Rolling Downhill and Uphill

The Physics of the "Let it go!" and "Defying Gravity" Demos

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The "Let it go!" Demo:

- Objects of different shapes are rolled downhill to see which shape reaches the bottom first
- Demonstration of the conservation of energy

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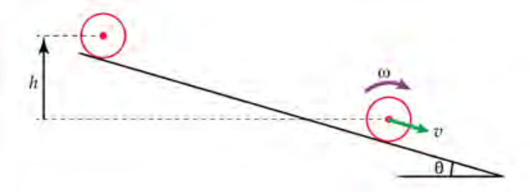
The “Defying Gravity” Demo:

- The double-cone is seen to roll UPHILL along a V-shaped track
- Demonstration of the concept of center-of-mass

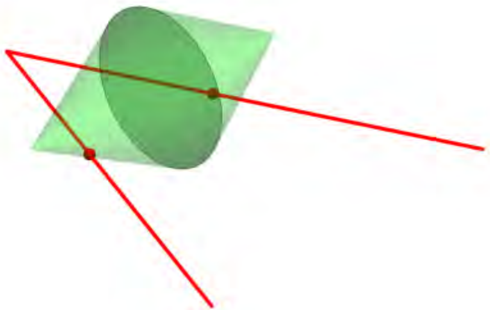
The “Defying Gravity” Demo:



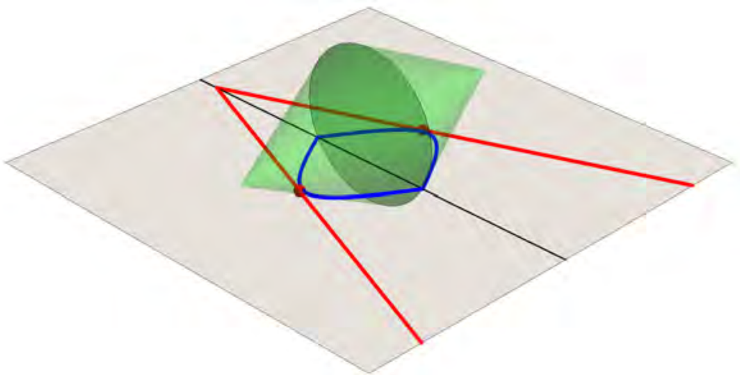
Object rolls without slipping: $v = R\omega$
Moment of inertia: $I_{CM} = \zeta m R^2$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I_{CM}\omega^2 = \frac{1}{2}(1+\zeta)mv^2$$
$$\downarrow$$
$$v = \sqrt{\frac{2gh}{1+\zeta}}$$

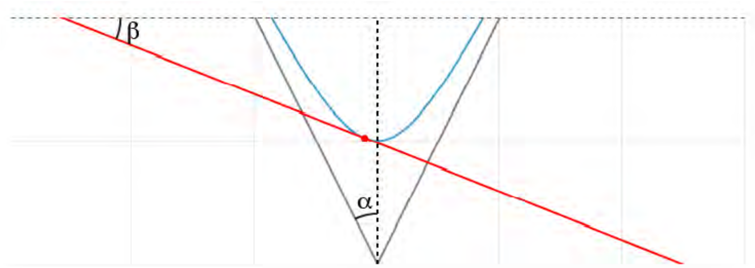
The "Defying Gravity" Demo : The Geometry



- We need to find the two points at which the double-cone is supported by the two tracks.

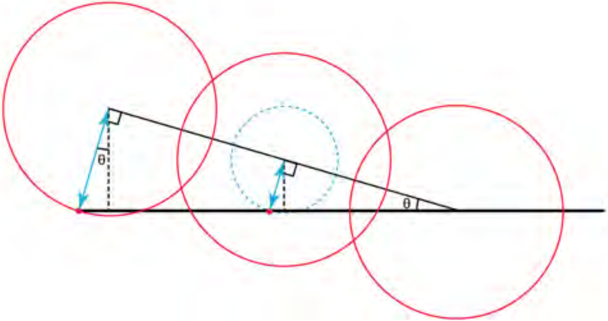


- Imagine slicing the cones with the plane which includes the two tracks. The shape of the cross section of each cone will be given by a hyperbola.



