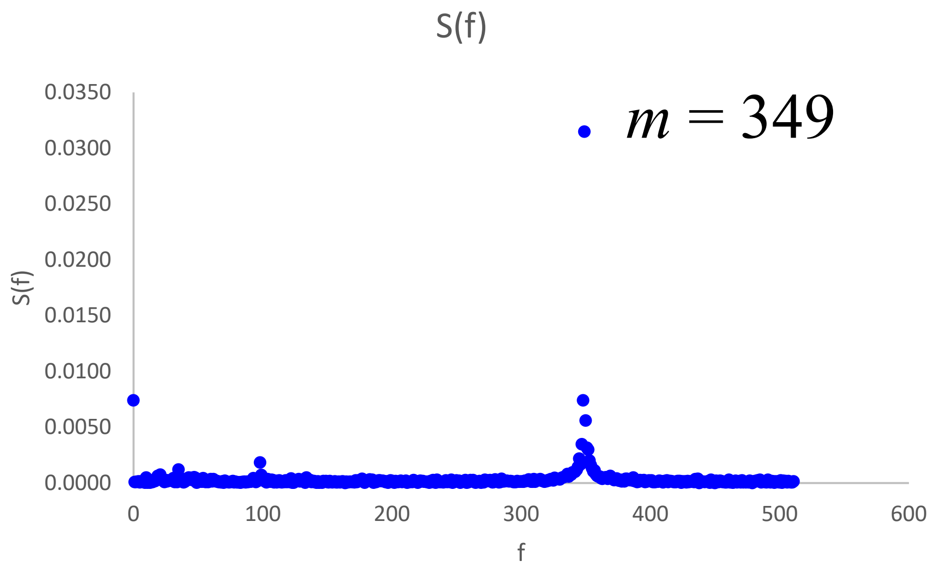
Software will take an input signal and multiply it with a window function then calculate the spectrum of the signal.

Students’ tasks are to collect the sound from a tuning fork and to analyze its spectrum for various window sizes.

