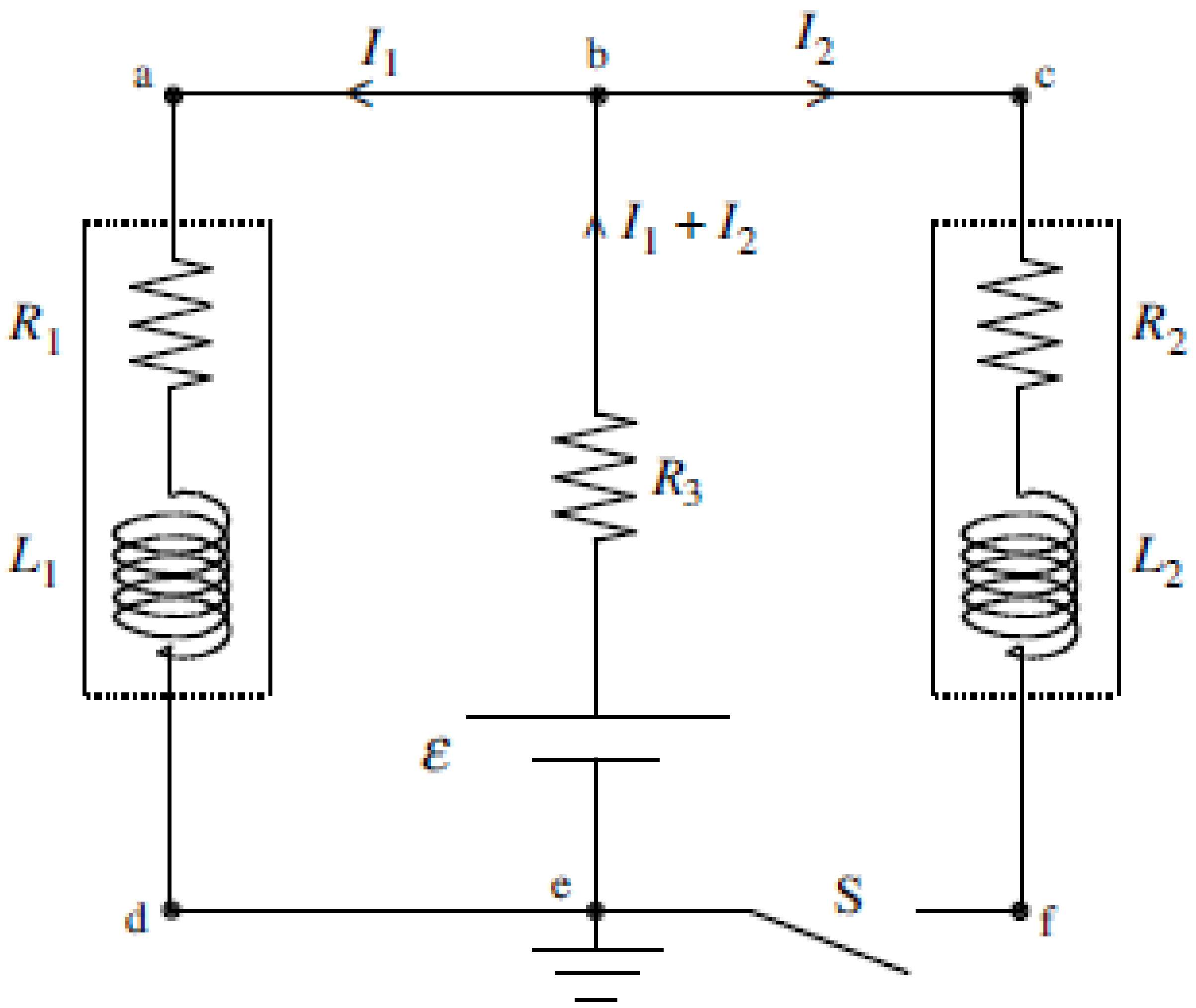
# Parallel *LR* Circuits

***Carl E. Mungan, Physics Dept, U.S. Naval Academy, Annapolis MD***

Fall Meeting of the Chesapeake Section
of the American Association of Physics Teachers