- The track will be tangent to the cross section at the point that supports the cone.
- The position of this point is determined by the angles α and β.
- Note that the point is displaced from the cone's axis.

When the V-shaped track is flat

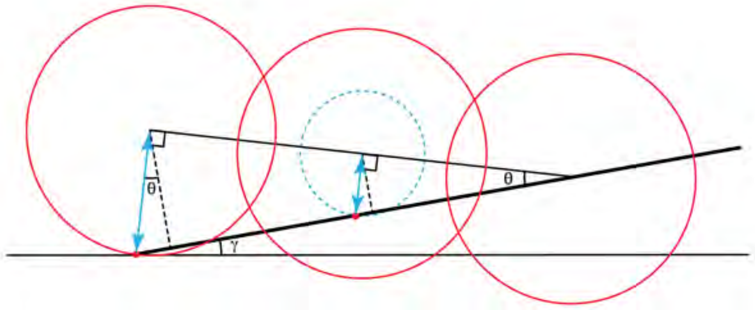


- As the double-cone rolls along the V-shaped track, its center-of-mass will move along an incline tilted by angle θ from the horizontal, where

$$\sin\theta \;=\; \tan\alpha\tan\beta\ .$$

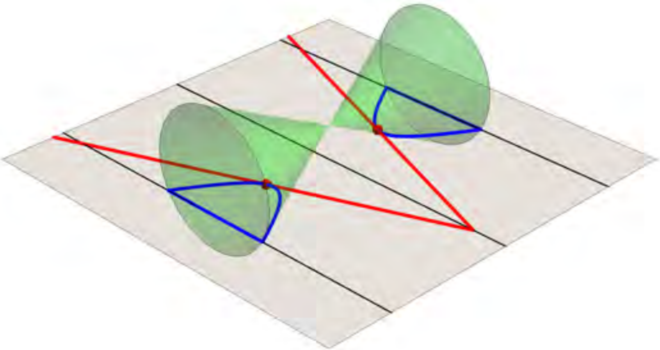
- The red points in the figure support the cone.

When the V-shaped track is tilted up by angle γ

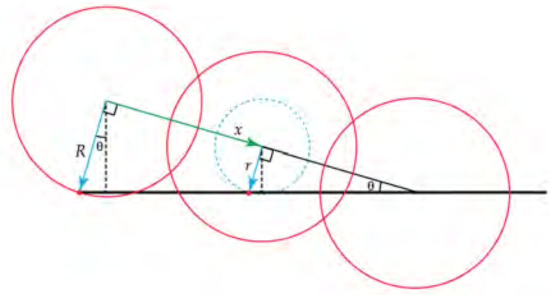


- Double-cone will continue to roll to the right as long as $\gamma < \theta$.

The "Defying Gravity" Demo : Alternative Setup



- If the two cones are joined at their tips, the cones will roll toward the vertex of the V.
- May be difficult to prevent the cones from veering left or right.

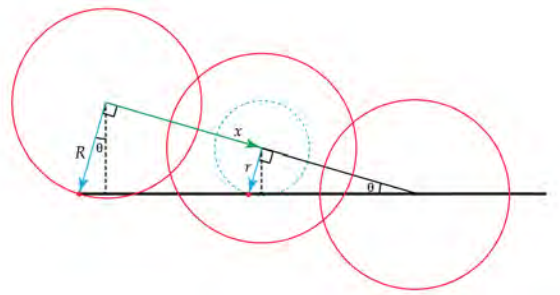


- The distance down the incline x and the effective radius of rolling r are related via

$$r \;=\; R - x\tan\theta \;,$$

where R is the radius of the base of the cone.