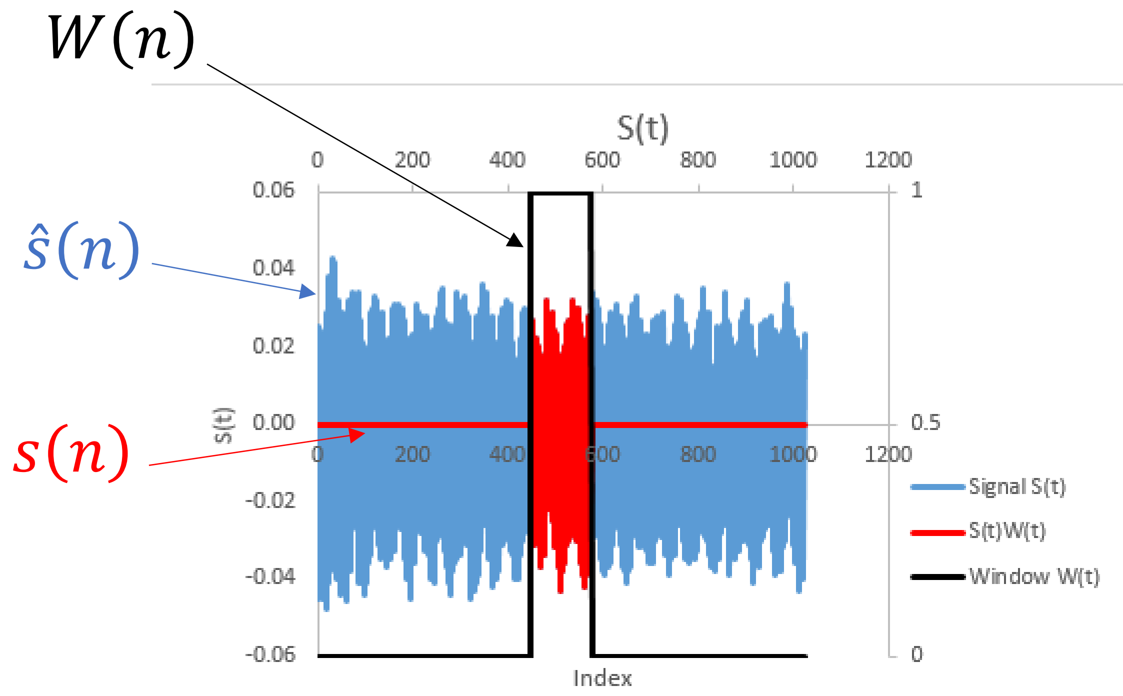
Spectrum of 341.3-Hz Tuning Fork

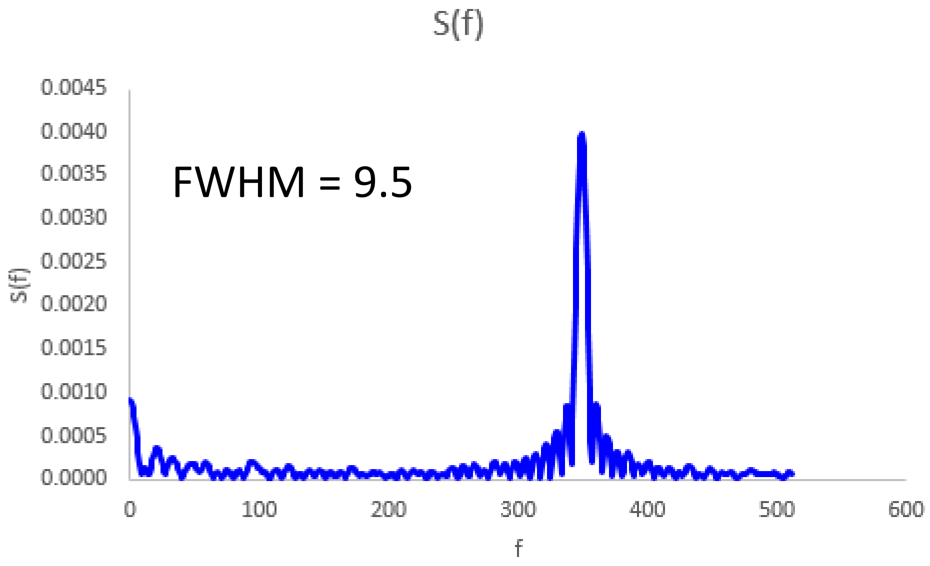
341.3Hz corresponds to f_m with $m = 349.5$