Saturday 11 October 2025 at Virginia Commonwealth University

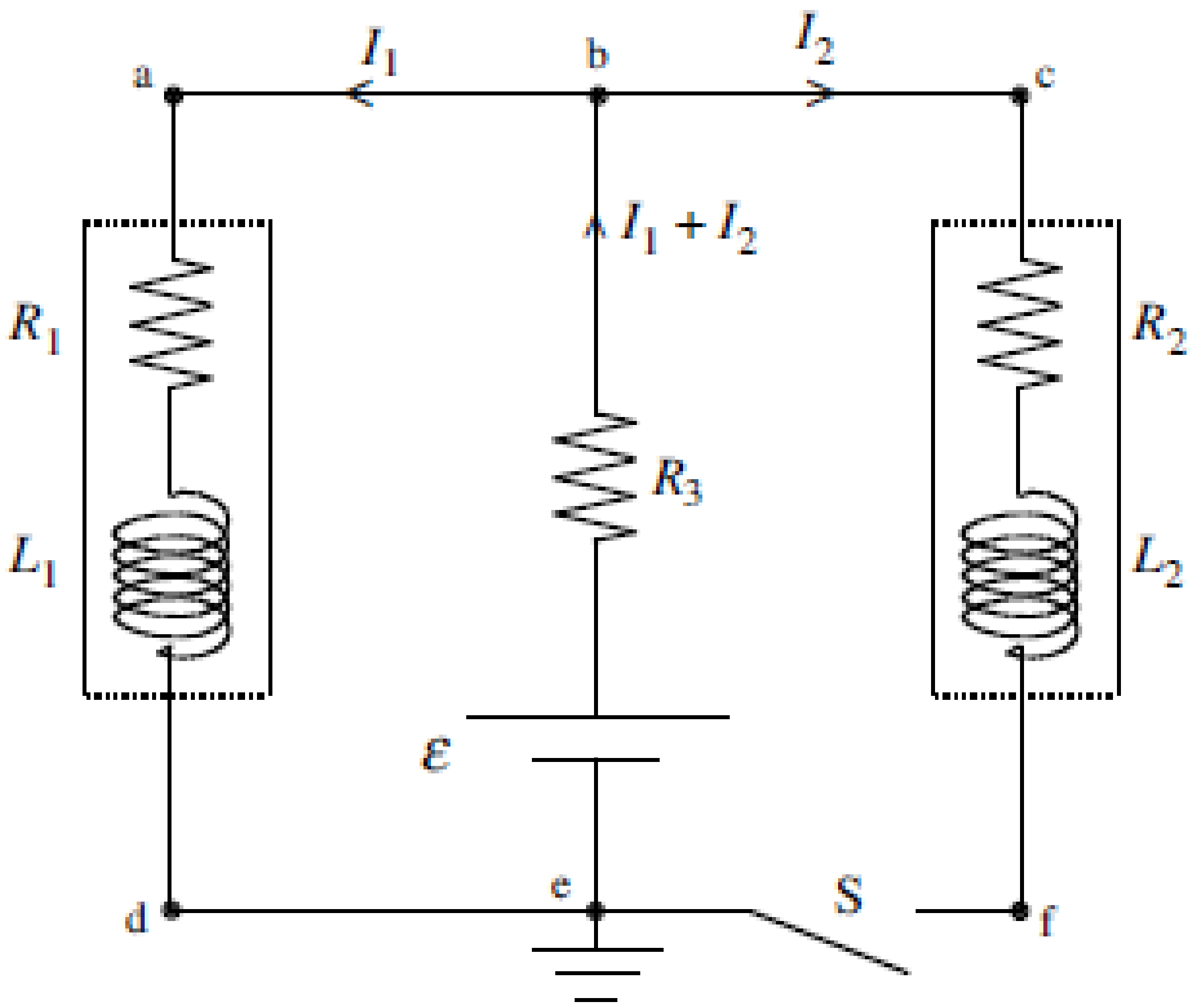
## First circuit




Two coils (indicated by the rectangles) each have inductance and resistance.

What happens if $R_3 = 0$?

## First circuit




Two coils (indicated by the rectangles) each have inductance and resistance.

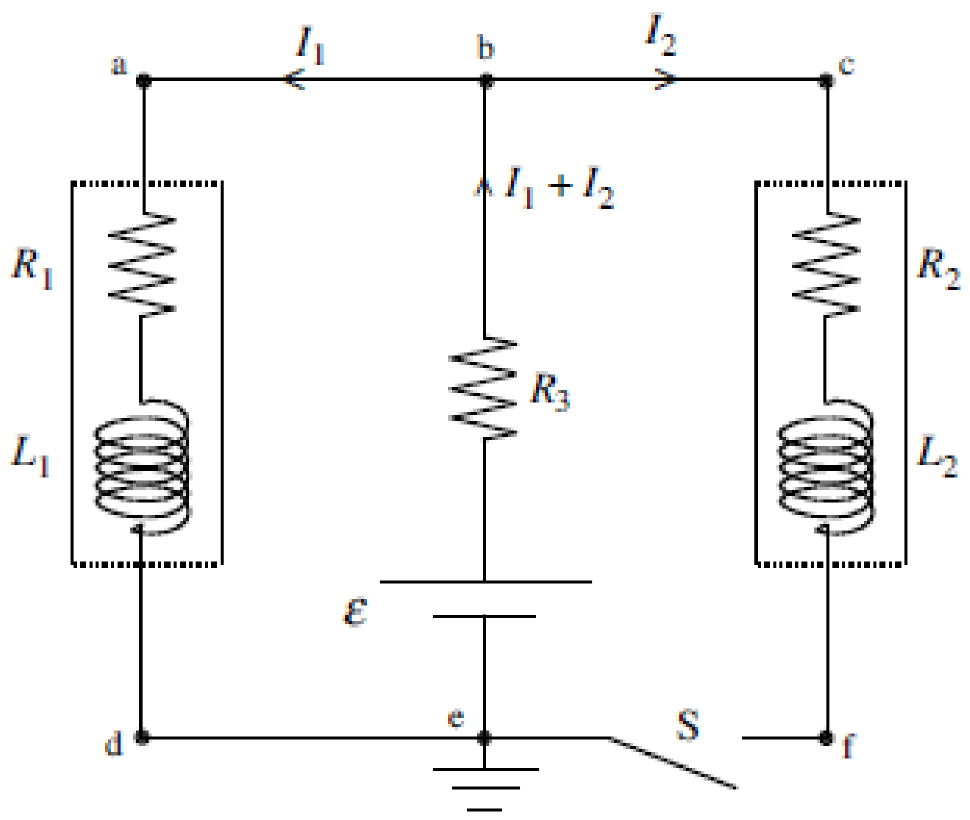
What happens if $R_3 = 0$?

Then the two coils *independently* have the battery connected (or not) across them.

Resistor $R_3$ *couples* together the two currents through the coils.

