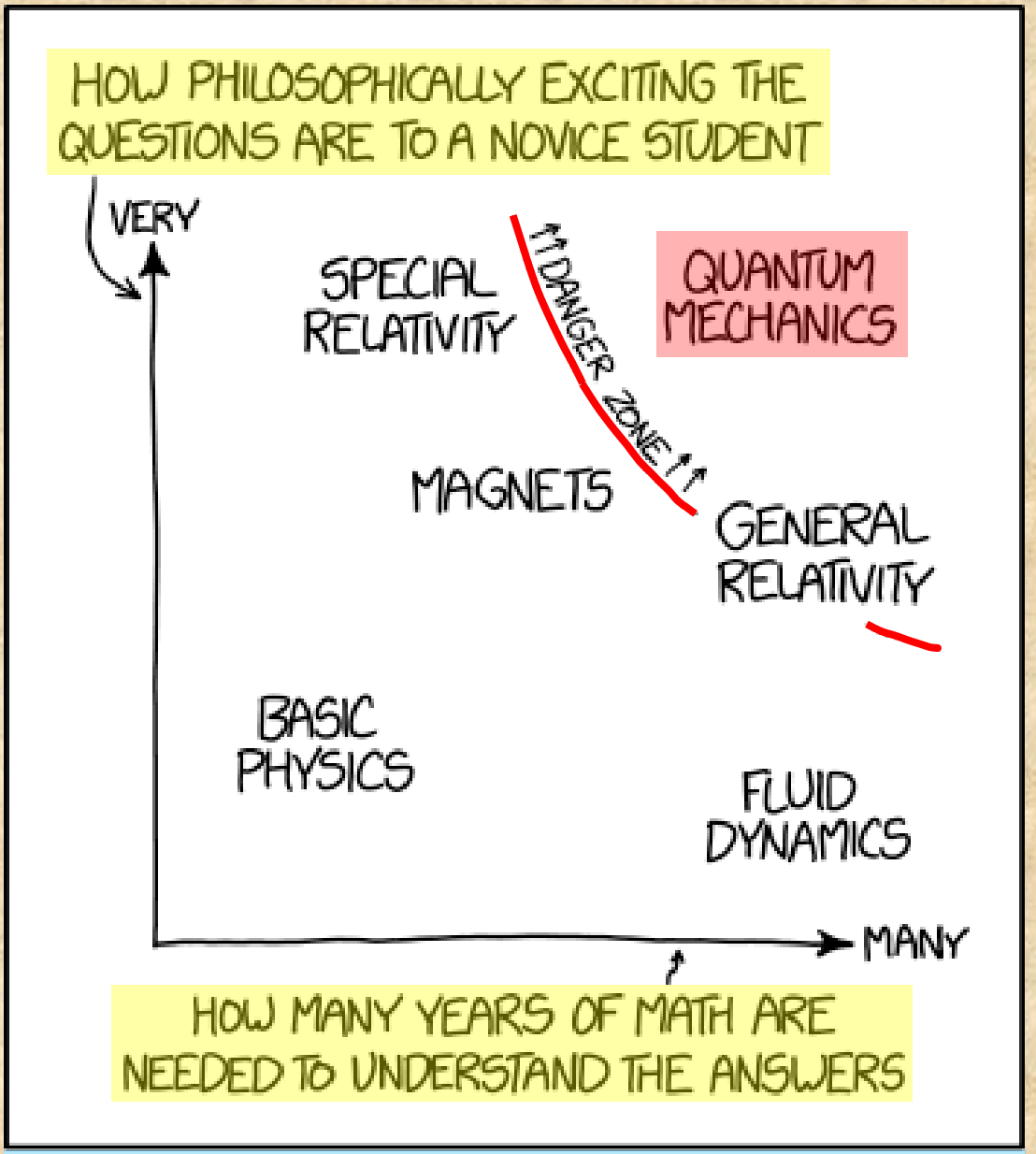
Escaping Schrödinger's Swamp: Putting the Right Spin on Quantum Mechanics

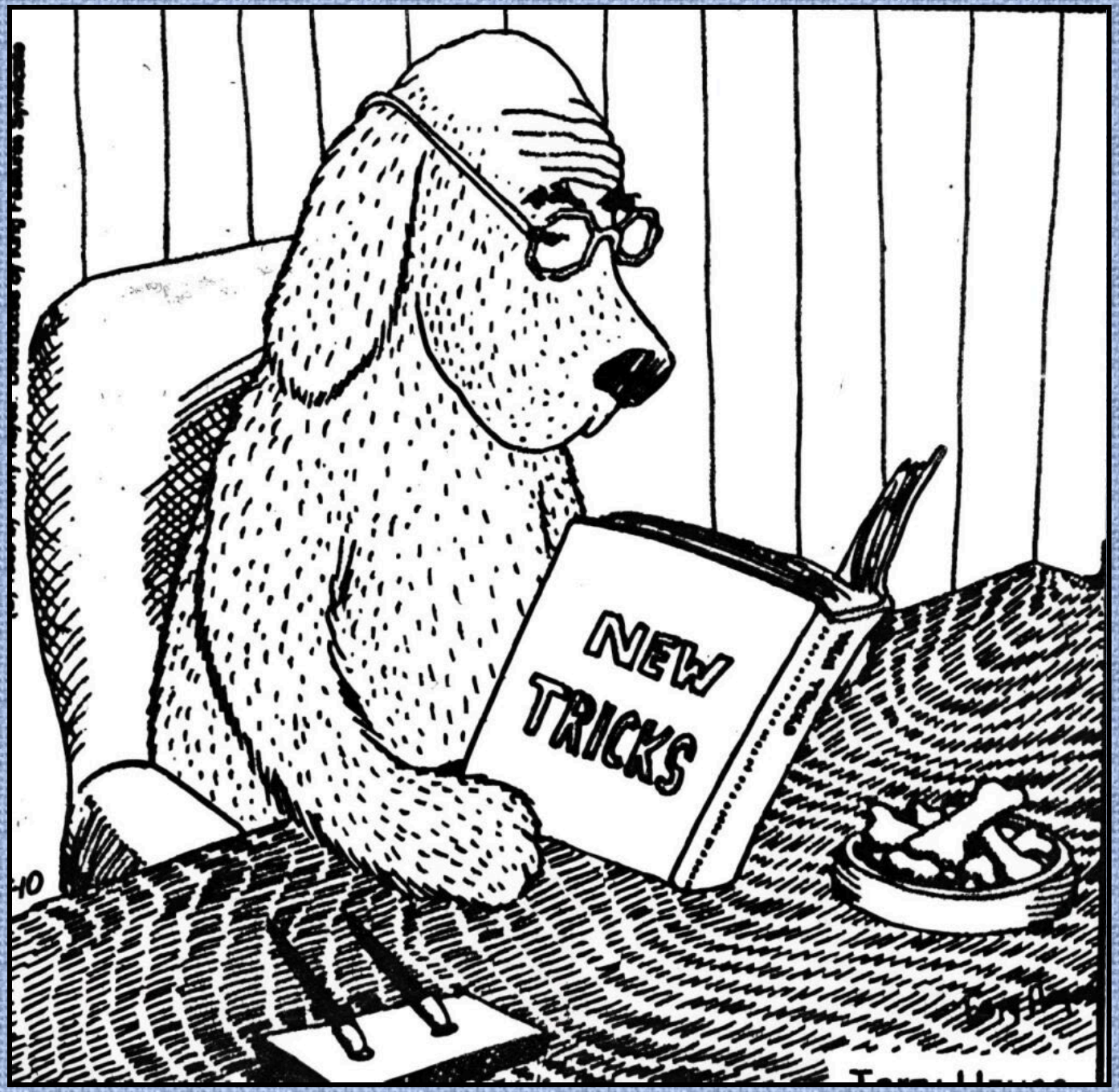
Gerald Feldman[1], Rahul Simha[2], Denis F. Cioffi[1]