Square W =128

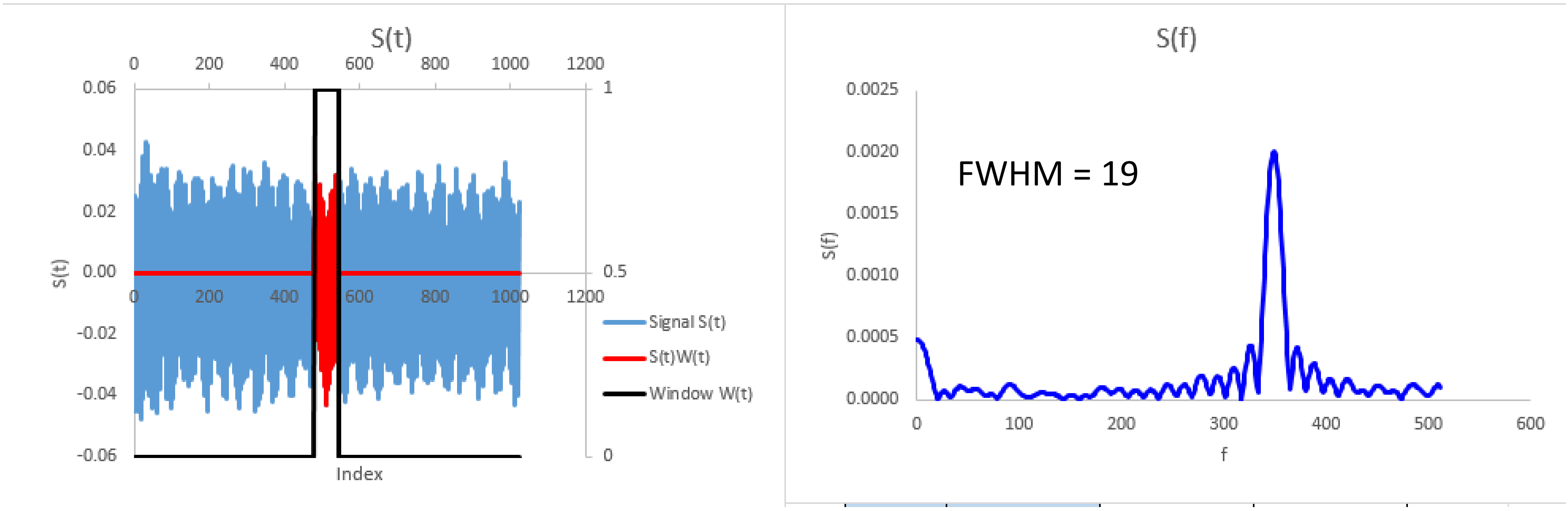




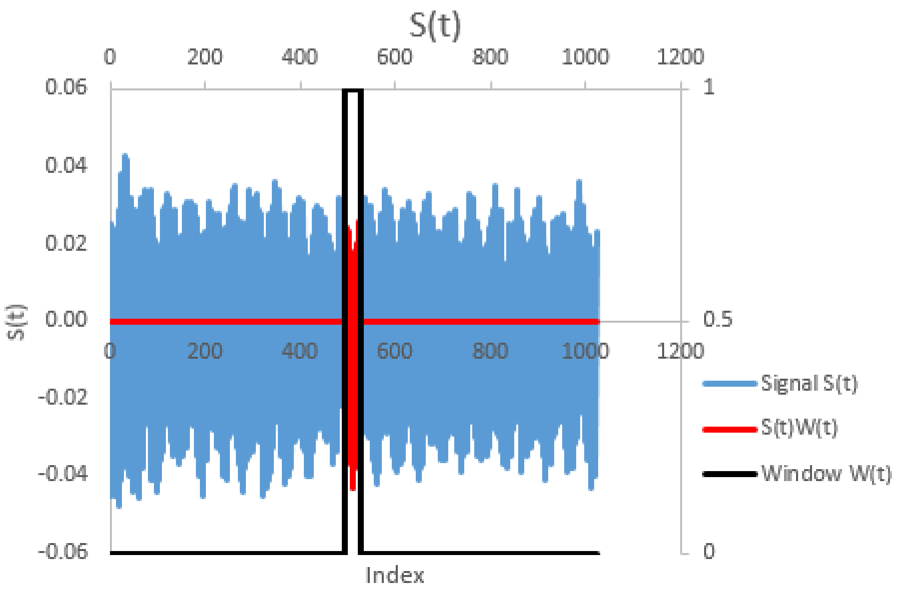
Δt = window size

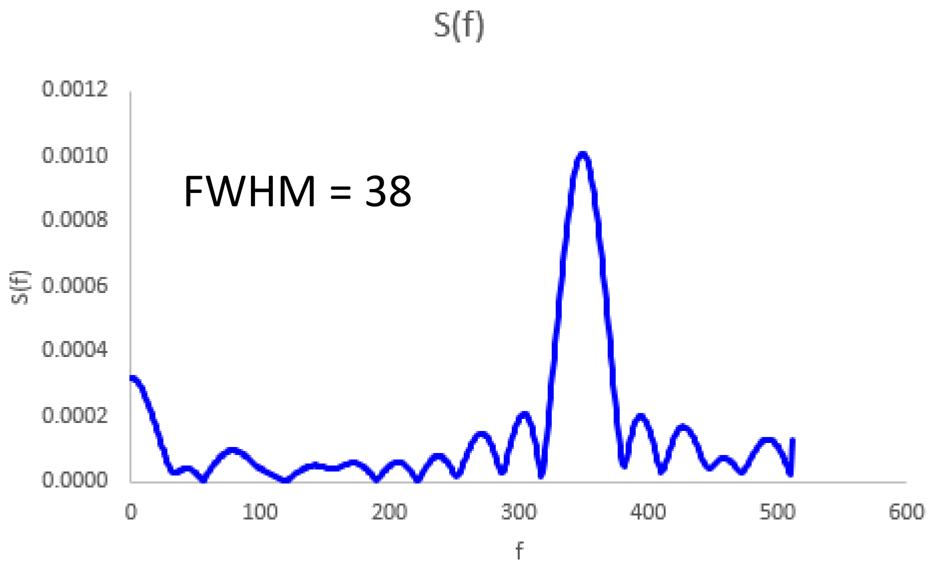
Δf = FWHM

Square W = 64

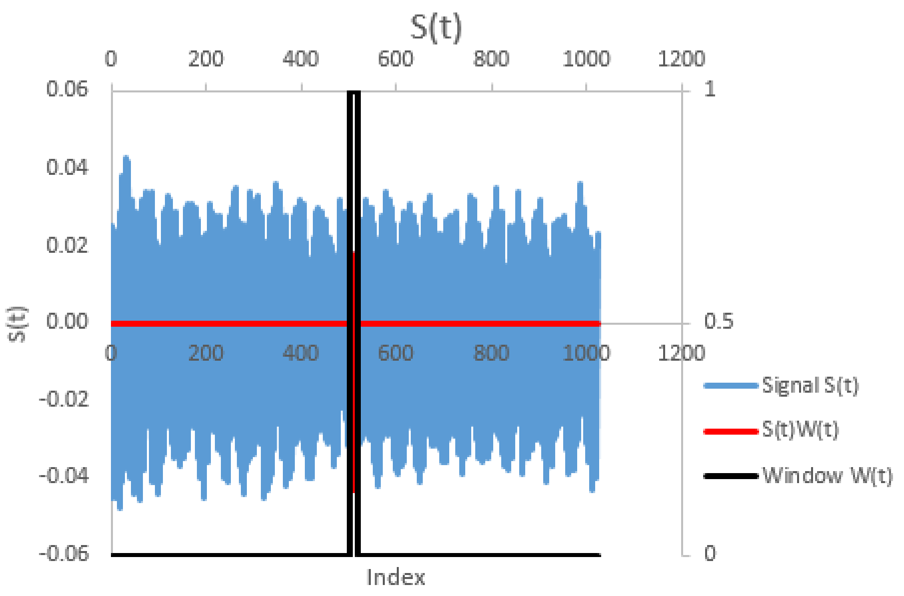

S(t)
0
200
400
600
800
1000
1200
0.06
0.04
0.02
0.00
-0.02
-0.04
-0.06
S(t)
1
0.5
0