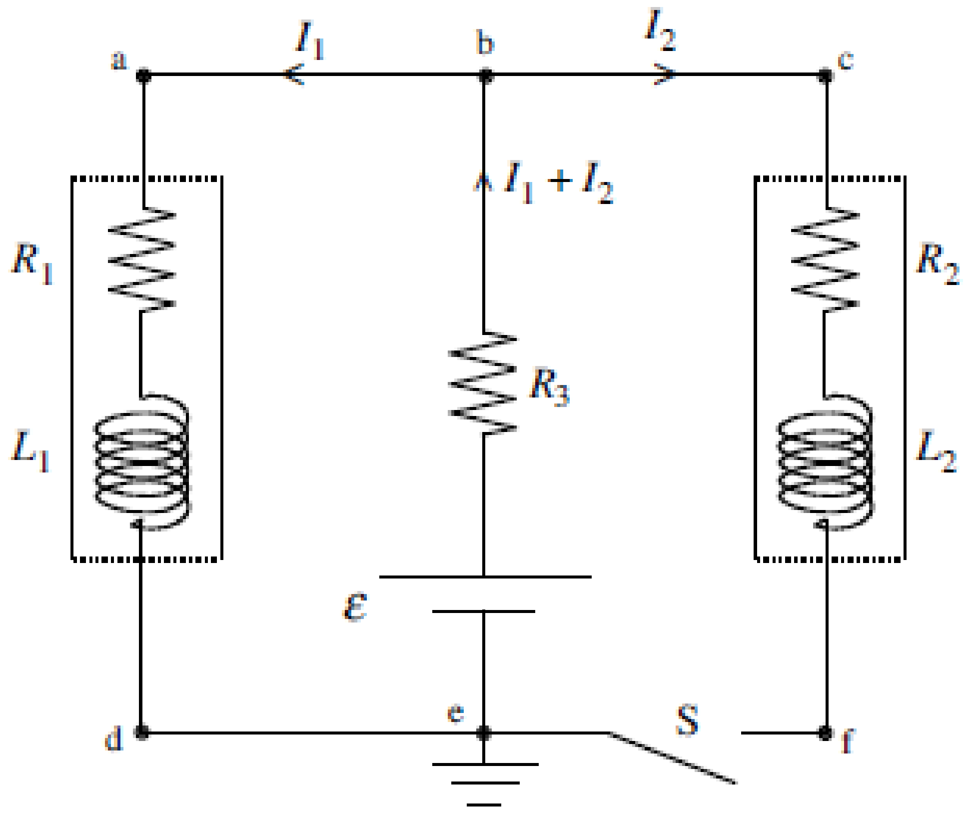
Consider the current through coil 1, starting from when we close S after it has been open for a long time:

$$I_1(0) = \frac{\varepsilon}{R_1 + R_2} \qquad \frac{dI_1}{dt}(0) = 0 \qquad I_1(\infty) = \frac{\varepsilon R_2}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$



Consider the current through coil 1, starting from when we close S after it has been open for a long time:

$$I_1(0) = \frac{\varepsilon}{R_1 + R_2} \qquad \frac{dI_1}{dt}(0) = 0 \qquad I_1(\infty) = \frac{\varepsilon R_2}{R_1R_2 + R_1R_3 + R_2R_3}$$




Write down KVL for the left and right loops. Eliminate $I_2$ between them to get a second order DE for $I_1$ whose solution is a double exponential:

$$I_1(t) = C_{\text{slow}} \exp(-k_{\text{slow}}t) + C_{\text{fast}} \exp(-k_{\text{fast}}t) + I_1(\infty)$$

where $C_{\text{slow}}$ is positive, $C_{\text{fast}}$ is negative, and $|C_{\text{slow}}| > |C_{\text{fast}}|$.

experimental values:

$$L_1 = L_2 = 4.0\ \text{H}$$

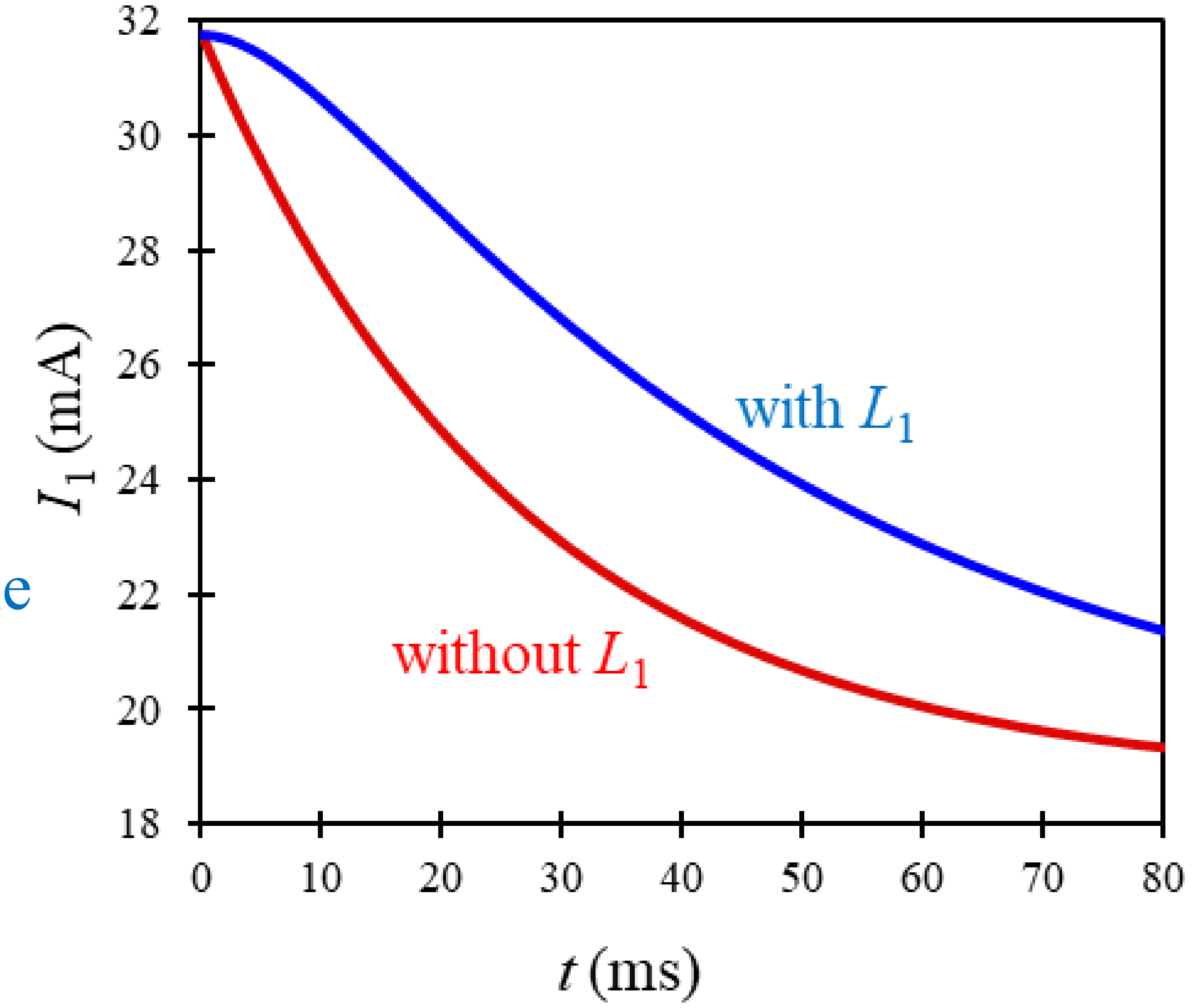
$$R_1 = R_2 = 89\ \Omega$$

$$R_3 = 200\ \Omega$$

$$\varepsilon = 9.3\ \text{V}$$

With coil 1 in place, we get a double exponential with zero initial slope.

Replace coil 1 with an 89 Ω resistor to get a single exponential.



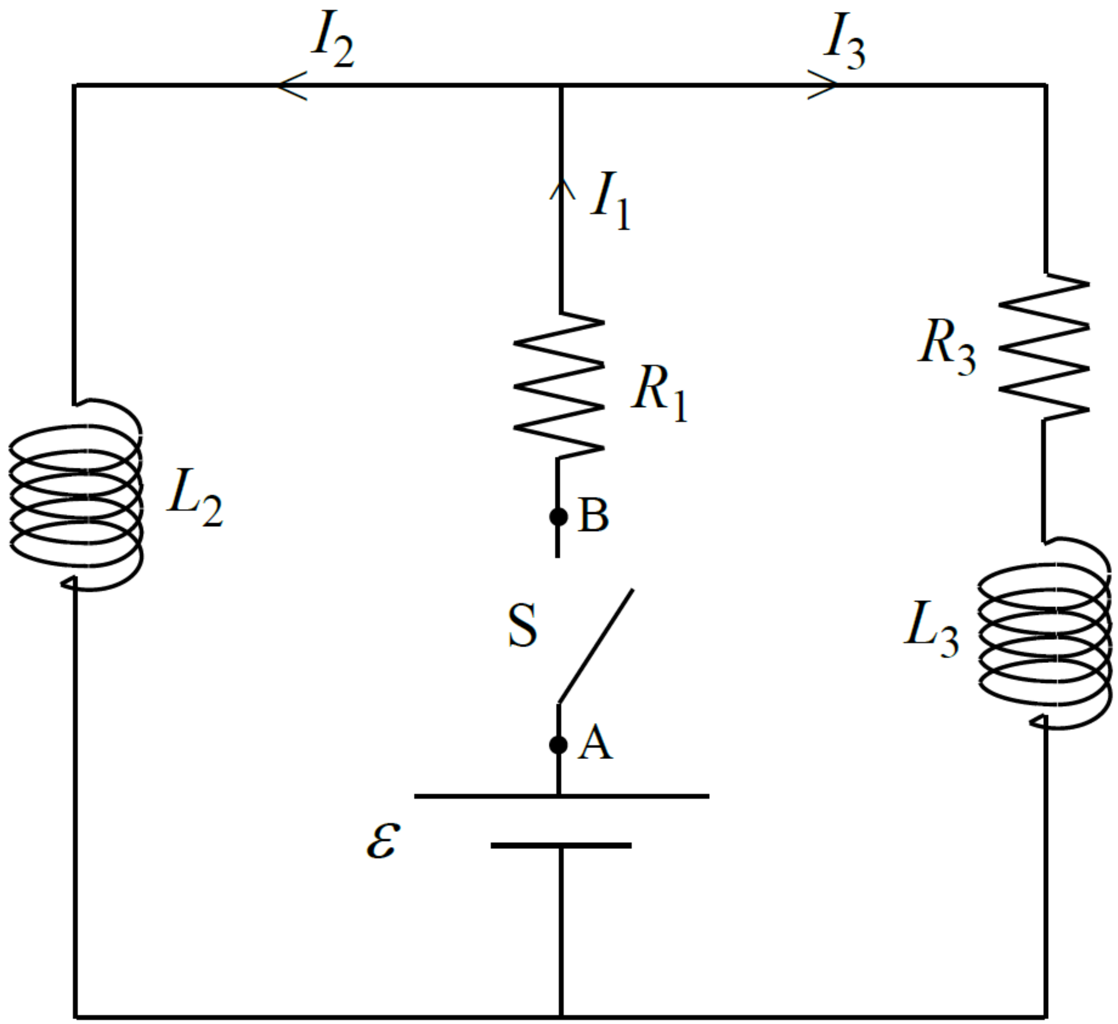
For such a case of identical coils with $L_1 = L_2 = L$ and $R_1 = R_2 = R$ we get symmetric and antisymmetric modes:

$$I_1 + I_2 \propto \exp(-k_{\text{fast}} t) \quad \text{and} \quad I_1 - I_2 \propto \exp(-k_{\text{slow}} t)$$

$$\text{where} \quad k_{\text{fast}} = \frac{R_{\text{equiv}}}{L_{\text{equiv}}} = \frac{R/2 + R_3}{L/2} \quad \text{and} \quad k_{\text{slow}} = \frac{R}{L}$$