[1]Dept. of Physics [2]Dept. of Computer Science





WHY SO MANY PEOPLE HAVE WEIRD IDEAS ABOUT QUANTUM MECHANICS

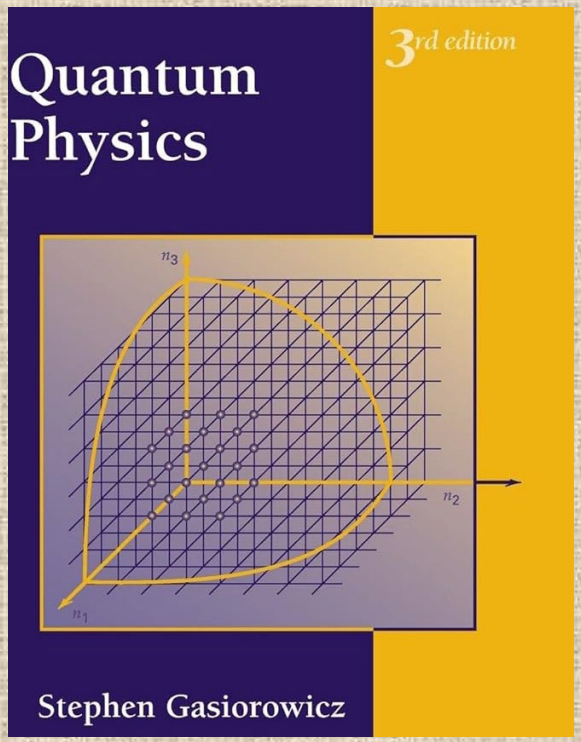

NEW
TRICKS

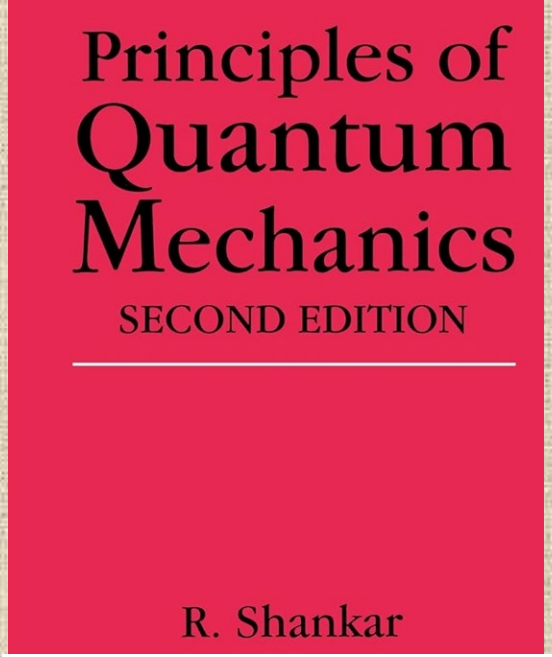
Basics of the Course

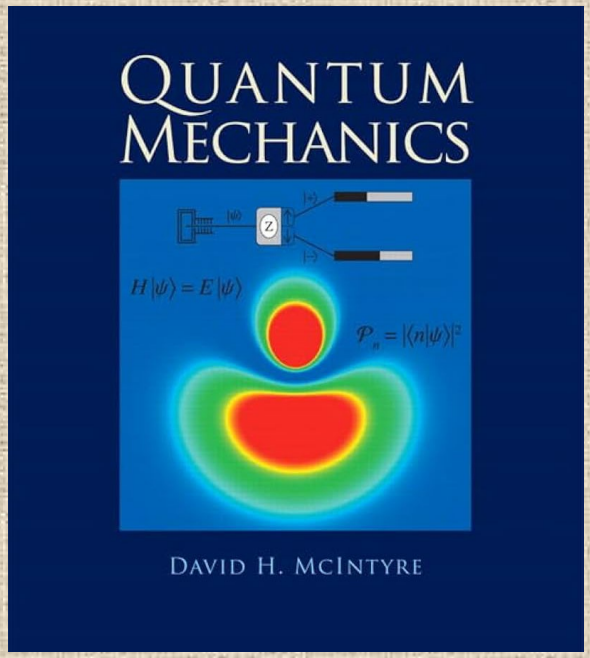
Quantum Mechanics

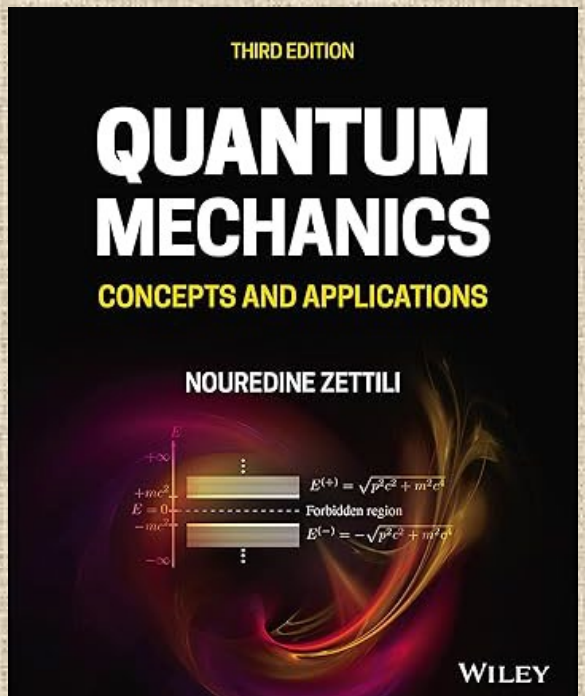
- **Modern Physics**: usually in 2nd undergraduate year
  - ✓ Special Relativity
  - ✓ **Quantum Physics**
  - ✓ Atomic Physics
  - ✓ Selected topics (nuclear, solid state, molecular, astro, etc.)

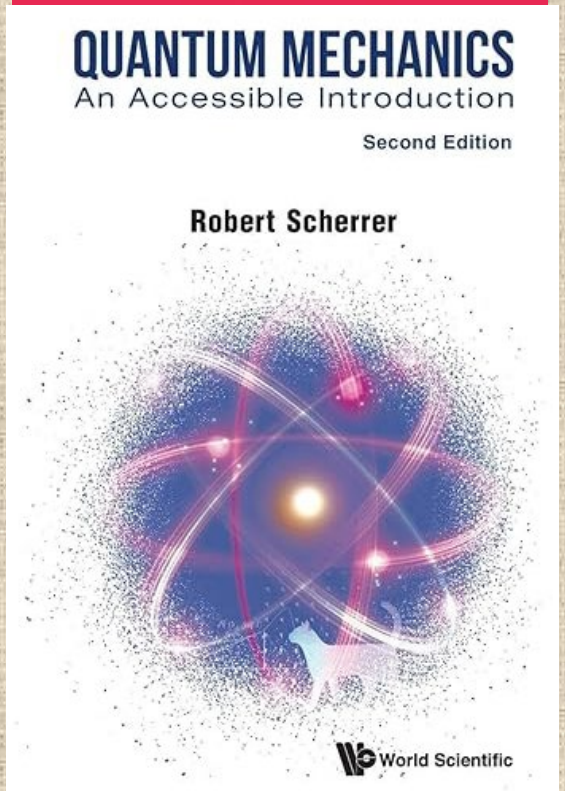
- **Quantum Mechanics**: usually in 3rd or 4th year
  - ✓ can be a one-semester or two-semester course
  - ✓ QM course at George Washington University
    - one semester – class scheduled twice a week (75 minutes)
    - primary instructor (GF), Friday tutor (RS), consultant (DFC)
    - students: four physics majors in Fall 2026 (six in Fall 2025)
    - three exams (70%), five quizzes (20%), weekly homework (10%)
    - supplementary session on Friday (90 minutes)