Index
Signal S(t)
S(t)W(t)
Window W(t)
S(f)
0.0025
0.0020
0.0015
0.0010
0.0005
0.0000
S(f)
FWHM = 19
0
100
200
300
400
500
600
f

Square W = 32

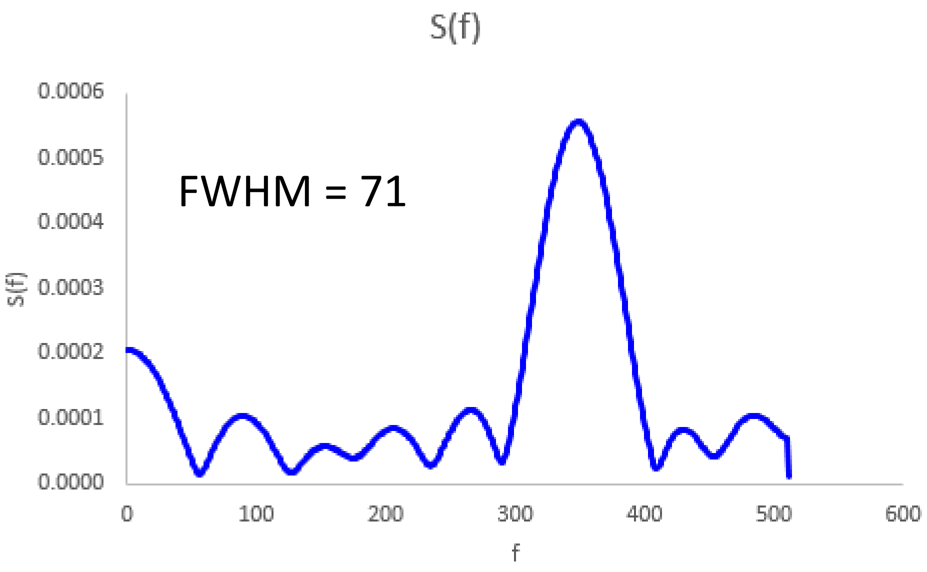

S(t)
0
200
400
600
800
1000
1200
0.06
0.04
0.02
0.00
-0.02
-0.04
-0.06
1
0.5
0
S(t)
Index
Signal S(t)
S(t)W(t)
Window W(t)


