A "Transformative" Assignment on Rotation: A Seemingly Simple Question Becomes an Epic Four-Part Exploration

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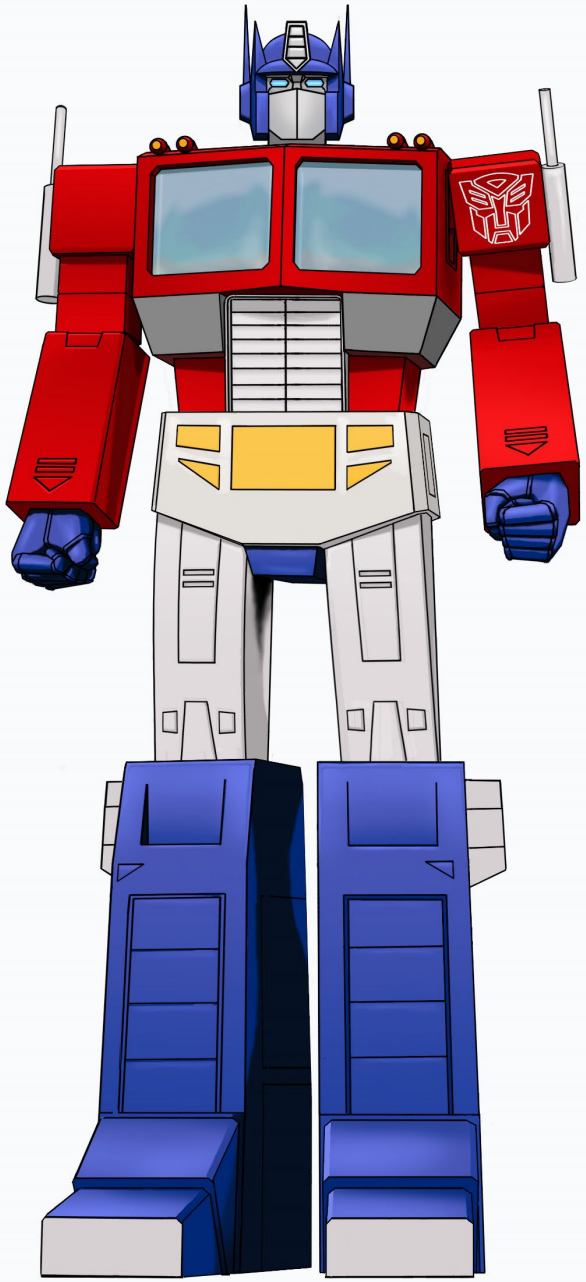
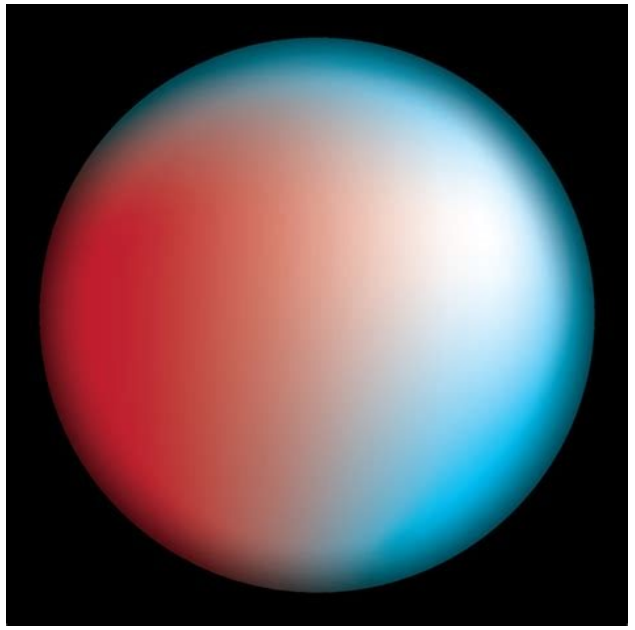