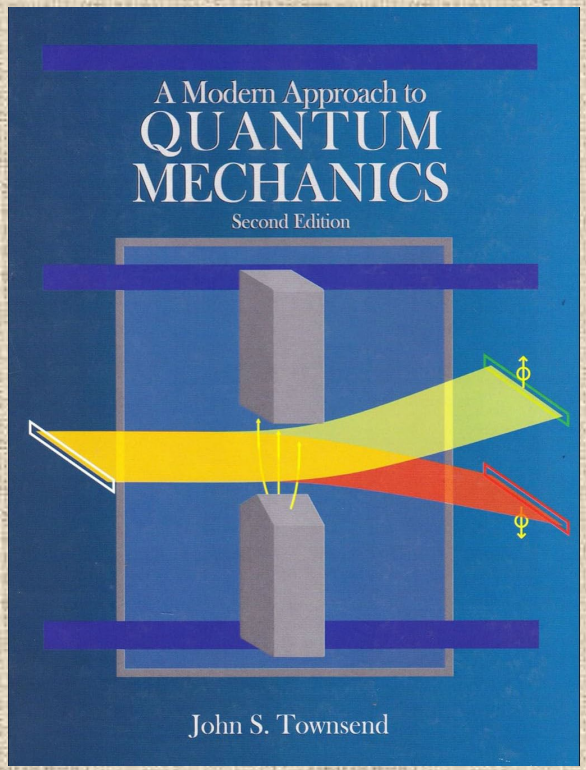

Quantum Physics
3rd edition
n_3
n_2
n_1
Stephen Gasiorowicz

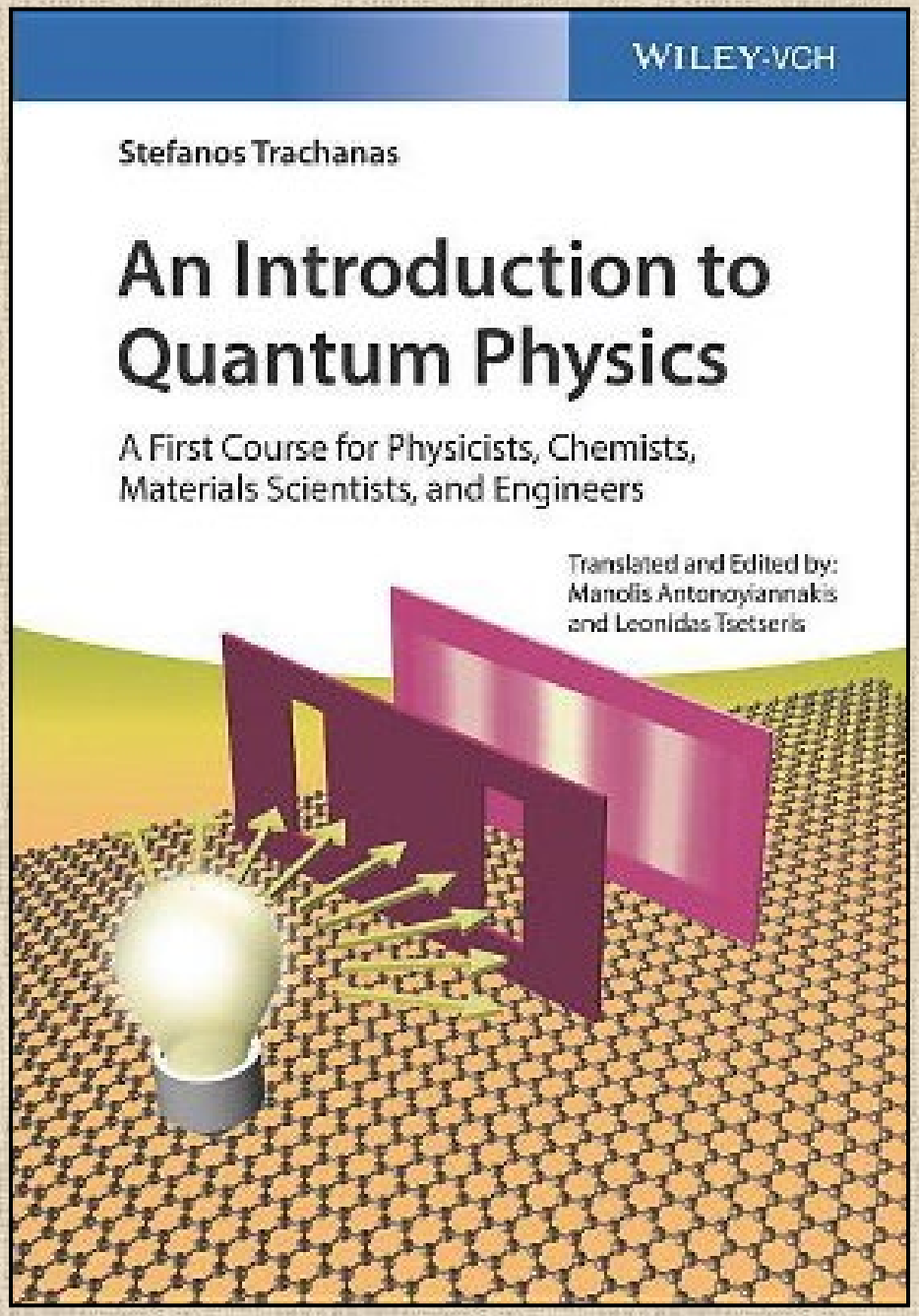

Principles of Quantum Mechanics
SECOND EDITION
R. Shankar


QUANTUM MECHANICS
$H|\psi\rangle = E|\psi\rangle$
$\mathcal{P}_n = |\langle n|\psi\rangle|^2$
DAVID H. MCINTYRE


THIRD EDITION
QUANTUM MECHANICS
CONCEPTS AND APPLICATIONS
NOUREDINE ZETTILI
$E^{(+)} = \sqrt{p^2c^2 + m^2c^4}$
Forbidden region
$E^{(-)} = -\sqrt{p^2c^2 + m^2c^4}$
WILEY


QUANTUM MECHANICS
An Accessible Introduction
Second Edition
Robert Scherrer
World Scientific


A Modern Approach to
QUANTUM MECHANICS
Second Edition
John S. Townsend


WILEY-VCH
Stefanos Trachanas
An Introduction to Quantum Physics
A First Course for Physicists, Chemists, Materials Scientists, and Engineers
Translated and Edited by:
Manolis Antonoyiannakis
and Leonidas Tsetseris