(see *Physics Challenges for Teachers and Students* in May 2004 issue of the Physics Teacher)

# Second circuit




compared to first circuit:

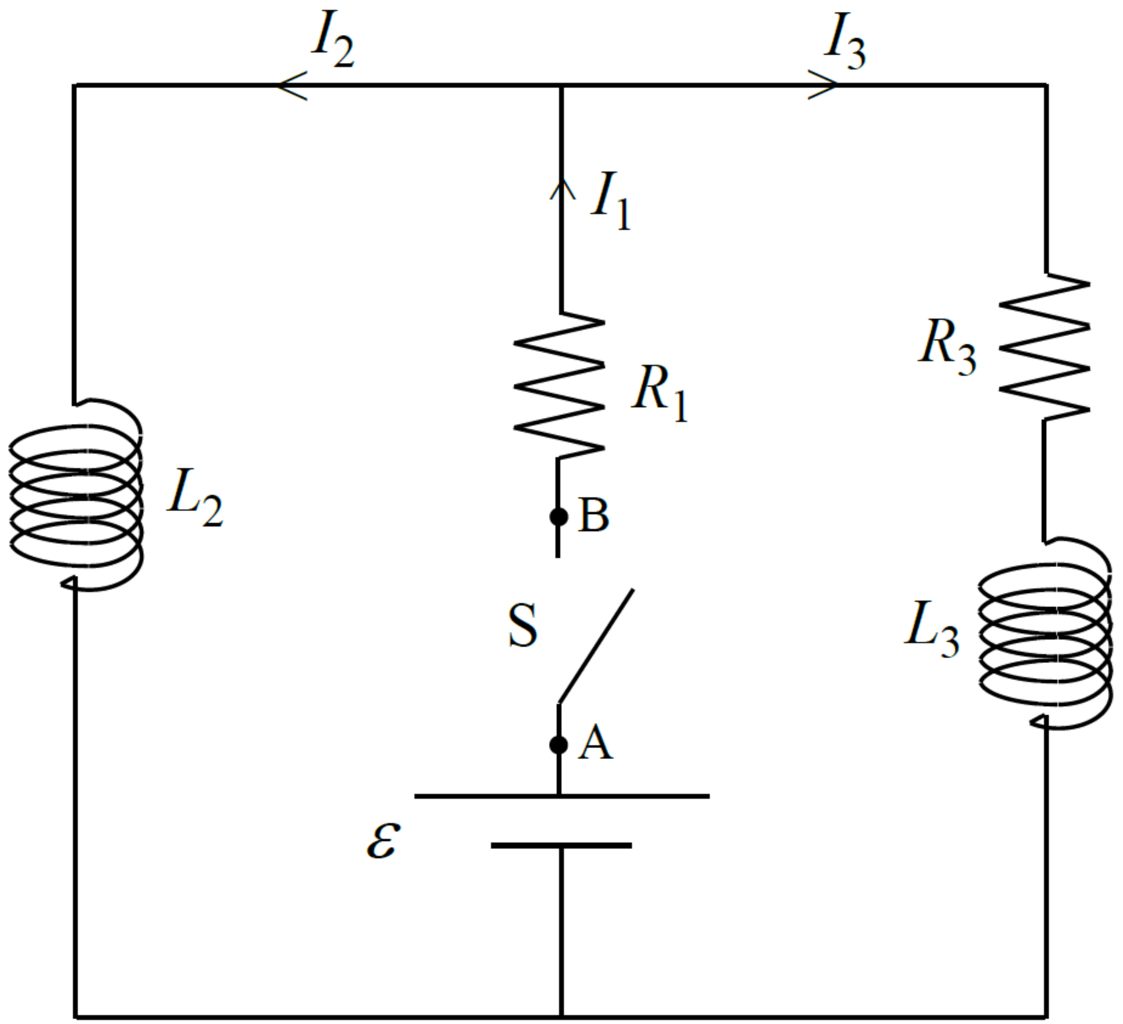
- switch moved into center branch
- resistance of left coil is negligible

switch S open for a long time:

$$I_1(0)=0,\ I_2(0)=0,\ I_3(0)=0$$

What will the currents be if S is closed for a long time?

# Second circuit




compared to first circuit:

- switch moved into center branch
- resistance of left coil is negligible

switch S open for a long time:

$$I_1(0) = 0,\ \ I_2(0) = 0,\ \ I_3(0) = 0$$

What will the currents be if S is closed for a long time?