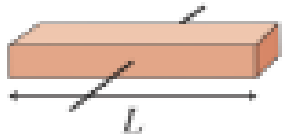
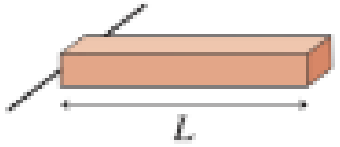
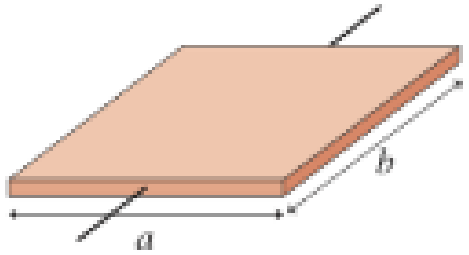
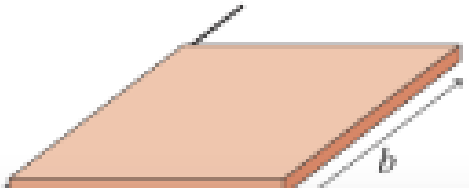
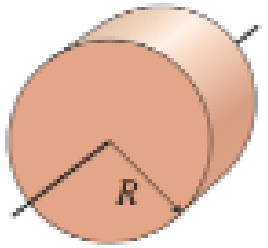
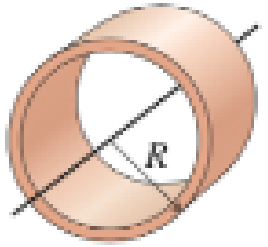
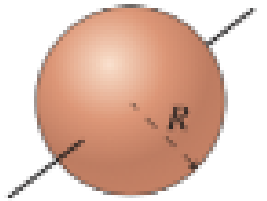
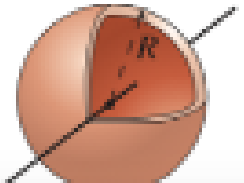
or

?

Optimus Prime, the Transformer hero, is rotating with negligible friction and negligible externally applied forces. He wants to rotate as fast as possible, so he transforms into a shape that is less spread out (like when you pull your arms in to rotate faster on a spinning chair). To end up rotating the fastest, AND to be able to angularly accelerate the fastest with a specific amount of torque, should he transform into a uniform disk / cylinder, a ring / hoop, or a uniform solid sphere?

(first result for “moment of inertia table” search):

TABLE 12.2 Moments of inertia of objects with uniform density

Object and axis	Picture	I
Thin rod, about center		$\frac{1}{12}ML^2$
Thin rod, about end		$\frac{1}{3}ML^2$
Plane or slab, about center		$\frac{1}{12}Ma^2$
Plane or slab, about edge		$\frac{1}{3}Ma^2$
Cylinder or disk, about center		$\frac{1}{2}MR^2$
Cylindrical hoop, about center		MR^2
Solid sphere, about diameter		$\frac{2}{5}MR^2$
Spherical shell, about diameter		$\frac{2}{3}MR^2$