A. Buzzell, T. Atherton, R. Barthelemy
(Phys. Rev. PER 2025)
Quantum Mechanics

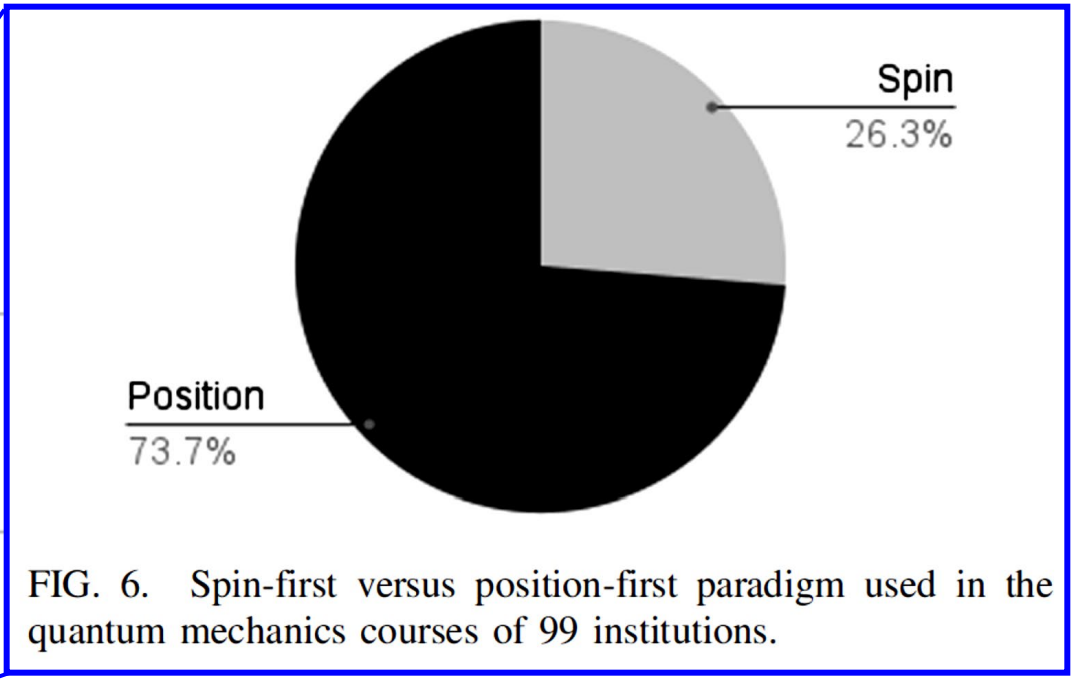
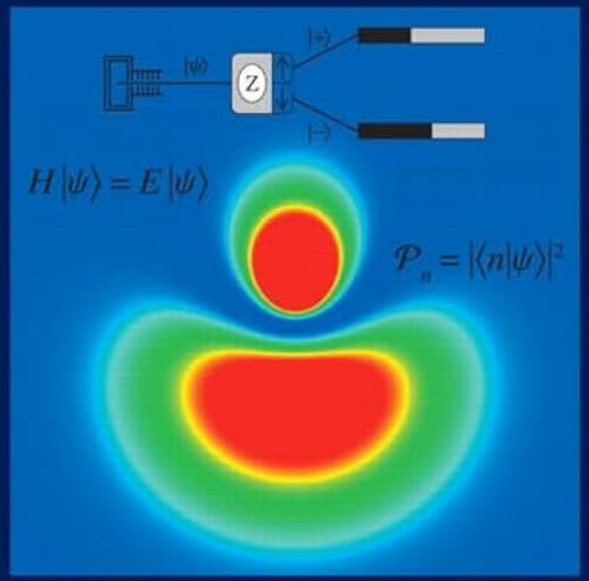


FIG. 6. Spin-first versus position-first paradigm used in the quantum mechanics courses of 99 institutions.

FIG. 7. Textbooks used by quantum mechanics courses at 99 institutions.

QUANTUM MECHANICS



DAVID H. MCINTYRE

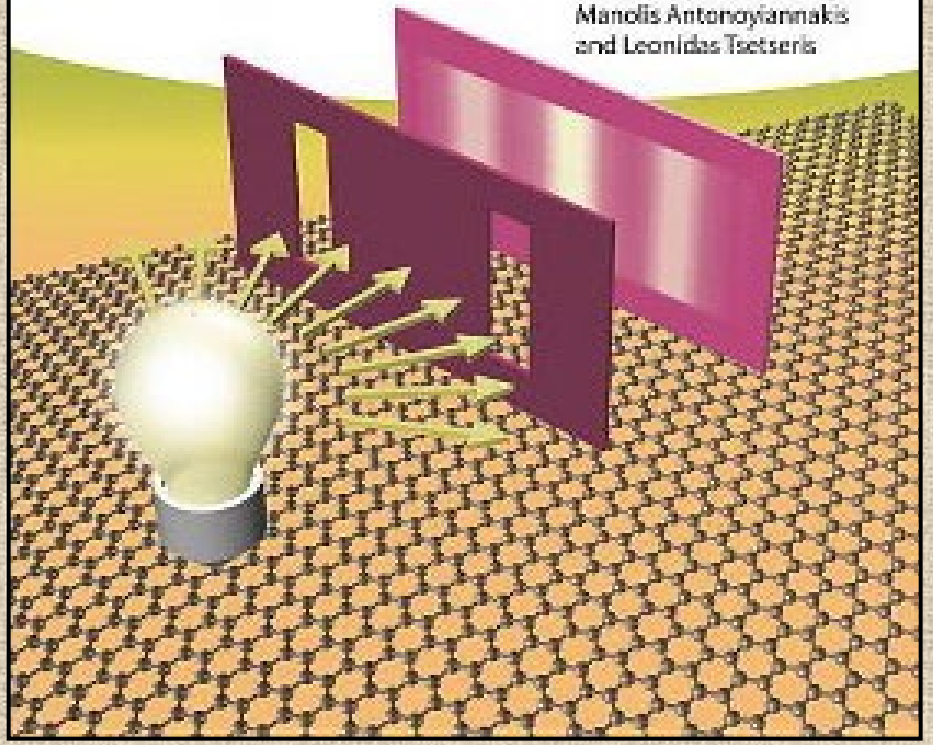
WILEY-VCH

Stefanos Trachanas

An Introduction to Quantum Physics

A First Course for Physicists, Chemists, Materials Scientists, and Engineers

Translated and Edited by:
Manolis Antonoyiannakis
and Leonidas Tsetseris



Why Spins First?

- **Two primary objectives**
  - ✓ to immerse students in **quantum aspects** of physics by focusing on simple measurements that have **no classical explanation**
  - ✓ to give students early experience and extensive practice with the mechanics of QM **using Dirac notation and matrices**
  - ✓ simplicity of spin-½ systems allows students to focus on these new features, which run counter to classical mechanics
- **Employ the Stern-Gerlach experiment for two reasons**
  - ✓ demonstrates **how QM works in principle** by illustrating the postulates of QM
  - ✓ demonstrates **how QM works in practice** through the use of Dirac and matrix notation to solve problems

Through simple examples, we focus on the principles, rather than having the complexity of the physics obscure these new aspects.

Being dissatisfied with teaching quantum mechanics in the traditional wave mechanics way, I have long been on the lookout for an up-to-date text **starting with discrete instead of continuous systems**, at a level suitable for physics undergraduates who have completed an introductory modern physics course.

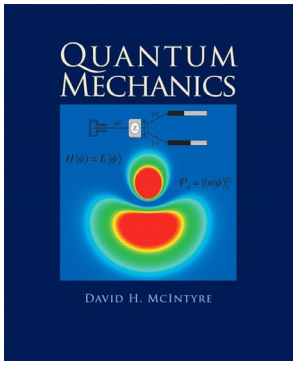
Starting with **simple two-level systems** such as photon polarization, the Mach-Zehnder interferometer, and **Stern-Gerlach experiments** may have multiple advantages compared with the traditional wave mechanical approach.