$I_2$ becomes constant $\Rightarrow$

voltage across $L_2$ becomes zero

$$\therefore I_3(\infty) = 0 \ \Rightarrow$$

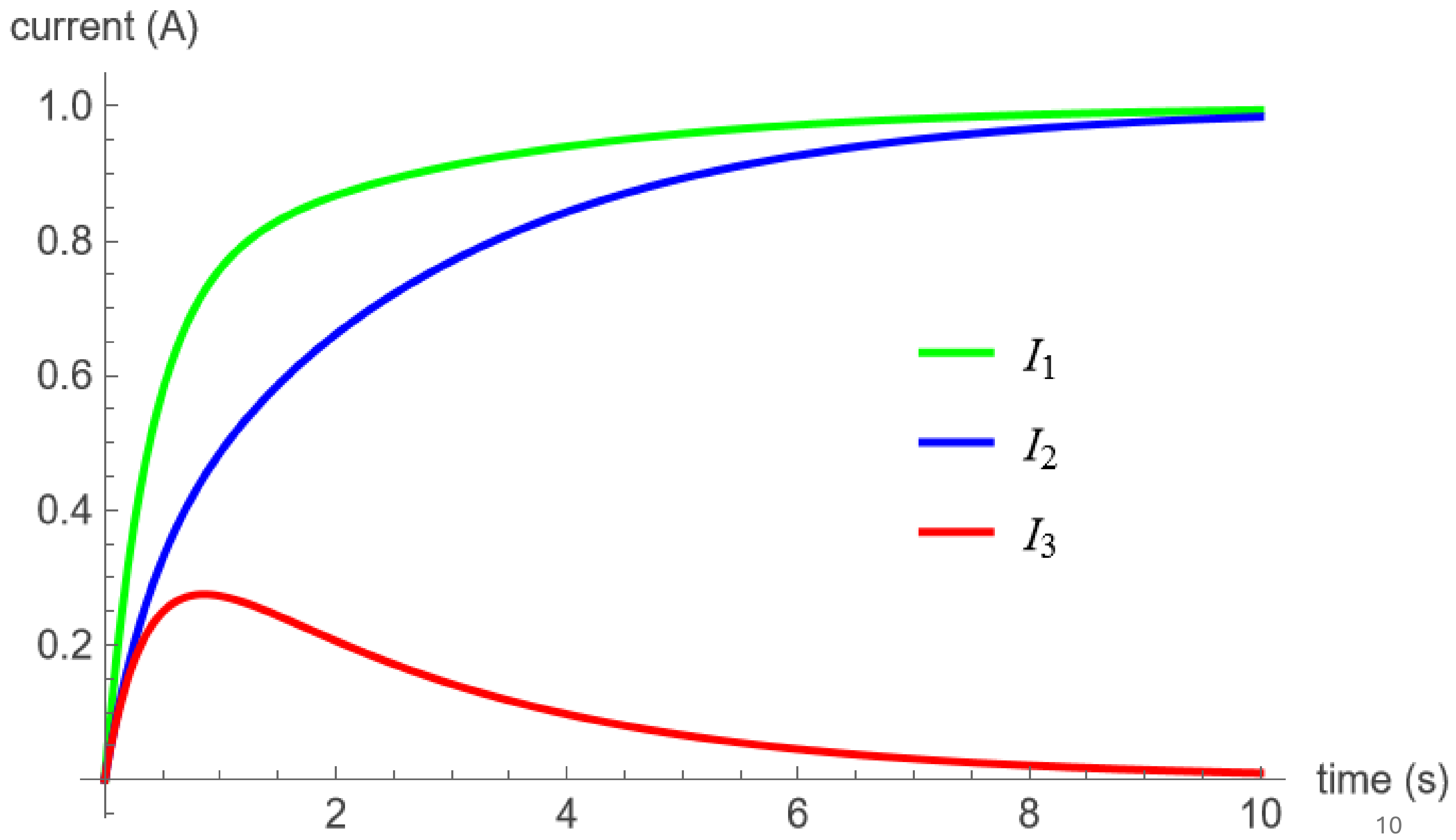
$$I_1(\infty) = I_2(\infty) = \varepsilon / R_1$$

Analyze circuit starting from the instant S is closed using Kirchhoff's laws to get:

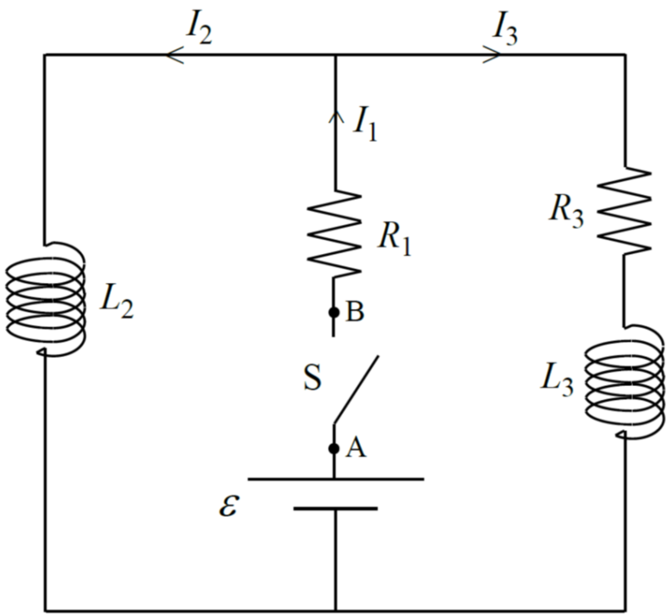
$R_1 = R_3 = 1\,\Omega$

$L_2 = L_3 = 1\,\text{H}$

$\varepsilon = 1\,\text{V}$

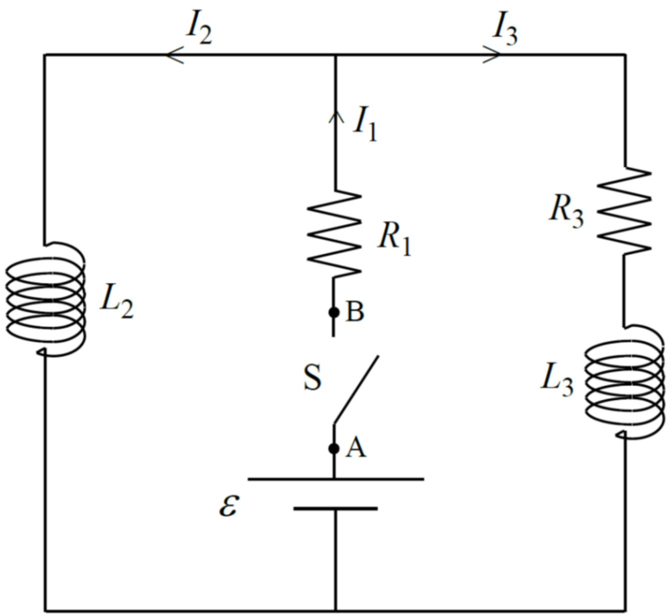


After S has been closed for a long time, open it.
Now what are the three time-dependent currents?