Energy Conservation when $\gamma = 0$, part 2



- The height difference after rolling a distance x is

$$h = x \sin\theta \,.$$

- The moment of inertia of the cone around its axis is

$$I_{CM} = \frac{3}{10} m R^2 \equiv m A^2 \,.$$

Energy Conservation when $\gamma = 0$, part 3

- Rolling without slipping condition

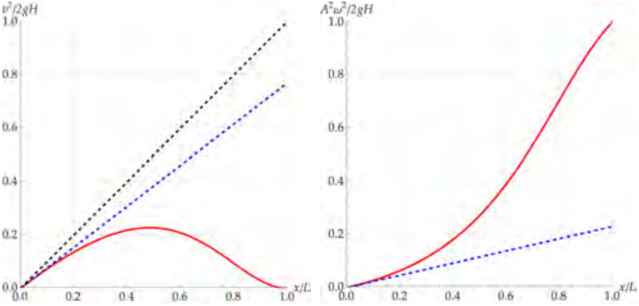
$$v = r\omega$$

- Energy conservation:

$$\begin{aligned} mgx\sin\theta &= \frac{1}{2}mv^2 + \frac{1}{2}I_{CM}\omega^2 \\ &= \frac{1}{2}m\left(1 + \frac{A^2}{r^2}\right)v^2 \\ &\downarrow \\ v^2 &= \frac{2gx\sin\theta}{1 + (A^2/r^2)} = \frac{2gx\sin\theta}{1 + \dfrac{A^2}{(R - x\tan\theta)^2}} \\ &\downarrow \\ \frac{v^2}{2gH} &= \frac{x/L}{1 + \dfrac{3}{10(1 - x/L)^2}} \end{aligned}$$

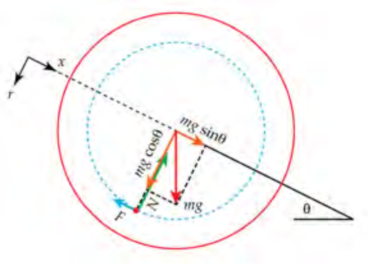
where $L = R/\tan\theta$. $H = R\cos\theta$.

Energy Conservation when $\gamma = 0$, part 4



- Due to the no-slipping condition $v = r\omega$, v starts to decrease after a certain point even though ω continues to increase. That is, r decreases faster that ω increases leading to $v = r\omega$ decreasing.

How can the Double-Cone slow down?



- Equation of motion:

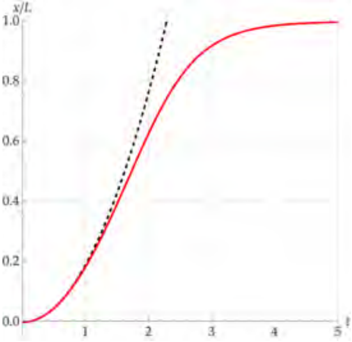
$$\begin{aligned} ma &= mg\sin\theta - F \\ I_{\mathrm{CM}}\alpha &= Fr \end{aligned}$$

- No-slipping condition

$$\begin{aligned} v &= r\omega \\ &\downarrow \\ a &= r\alpha + \dot{r}\omega \\ &= r\alpha - v\omega\tan\theta \end{aligned}$$

- At some point, the force F becomes larger than $mg\sin\theta$ and the Double-Cone will start to slow down.

Time dependence of x

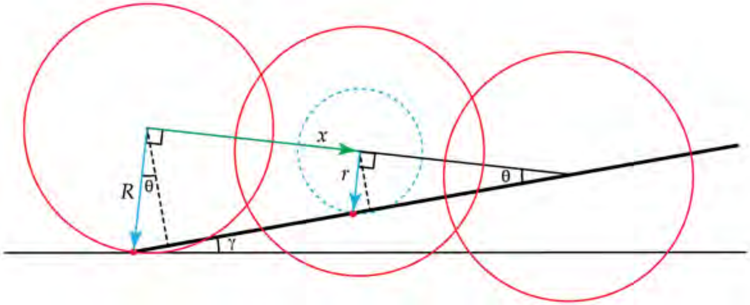


- Time-dependence of x can be obtained by integrating $\dot{x}$:

$$\begin{aligned} \dot{x}^2 &= f(x) \\ &\downarrow \\ \dot{x} = \frac{dx}{dt} &= \sqrt{f(x)} \\ &\downarrow \\ \int_{x_{\text{init}}}^{x} \frac{dx}{\sqrt{f(x)}} &= \int_0^t dt \end{aligned}$$

- $x = L$ in the graph is where the double-cone falls off the V-shaped track, but it takes forever to get there.

Energy Conservation when $\gamma \neq 0$



- Only thing that changes is the relation between the height difference and x

$$h = x \sin(\theta - \gamma) .$$

- Can be absorbed into the redefinition of H.

The End