One limitation of such an approach is that **it can appear more abstract** and link less easily to previously encountered physics such as classical waves. **However, this may be an advantage as analogy with classical systems can lead to incorrect, classical interpretations of quantum phenomena.**

book review by Antje Kohnle (American Journal of Physics 2012)

1 ■ Stern-Gerlach Experiments

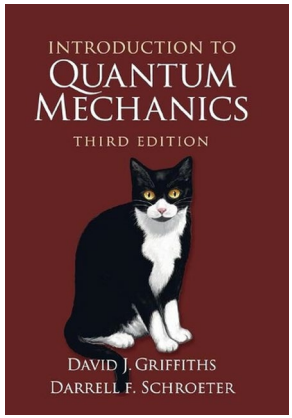
1.1 Stern-Gerlach Experiment 1
1.1.1 Experiment 1 5
1.1.2 Experiment 2 6
1.1.3 Experiment 3 7
1.1.4 Experiment 4 8
1.2 Quantum State Vectors 10
1.2.1 Analysis of Experiment 1 16
1.2.2 Analysis of Experiment 2 16
1.2.3 Superposition States 19
1.3 Matrix Notation 22
1.4 General Quantum Systems 25
1.5 Postulates 27
Summary 28
Problems 29
Resources 32
Activities 32
Further Reading 33



2 ■ Operators and Measurement

2.1 Operators, Eigenvalues, and Eigenvectors 34
2.1.1 Matrix Representation of Operators 37
2.1.2 Diagonalization of Operators 38
2.2 New Operators 41
2.2.1 Spin Component in a General Direction 41
2.2.2 Hermitian Operators 44
2.2.3 Projection Operators 44
2.2.4 Analysis of Experiments 3 and 4 47
2.3 Measurement 50
2.4 Commuting Observables 54
2.5 Uncertainty Principle 56
2.6 S^2 Operator 57
2.7 Spin-1 System 59

1 The Wave Function
1.1 The Schrödinger Equation
1.2 The Statistical Interpretation
1.3 Probability
1.3.1 Discrete Variables
1.3.2 Continuous Variables
1.4 Normalization
1.5 Momentum
1.6 The Uncertainty Principle
Further Problems on Chapter 1

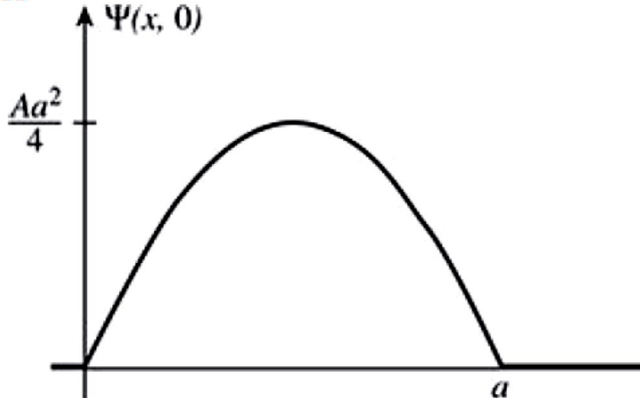


2 Time-Independent Schrödinger Equation
2.1 Stationary States
2.2 The Infinite Square Well
2.3 The Harmonic Oscillator
2.3.1 Algebraic Method
2.3.2 Analytic Method
2.4 The Free Particle
2.5 The Delta-Function Potential
2.5.1 Bound States and Scattering States
2.5.2 The Delta-Function Well
2.6 The Finite Square Well
Further Problems on Chapter 2

Example 2.2

A particle in the infinite square well has the initial wave function

$$\Psi(x,0) = Ax(a-x), \quad (0 \le x \le a)$$



$$c_n = \sqrt{\frac{2}{a}}\int_0^a \sin\left(\frac{n\pi}{a}x\right)\sqrt{\frac{30}{a^5}}\,x(a-x)\,dx$$

$$= \frac{2\sqrt{15}}{a^3}\left[a\int_0^a x\sin\left(\frac{n\pi}{a}x\right)dx - \int_0^a x^2\sin\left(\frac{n\pi}{a}x\right)dx\right]$$

$$= \frac{2\sqrt{15}}{a^3}\left\{a\left[\left(\frac{a}{n\pi}\right)^2\sin\left(\frac{n\pi}{a}x\right) - \frac{ax}{n\pi}\cos\left(\frac{n\pi}{a}x\right)\right]\Bigg|_0^a\right.$$

$$\left. - \left[2\left(\frac{a}{n\pi}\right)^2 x\sin\left(\frac{n\pi}{a}x\right) - \frac{(n\pi x/a)^2 - 2}{(n\pi/a)^3}\cos\left(\frac{n\pi}{a}x\right)\right]\Bigg|_0^a\right\}$$

$$= \frac{2\sqrt{15}}{a^3}\left[-\frac{a^3}{n\pi}\cos(n\pi) + a^3\frac{(n\pi)^2-2}{(n\pi)^3}\cos(n\pi) + a^3\frac{2}{(n\pi)^3}\cos(0)\right]$$

$$= \frac{4\sqrt{15}}{(n\pi)^3}[\cos(0) - \cos(n\pi)]$$

$$= \begin{cases} 0, & n \text{ even}, \\ 8\sqrt{15}/(n\pi)^3, & n \text{ odd}. \end{cases}$$