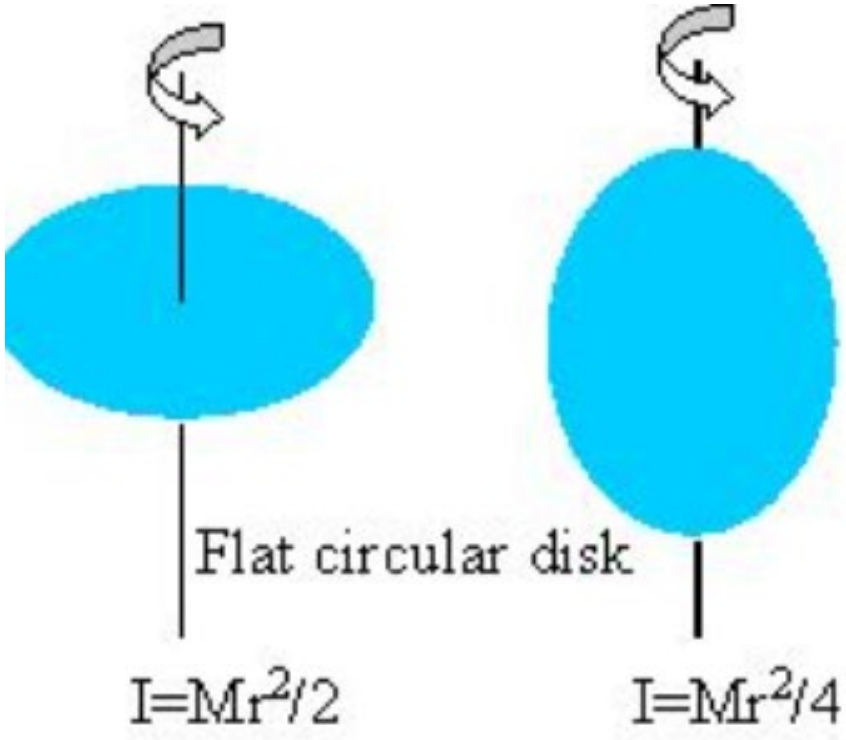
Must understand that angular momentum is conserved and a lower I means higher angular velocity.

Results of Part One (Students' First Attempt):

-Most pick sphere b/c of smaller fraction in eqn for I in table.

-Significant contingent say disk because "higher I means higher angular momentum"

Part 2: Look at this and reconsider.

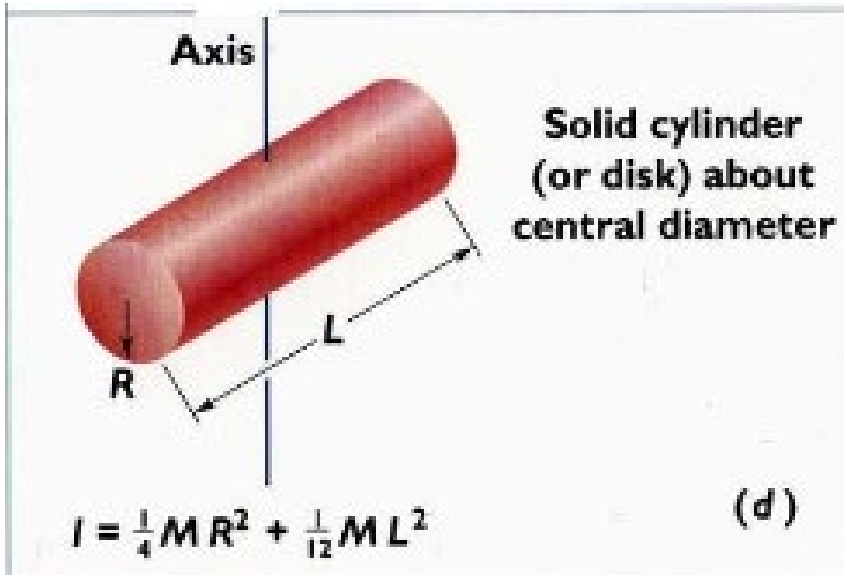


A disk can rotate around multiple axes!

Results:

On Part 2, vast majority of students change their answer to disk, since ¼ is lower than 2/5.

Part 3: Again, reconsider your answer. Realize that a disk is actually just a very short cylinder. For disk / cylinder of non-negligible length:



Qs: In order for the equation I=1/4 MR^2 to apply, what is the assumed length of the disk / cylinder? a. pretty much zero b. equal to R c. 12 times R

If the length of a disk / cylinder approaches zero, its R must approach infinity.

How would a higher R value affect the moment of inertia of a disk, the mass being the same?

If Optimus Prime were to have a much higher R value as a disk rather than a sphere (lets say at least 2 times higher), could he possibly have a lower moment of inertia as a thin disk shape than as a sphere?

Results:

On Part 3, a small minority of students are STILL convinced that he could have a smaller I as a thin disk!