# After S has been closed for a long time, open it.
# Now what are the three time-dependent currents?

redefine $t = \infty$ as $t = 0$ so $I_2(0) = \varepsilon / R_1$ and $I_3(0) = 0$ maintained thru the inductors

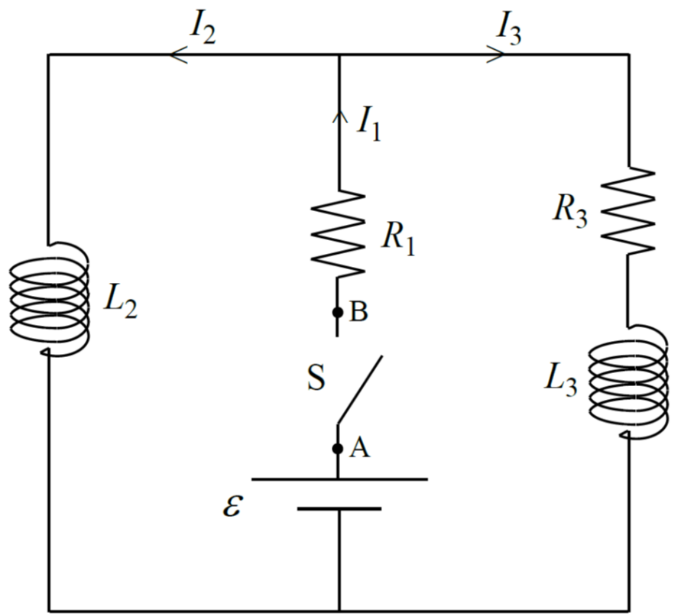



but S interrupts the middle current so $I_1(0) = 0$

contradiction! those currents violate $I_1 = I_2 + I_3$

After S has been closed for a long time, open it.
Now what are the three time-dependent currents?

redefine $t = \infty$ as $t = 0$ so $I_2(0) = \varepsilon / R_1$ and $I_3(0) = 0$ maintained thru the inductors




but S interrupts the middle current so $I_1(0) = 0$

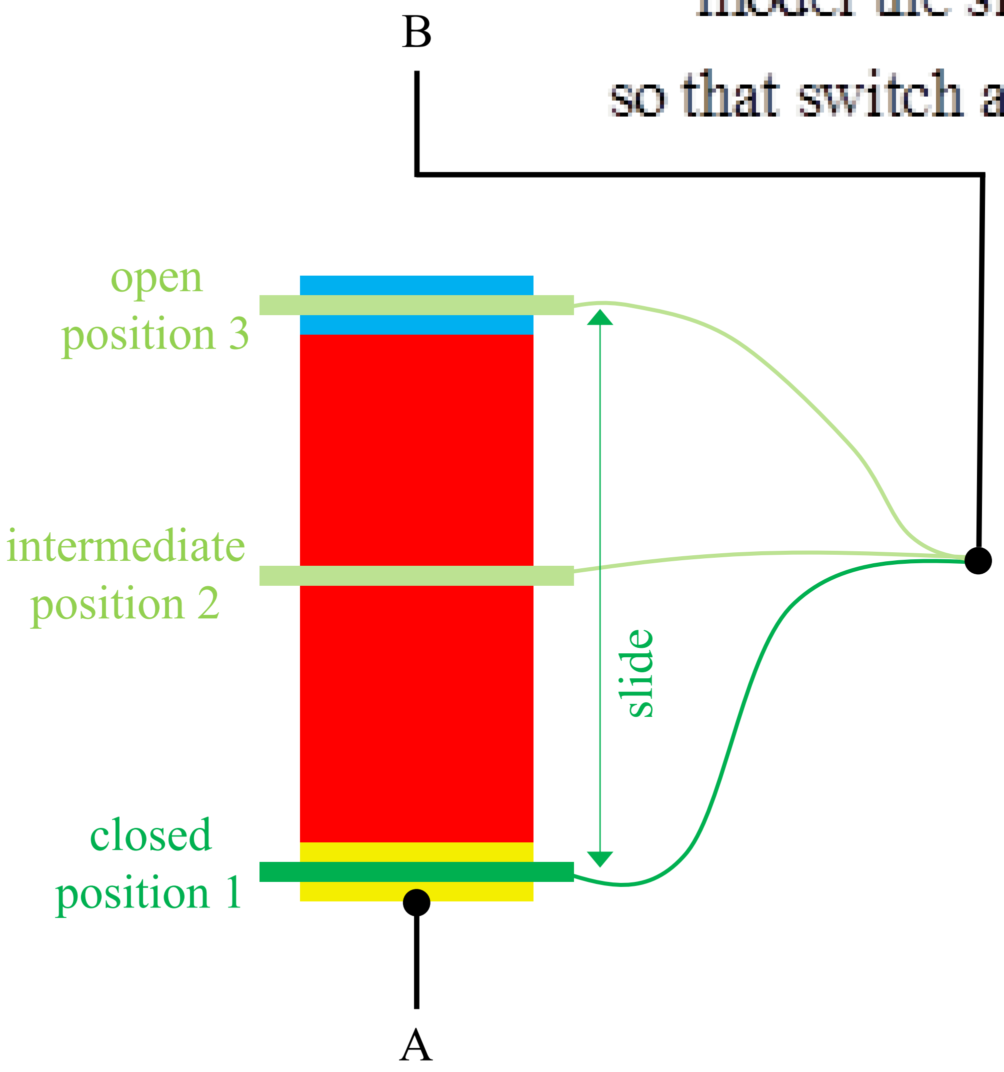
contradiction! those currents violate $I_1 = I_2 + I_3$

If you tried this with a real circuit,
a spark would arc across S to try to maintain $I_1$.
So the fix is to replace S with a switch that
turns off $I_1$ gradually instead of instantly.