A. Buzzell, T. Atherton, R. Barthelemy
(Phys. Rev. PER 2025)
Modern Physics

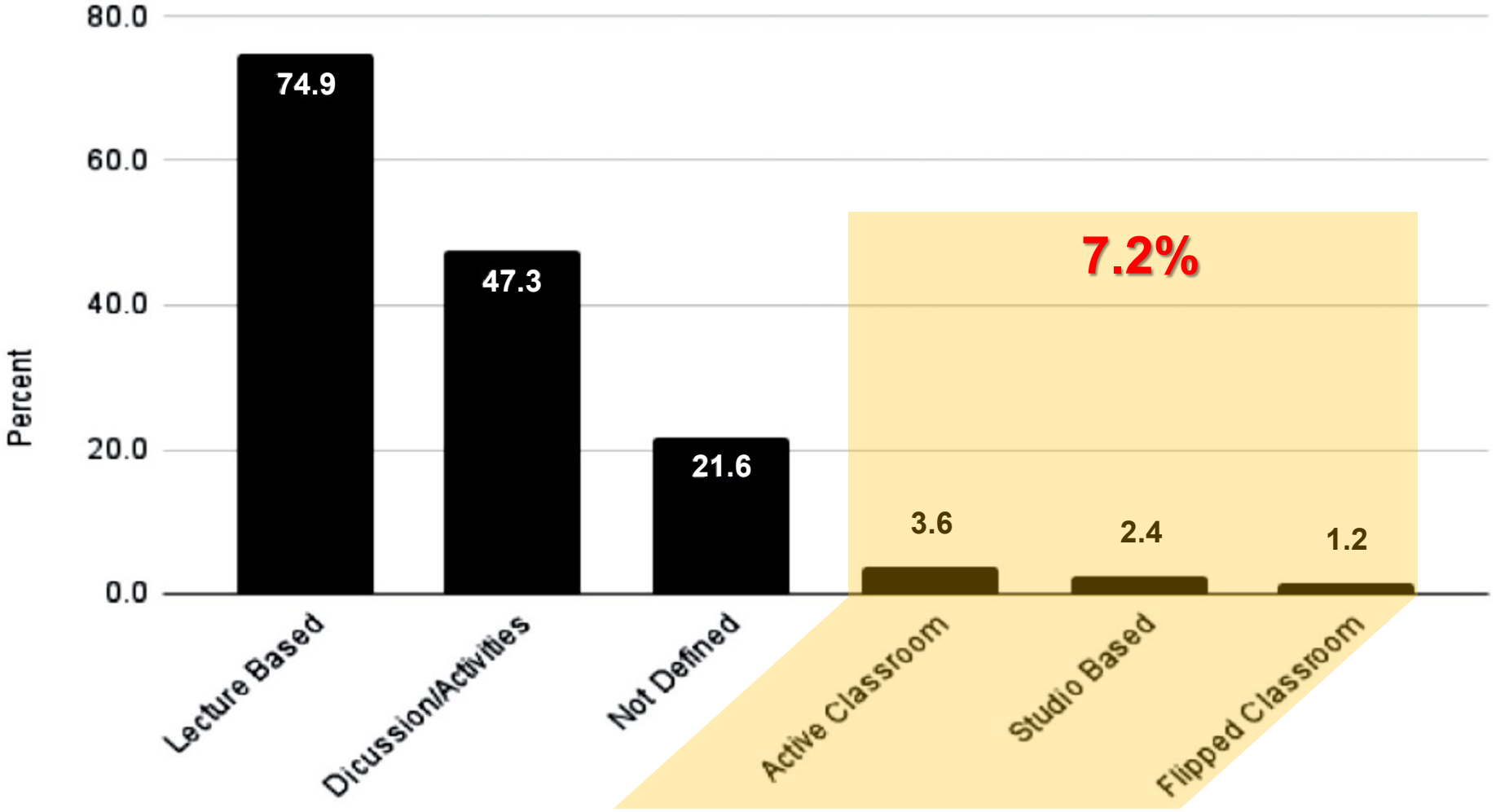


FIG. 7. Pedagogical approach utilized.

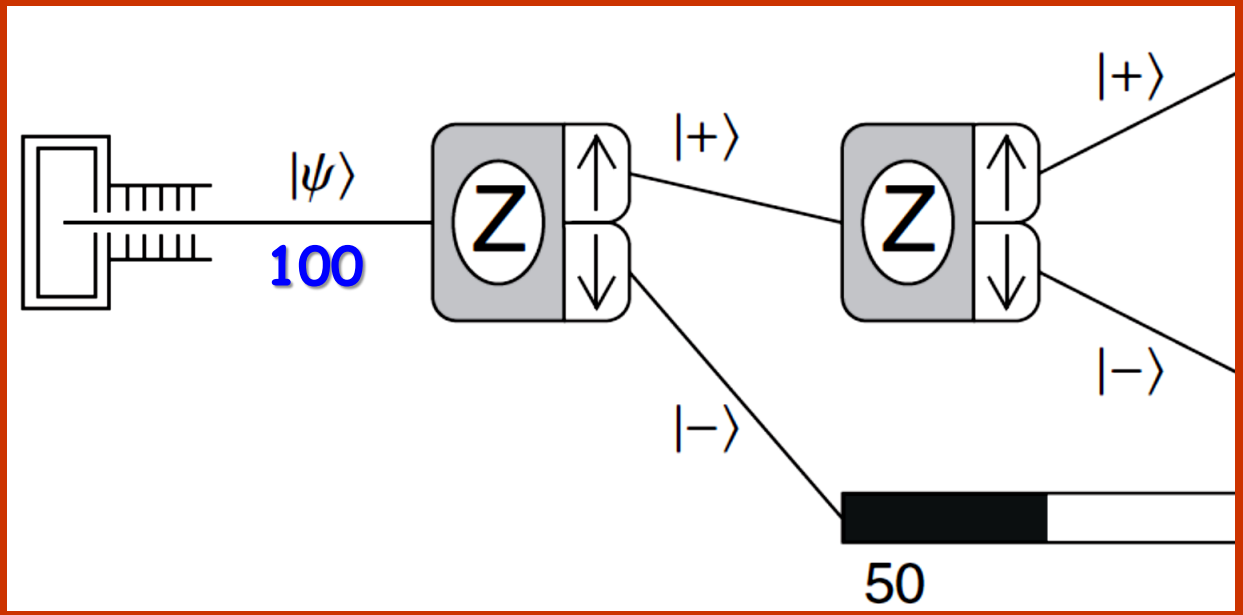
Activities in the Course

ConcepTest 2 Spin and SG Devices

What is the result of this Experiment #1 with a spin ½ beam for the 2nd SG device?

1. 50, 0
2. 50, 50
3. 25, 25
4. 0, 50
5. 25, 0

$$Probability = |\langle final|initial\rangle|^2$$



$$P_+ = |\langle +|+\rangle|^2 = 1$$

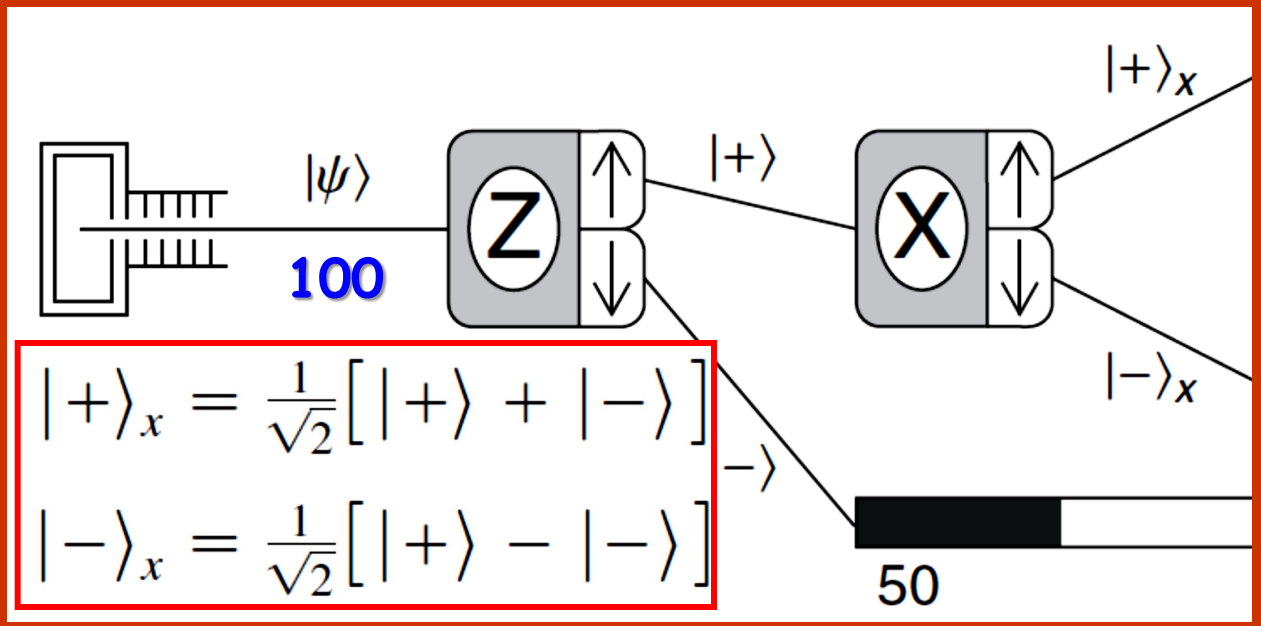
$$P_- = |\langle -|+\rangle|^2 = 0$$

ConcepTest 3 Spin and SG Devices

What is the result of this Experiment #2 with a spin ½ beam for the 2nd SG device?