Part 4: … but he would not have a negligibly small length as a cylinder!

Maybe R=L?

If R=L for our cylinder, we could substitute L for R in the above equation and get: $I = 1/4\ MR^2 + 1/12\ MR^2$ **$= 1/3MR^2$**

1/3 < 2/5 so we are back to disk again as our answer?

… but R for this cylinder would not = R for sphere.

Compare I for sphere to I for disk with equal R and L. The math:

Assume an arbitrary volume of 100 arbitrary units for Optimus Prime. Find the radius he would have as a **sphere** by solving $V=4/3\ \pi R^3$ for R: $R = (100*(3/4)/\pi)^{(1/3)}$ = **2.879**

Next, find the L and R values that Optimus as a **cylinder** must have if L=R: $V=L\pi R^2$ becomes $V=\pi R^3$, and $L= R = (100/\pi)^{(1/3)}$ = **3.169**

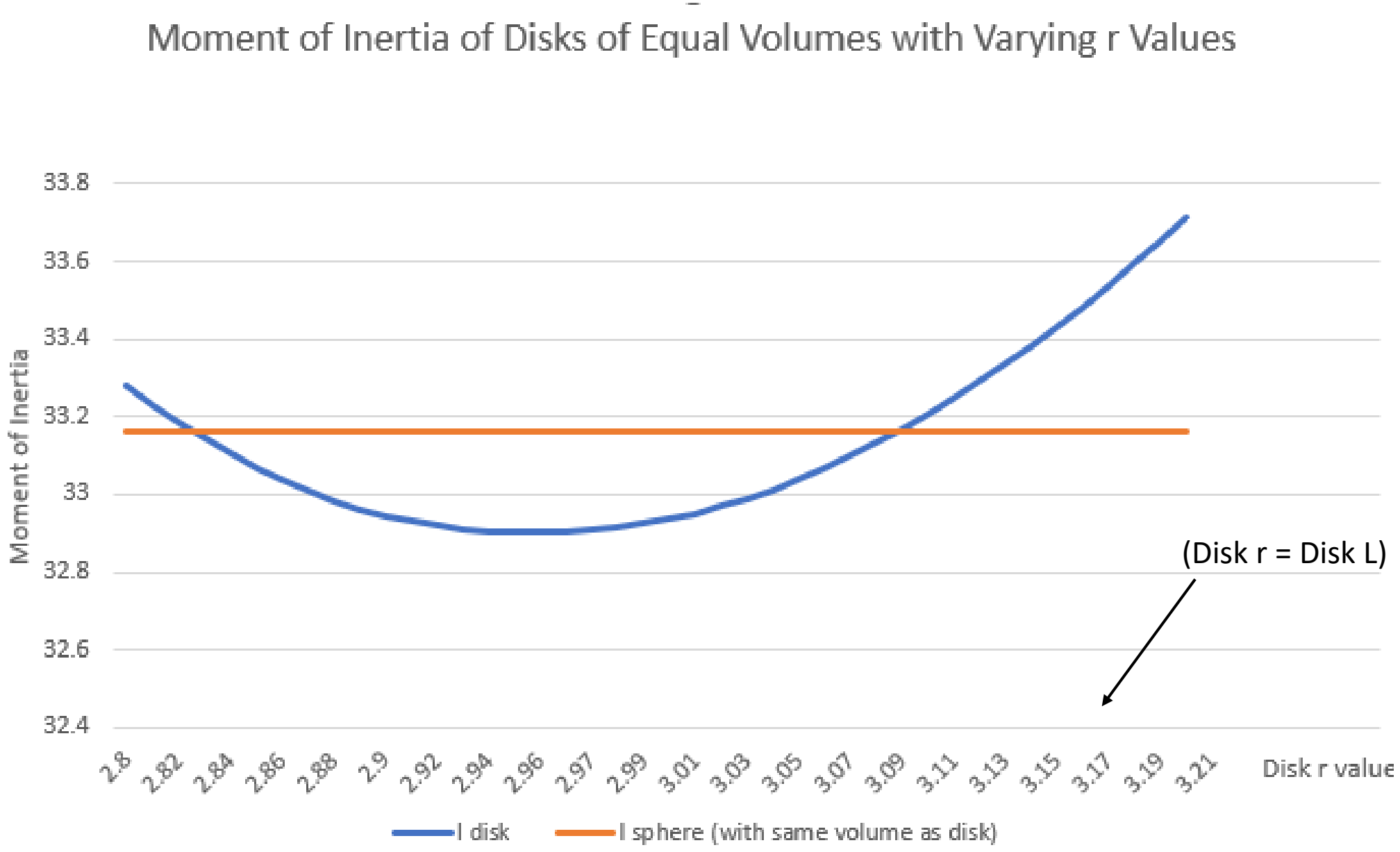
Now, moments of inertia can be compared. For the sphere, we find $I=2/5\ M*2.879^2$ = **3.316M**. For the cylinder, we find $I = 1/4\ M*3.169^2 + 1/12\ M*3.169^2 = 1/3\ M*3.169^2$ = **3.348M**.