S(f)
0.0012
0.0010
0.0008
0.0006
0.0004
0.0002
0.0000
FWHM = 38
S(f)
0
100
200
300
400
500
600
f

Square W = 16

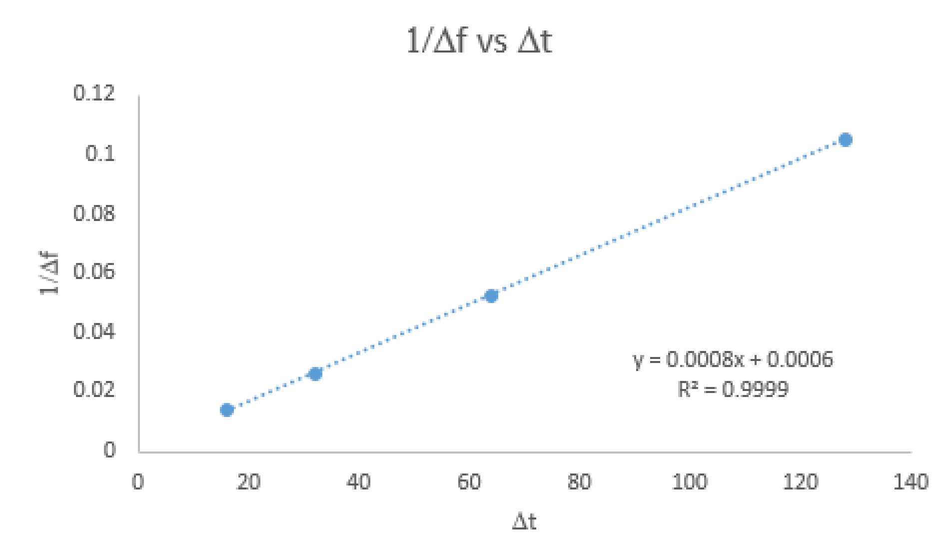


S(t)
0
200
400
600
800
1000
1200
0.06
0.04
0.02
0.00
-0.02
-0.04
-0.06
1
0.5
0
S(t)
Index
Signal S(t)
S(t)W(t)
Window W(t)



S(f)
FWHM = 71
0.0006
0.0005
0.0004
0.0003
0.0002
0.0001
0.0000
S(f)
0
100
200
300
400
500
600
f