1. 50, 0
2. 50, 50
3. 25, 25
4. 0, 50
5. 0, 0

$$Probability = |\langle final|initial\rangle|^2$$



$$P_{+x} = |\langle +_x|+\rangle|^2 = \frac{1}{2}$$

$$P_{-x} = |\langle -_x|+\rangle|^2 = \frac{1}{2}$$

$$\hat{\mathbf{i}}\cdot\hat{\mathbf{i}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{k}}\cdot\hat{\mathbf{k}} = 1 \qquad \textit{normalization}$$

$$\hat{\mathbf{i}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{i}}\cdot\hat{\mathbf{k}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{k}} = 0 \qquad \textit{orthogonality}$$

$$\mathbf{A} = a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}} + a_z\hat{\mathbf{k}} \qquad \textit{completeness}$$

spin basis vectors:

$|+\rangle$ **spin up**

$|-\rangle$ **spin down**

$$\left.\begin{aligned}\langle +|+\rangle &= 1\\ \langle -|-\rangle &= 1\end{aligned}\right\} \qquad \textit{normalization}$$

$$\left.\begin{aligned}\langle +|-\rangle &= 0\\ \langle -|+\rangle &= 0\end{aligned}\right\} \qquad \textit{orthogonality}$$

$$|\psi\rangle = a|+\rangle + b|-\rangle \qquad \textit{completeness}$$

Determine the initial spin state $|\psi\rangle$ of the particle.

$$\mathcal{P}_{+} = 0.9 \qquad \mathcal{P}_{+y} = 0.2$$

$$\mathcal{P}_{-} = 0.1 \qquad \mathcal{P}_{-y} = 0.8$$

The initial state vector can be written in a general form: $|\psi\rangle = a|+\rangle + b|-\rangle$

$$|\langle +|\psi\rangle|^2 = |\langle +|(a|+\rangle + b|-\rangle)\rangle|^2 = 0.9 \quad \Rightarrow\Rightarrow\Rightarrow \quad |a+0|^2 = 0.9 \quad \Rightarrow\Rightarrow\Rightarrow \quad a = \frac{3}{\sqrt{10}}$$

$$|\langle -|\psi\rangle|^2 = |\langle -|(a|+\rangle + b|-\rangle)\rangle|^2 = 0.1 \quad \Rightarrow\Rightarrow\Rightarrow \quad |0+b|^2 = 0.1 \quad \Rightarrow\Rightarrow\Rightarrow \quad b = \frac{1}{\sqrt{10}}e^{i\phi}$$

$$|\langle +y|\psi\rangle|^2 = \left|\left\langle \frac{1}{\sqrt{2}}(\langle +| - i\langle -|)\left|\frac{1}{\sqrt{10}}(3|+\rangle + e^{i\phi}|-\rangle)\right\rangle\right|^2 = 0.2$$

$$= \frac{1}{20}\left|3 - ie^{i\phi}\right|^2 = \frac{1}{20}(3 - ie^{i\phi})(3 + ie^{-i\phi}) = \frac{1}{20}\left(9 + 1 - 3i\left(e^{i\phi} - e^{-i\phi}\right)\right) = 0.2$$

$$= \frac{1}{20}(10 - 3i \cdot 2i\sin\phi) = \frac{1}{20}(10 + 6\sin\phi) = 0.2$$

$$10 + 6\sin\phi = 4 \quad \Rightarrow\Rightarrow\Rightarrow \quad 6\sin\phi = -6 \quad \Rightarrow\Rightarrow\Rightarrow \quad \sin\phi = -1 \quad \Rightarrow\Rightarrow\Rightarrow \quad \phi = -\frac{\pi}{2}$$

The final form of the state vector is given as: $|\psi\rangle = \frac{1}{\sqrt{10}}(3|+\rangle + e^{i\phi}|-\rangle) = \frac{1}{\sqrt{10}}(\mathbf{3}|+\rangle - \boldsymbol{i}|-\rangle)$

9. (3 pts) The state of a spin $\frac{1}{2}$ particle is described by: $\psi = N\begin{pmatrix} i \\ 2 - i \end{pmatrix}$ where N is a normalization constant. What is the expectation value $\langle s_x \rangle$ of the spin along the x-axis?

a) $-\hbar$ b) $\hbar/2$ c) 0 d) $-\hbar/6$ e) $-\hbar/3$

---

10. (3 pts) For a given state vector of a spin $\frac{1}{2}$ particle, you find that the expectation value for a measurement of the mean value along the x-axis is $\langle s_x \rangle = 0$. If that is the case, then what is the probability $P_{x\uparrow}$ of measuring **spin up** along the x-axis for this state?

a) 0 b) 1/4 c) 1/2 d) 2/3 e) 1

---

11. (3 pts) For a given state vector of a spin $\frac{1}{2}$ particle, you find that the expectation value for a measurement of the mean value along the x-axis is $\langle s_x \rangle = +\frac{\hbar}{2}$. If that is the case, then what is the probability $P_{z\downarrow}$ of measuring **spin down** along the z-axis for this state?

a) 0 b) 1/4 c) 1/2 d) 2/3 e) 1

Friday Sessions -- motivation

- Low-stakes supplemental recitation

- Key concepts rooted in complex-vector linear algebra
  - ✓ abstract state vectors, operators, measurement outcomes, amplitudes/probability, basis states, projection operators
  - ✓ standard linear algebra course tends to focus on row reduction, transposes, inverses, determinants

Contextualize/review linear algebra background for complex vectors.

Friday Sessions -- approach

- Review (gently) the necessary linear algebra background
  - standard linear algebra course hardly covers what is needed
- Review other useful (possibly forgotten?) math info
  - ✓ complex numbers, differential equations, waves, delta functions
- Reinforce core concepts by seeing them in a different way and at a different (later) time $\rightarrow\rightarrow$ **spaced repetition**
- Encourage accessible, low-stakes friendly atmosphere, with **plenty of Q&A**
- Cover interesting QM aspects outside of class!
  - ✓ key experiments, entanglement, EPR, delayed choice

Conclusions

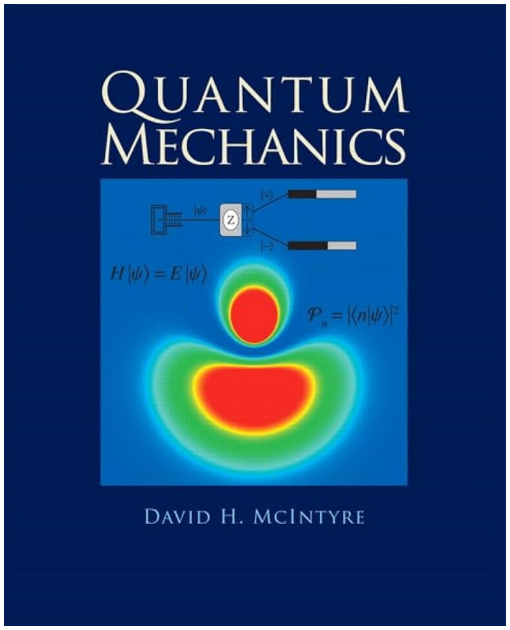
- Switched to a spins-first approach for upper-level QM
  - Initially treat very basic two-state systems
  - introduce QM concepts without relying on calculus
  - discrete spin-1/2 systems are easier to handle
  - enables a comprehensive coverage of basic QM concepts
- Supplemental Friday enrichment sessions
  - review linear algebra necessary for QM
  - informal atmosphere encourages student interaction
  - student engagement is (very) high!
- Personal conclusions (my own…)
  - students' facility with the QM material is really improved
  - this actually seems to be working!