The disk/cylinder has 1% higher moment of inertia and we can't conclude it is the best answer!

Part 5: Function Optimization

For Algebra-Based Class:

	A	B	C	D	E	F	G	H	I	J	K
1	For vol of 100	r disk (by increment)	L disk (solved from v=pi r^2 * L)		I values calculated		I disk	I sphere (with same volume as disk)			r sphere
2		2.8	4.060075079		based on arbitrary		33.33684	33.16405			2.879412
3	v=pi r^2 * L	2.81	4.031229166		mass of 10 arbitrary		33.28259	33.16405			2.879412
4	V=4/3 pi r^3	2.82	4.00268958		units:		33.23227	33.16405			2.879412
5		2.83	3.974451999				33.18581	33.16405			2.879412
6		2.84	3.946512177				33.14313	33.16405			2.879412
7		2.85	3.918865943				33.10418	33.16405			2.879412
8		2.86	3.891509196				33.06887	33.16405			2.879412
9		2.87	3.86443791				33.03715	33.16405			2.879412
10		2.88	3.837648126				33.00895	33.16405			2.879412
11		2.89	3.811135956				32.98421	33.16405			2.879412
12		2.9	3.784897577				32.96287	33.16405			2.879412
13		2.91	3.758929231				32.94487	33.16405			2.879412
14		2.92	3.733227226				32.93015	33.16405			2.879412
15		2.93	3.707787932				32.91866	33.16405			2.879412
16		2.94	3.682607781				32.91033	33.16405			2.879412
17		2.95	3.657683266				32.90512	33.16405			2.879412
18		2.96	3.633010936				32.90297	33.16405			2.879412
19	this one	2.962	3.628106433				32.90291	33.16405			2.879412
20		2.97	3.608587402				32.90384	33.16405			2.879412
21		2.98	3.58440933				32.90766	33.16405			2.879412
22		2.99	3.560473442				32.91439	33.16405			2.879412
23		3	3.536776513				32.92399	33.16405			2.879412
24		3.01	3.513315374				32.9364	33.16405			2.879412
25		3.02	3.490086906				32.95159	33.16405			2.879412
26		3.03	3.467088043				32.9695	33.16405			2.879412
27		3.04	3.44431577				32.99009	33.16405			2.879412
28		3.05	3.421767118				33.01333	33.16405			2.879412


Moment of Inertia of Disks of Equal Volumes with Varying r Values
Moment of Inertia
33.8
33.6
33.4
33.2
33
32.8
32.6
32.4
2.8
2.82
2.84
2.86
2.88
2.9
2.92
2.94
2.96
2.97
2.99
3.01
3.03
3.05
3.07
3.09
3.11
3.13
3.15
3.17
3.19
3.21
Disk r value
(Disk r = Disk L)
I disk
I sphere (with same volume as disk)

Part 5: Function Optimization For Calculus-Based Class:

Find the optimum dimensions of a cylinder by finding the minimum of the function $I=1/4\ MR^2 + 1/12\ ML^2$. First, define L in terms of R using $V=L\pi R^2$: $L=V/\pi R^2$.