$$\frac{1}{\Delta f} \; vs \; \Delta t$$

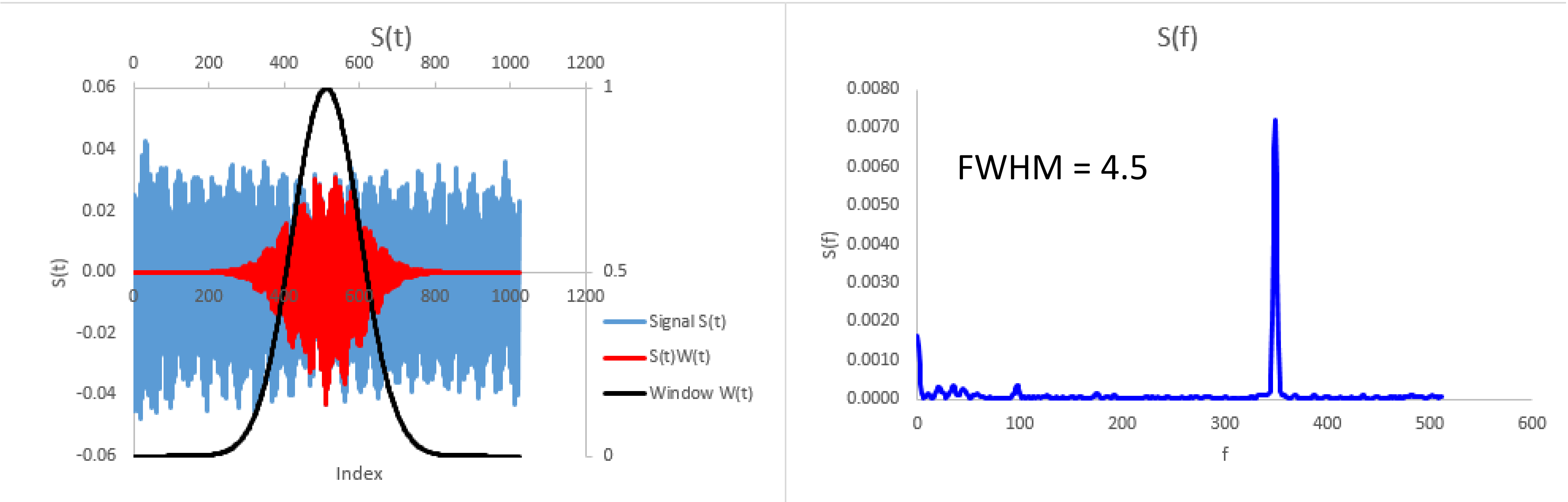


Gaussian Window

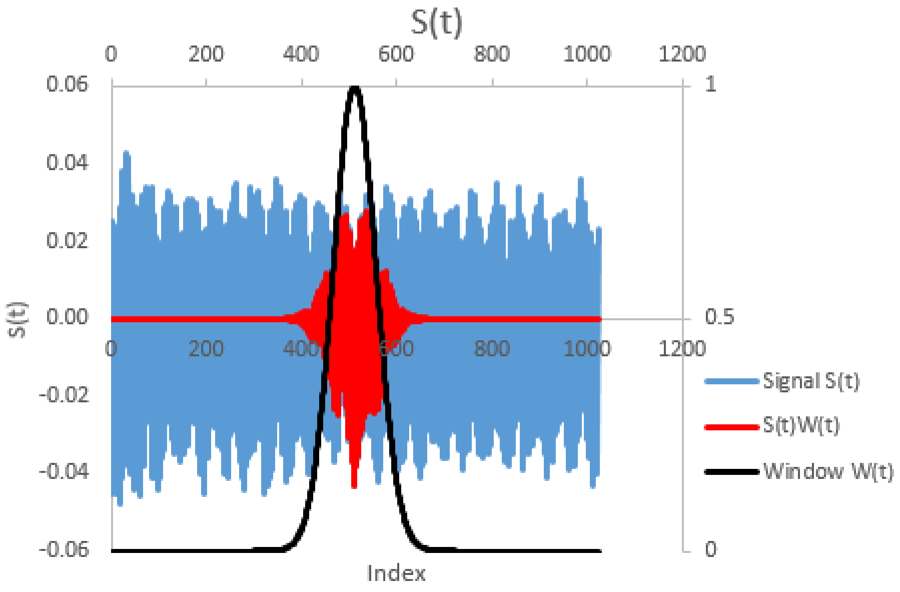
$$W(n) = \exp\left[-\frac{(n-512)^2}{w^2}\right]$$