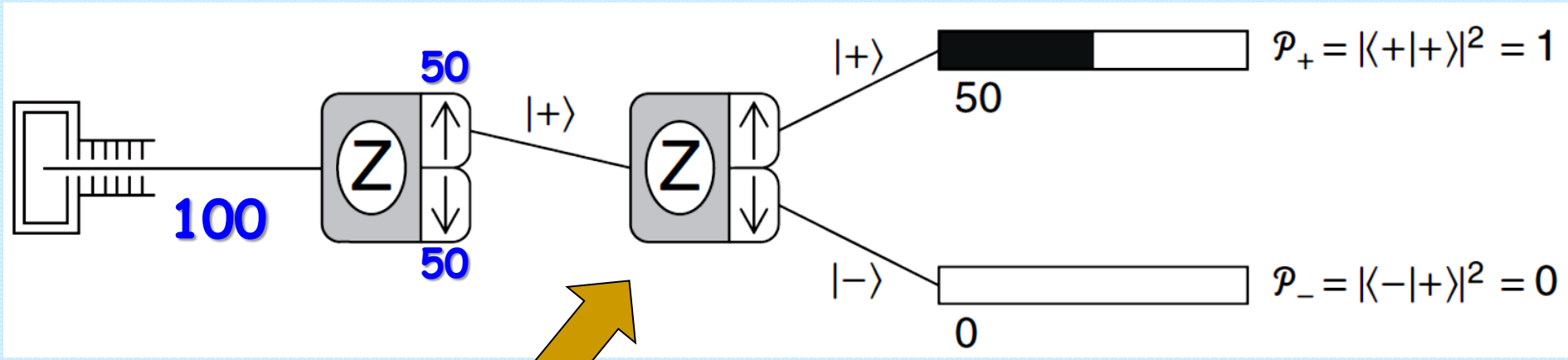
The End

The aim of the spins-first approach is twofold: (1) To immediately immerse students in the inherently **quantum** mechanical aspects of physics by focusing on simple measurements that have no classical explanation, and (2) To give students early and extensive experience with the **mechanics** of quantum mechanics in the forms of Dirac and matrix notation.

The simplicity of the spin-1/2 and spin-1 systems allows the students to focus on these new features, which run counter to classical mechanics.

We use the Stern-Gerlach experiment to learn about quantum mechanics theory for two primary reasons: (1) It demonstrates how quantum mechanics works in principle by illustrating the postulates of quantum mechanics, and (2) it demonstrates how quantum mechanics works in practice through the use of Dirac notation and matrix mechanics to solve problems. By using a simple example, we can focus on the principles and the new mathematics, rather than having the complexity of the physics obscure these new aspects.

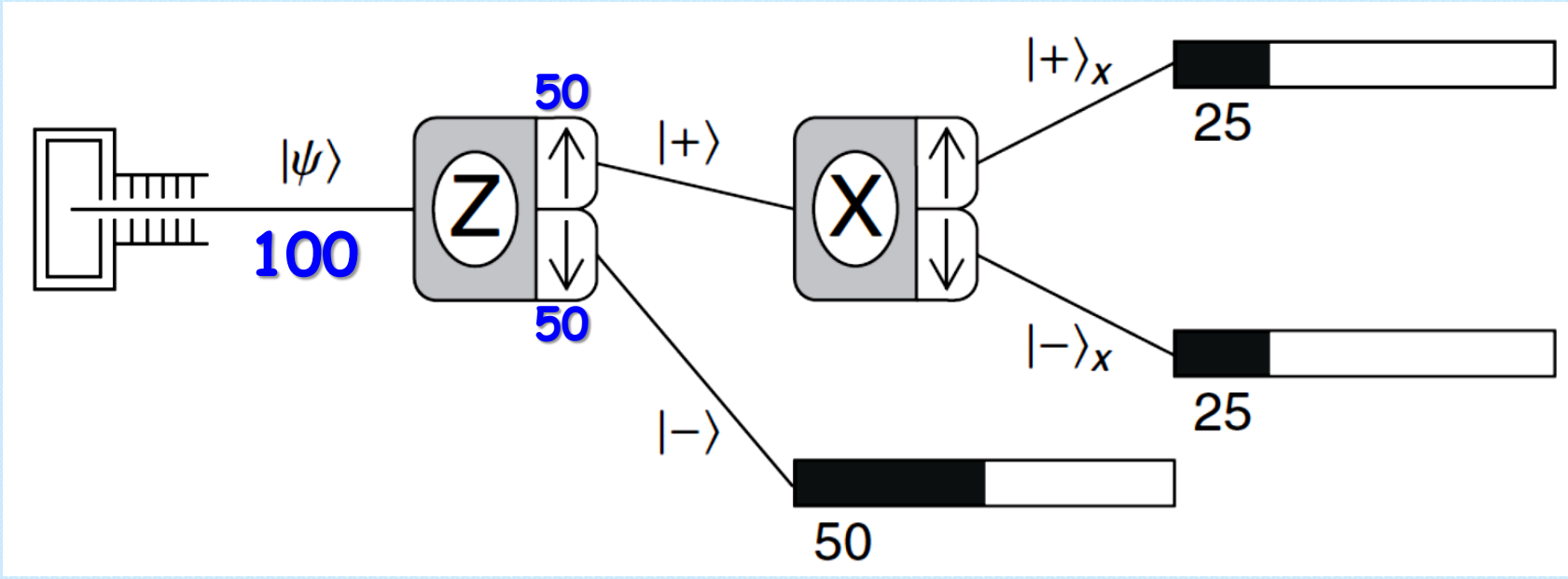




$$Probability = |\langle final|initial\rangle|^2$$

$$P_+ = |\langle +|+\rangle|^2 = 1$$

$$P_- = |\langle -|+\rangle|^2 = 0$$



$$|+\rangle_x = \frac{1}{\sqrt{2}}\left[|+\rangle + |-\rangle\right]$$

$$|-\rangle_x = \frac{1}{\sqrt{2}}\left[|+\rangle - |-\rangle\right]$$

$$Probability = |\langle final|initial\rangle|^2$$

$$P_{+x} = |\langle +_x|+\rangle|^2 = \frac{1}{2}$$

$$P_{-x} = |\langle -_x|+\rangle|^2 = \frac{1}{2}$$

Our approach: a low-stakes supplemental recitation

- Motivation: many of the key concepts are rooted in **complex-vector linear algebra**
  - State as an abstract vector, measurement bases/outcomes, amplitudes, operators, the operator method, projectors, unitary evolution
- Unfortunately, their standard linear algebra course (if they even remember) focused on calculations with:
  - Row-reduction, inverses, determinants, eigenvalues
- Thus, our **Goal #1**: ***supply the complex-vector linear algebra background***
  - Abstract vectors/operators, bases, orthonormality, coefficient determination, Dirac notation, inner/outer products, Hermitian/unitary operators, continuous vectors and operators, Dirac-delta
  - Covered in Chapter 1 of Shankar's book, but that's too intimidating

Our approach: recitation, 90 minutes Friday mornings

- (Revised) **Goal #1**: **gently** *supply the complex-vector linear algebra background*
- **Goal #2**: *Review useful (forgotten? weakly understood?) background* - diff-eq, delta-functions, waves, simple harmonic oscillator
- **Goal #3**: *reinforce main lecture section* by seeing core concepts a different way and later (the power of spaced repetition)
- **Goal #4**: Encourage an accessible, office-hours-like *low-stakes friendly atmosphere*, with plenty of Q&A
- **Goal #5**: Cover interesting QM the main lectures don't have room for:
  - Examples: the key experiments, entanglement, EPR, delayed-choice etc.

Did it work (iteration #2)?

- Fall-2026: Students are responding quite positively!

Developing introductory quantum mechanics with two-level systems (two-level atoms, spin ½ particles, interferometers, qubits) has multiple advantages:

- It immediately immerses students in inherently quantum mechanical aspects of physics (complementarity, incompatible observables, single photon interference) by focusing on experiments that have no classical explanation.
- It is mathematically less challenging, requiring only basic linear algebra such as the manipulation of 2×2 matrices instead of solving integrals and differential equations.
- It allows a direct discussion of interpretive aspects of quantum mechanics (EPR paradox, entanglement, completeness of quantum theory, local hidden variables).
- It allows an inclusion from the start of aspects of quantum information theory (quantum key distribution, teleportation, quantum computing).

Antje Kohnle (EJP 2014)