$I=1/4\ MR^2 + 1/12\ M\ (V/\pi R^2)^2$.

Next, set the derivative of the function, dI/dr, equal to 0, to find the optimal R at the minimum I value, where the tangent line to the curve is horizontal:

$dI/dr = 1/2\ MR + -4/12\ M\ (V^2/\pi^2)R^{-5} = 0$ then cancel M:

$1/2\ R = 1/3\ (V^2/\pi^2)R^{-5}$; $R^{-6} = 3/2\ (V^2/\pi^2)$; $R = (3/2\ (V^2/\pi^2))^{-1/6}$.

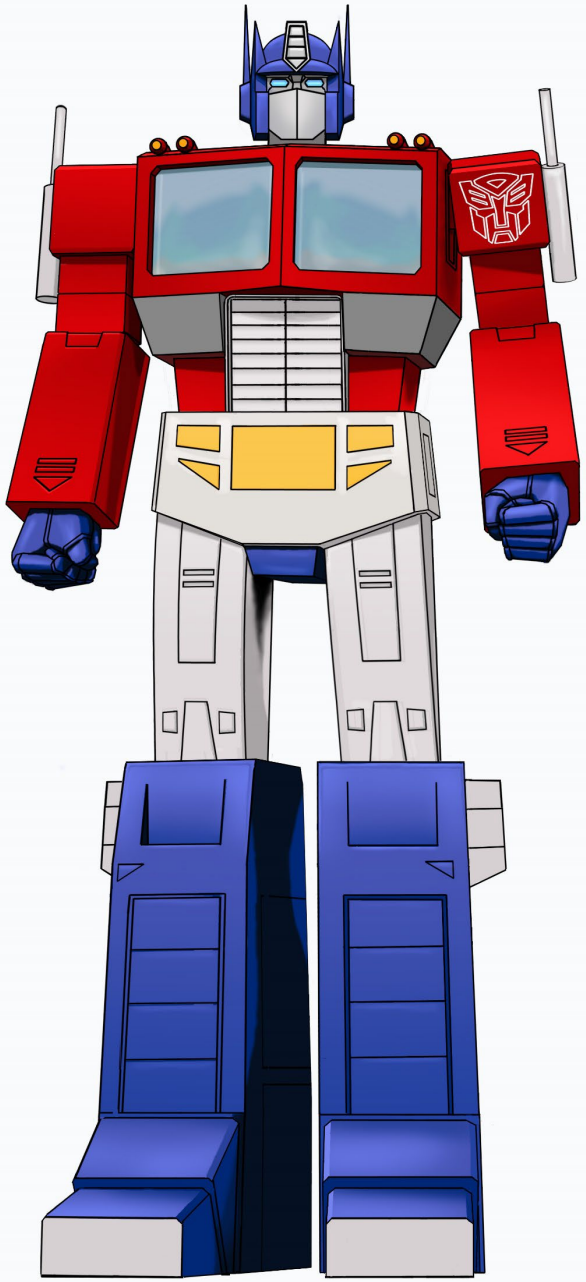
Keeping the value of 100 for the volume gives:

R=**2.962** Length in this case is $L=V/\pi r^2 = 100/(\pi * 2.962^2) = 3.628$.

The moment of inertia of this cylinder comes out to:

$I=1/4\ MR^2 + 1/12\ ML^2$ = **3.290M**

Compare to I of sphere **(3.316M)**



The Final Answer:

Disk wins by < 1% !!!

!!

Results:

Students mostly get walked through parts 4 and 5.

Student Feedback:

Mostly: “Breaking the question down made it easier to understand”

Negative feedback: “Too many parts of the assignment due one after the other on the same weekend I have a (sports game thing).”