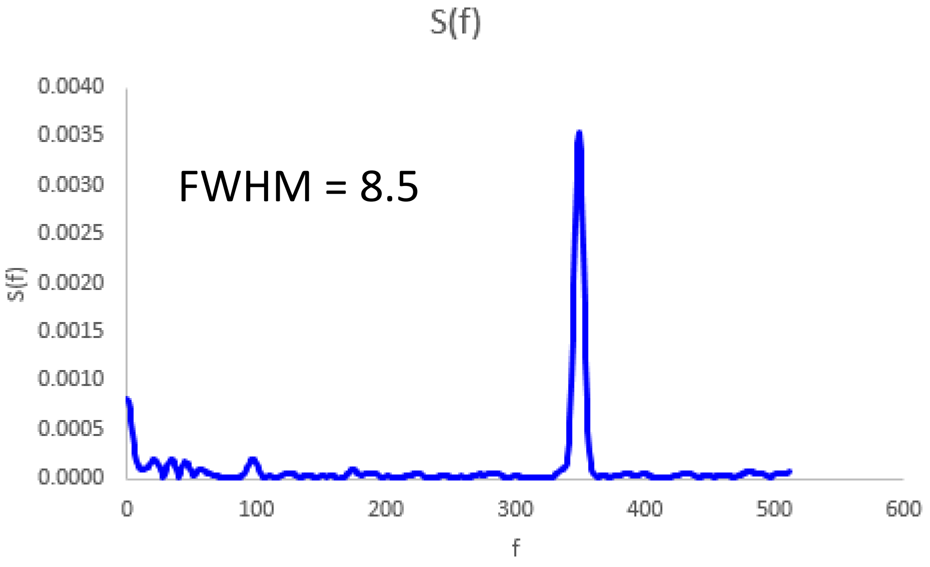
$$n = 0, 1, 2 \ldots 1023$$

Gaussian W = 128

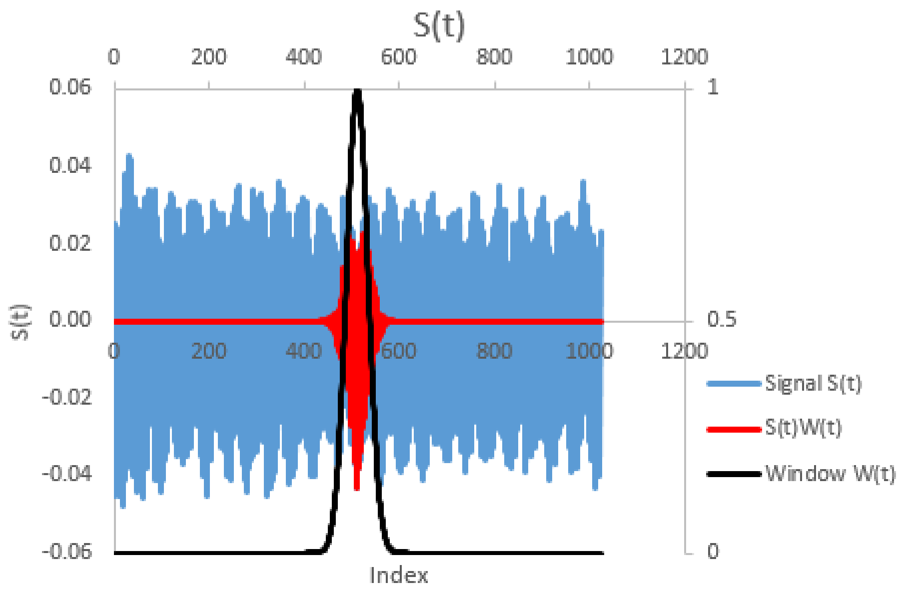


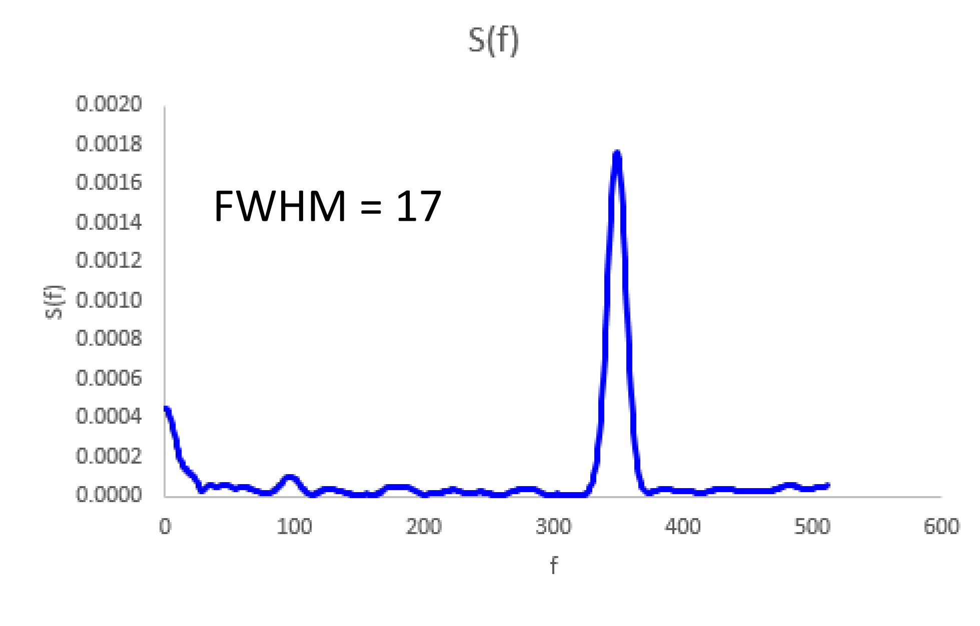