# Modified switch: zero resistance in position 1 and infinite resistance in position 3.

model the slide as a linear increase at rate constant $k$

so that switch and resistor together has resistance $(1+kt)R_1$




$\therefore$ resistance is $R_1$ at $t = 0$

and infinite at $t = \infty$

(as desired)

# Reanalyze circuit numerically starting from the instant S is opened using Kirchhoff's laws to get:

$R_1 = 1\ \Omega$

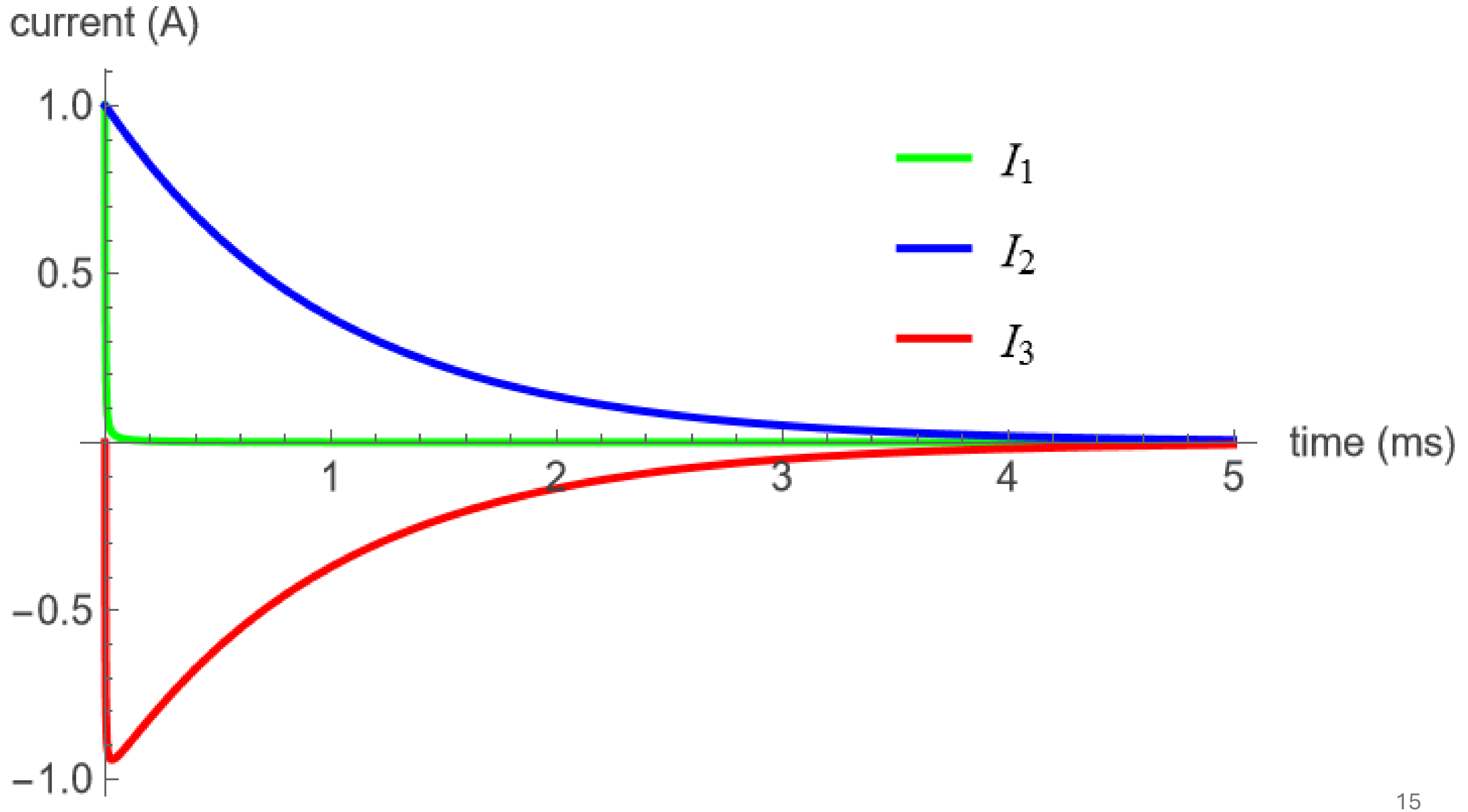
$R_3 = 1\ \text{k}\Omega$

$L_2 = 1\ \text{H}$

$L_3 = 1\ \text{mH}$

$\varepsilon = 1\ \text{V}$

$k = 10^9\ \text{s}^{-1}$



# CONCLUSIONS

## Circuit 1

Connecting two *LR* circuits in parallel with a battery and resistor gives coupled double-exponential currents. For the special case of identical coils we get symmetric and antisymmetric modes. We can decouple the two circuits by setting the battery resistance to zero.

## Circuit 2

*LR* circuits often require a “Make Before Break” switch to avoid problems. Another special one may be needed that could be called a “Smooth Transition” switch to avoid discontinuously jumping between zero and infinite resistance when it is opened or closed.

# Comments or questions?

email: mungan@usna.edu

webpage: usna.edu/Users/physics/mungan

where you can find the papers on which this presentation is based:

- *Physics Teacher* **43**, 519 (2005)
- *Physics Education* **59**, 023005 (2024)