Gaussian W = 64



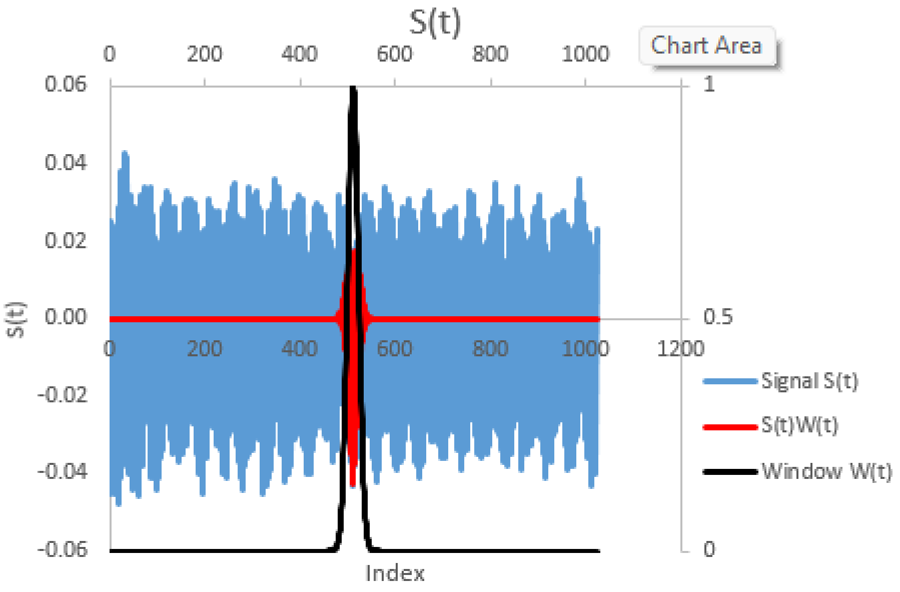


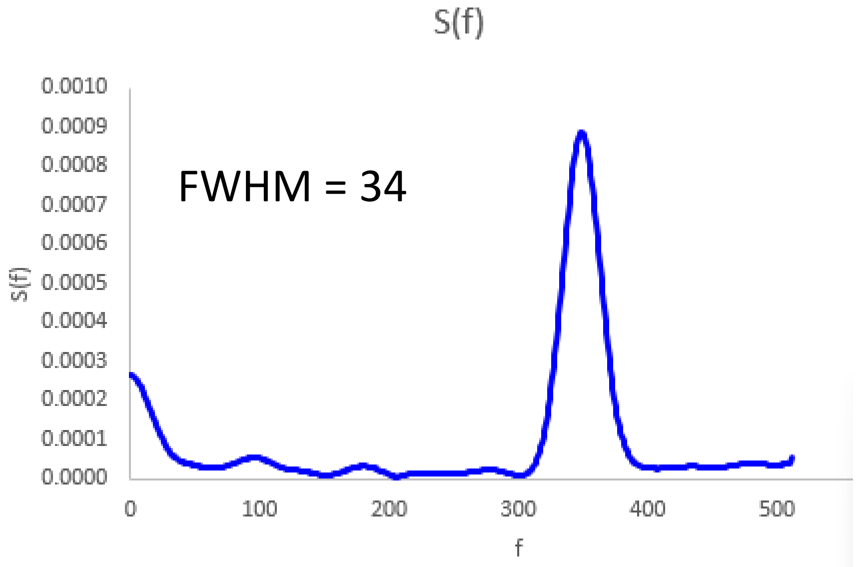
Gaussian W = 32





Gaussian W = 16





$$\frac{1}{\Delta f}\ vs\ \Delta t$$