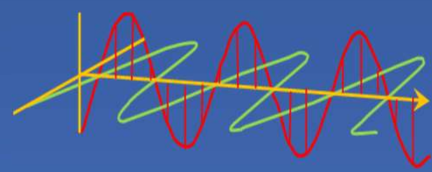
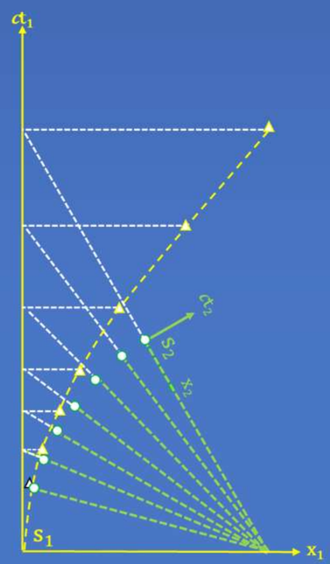
Riding on a Light Beam

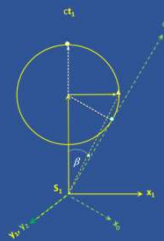


Acceleration and Mass Rise

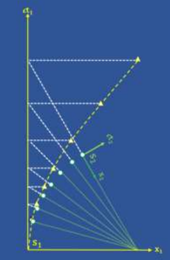


Lewis F. McIntyre
CS-AAPT Spring Meeting
April 1, 2023

mcintyrel@verizon.net

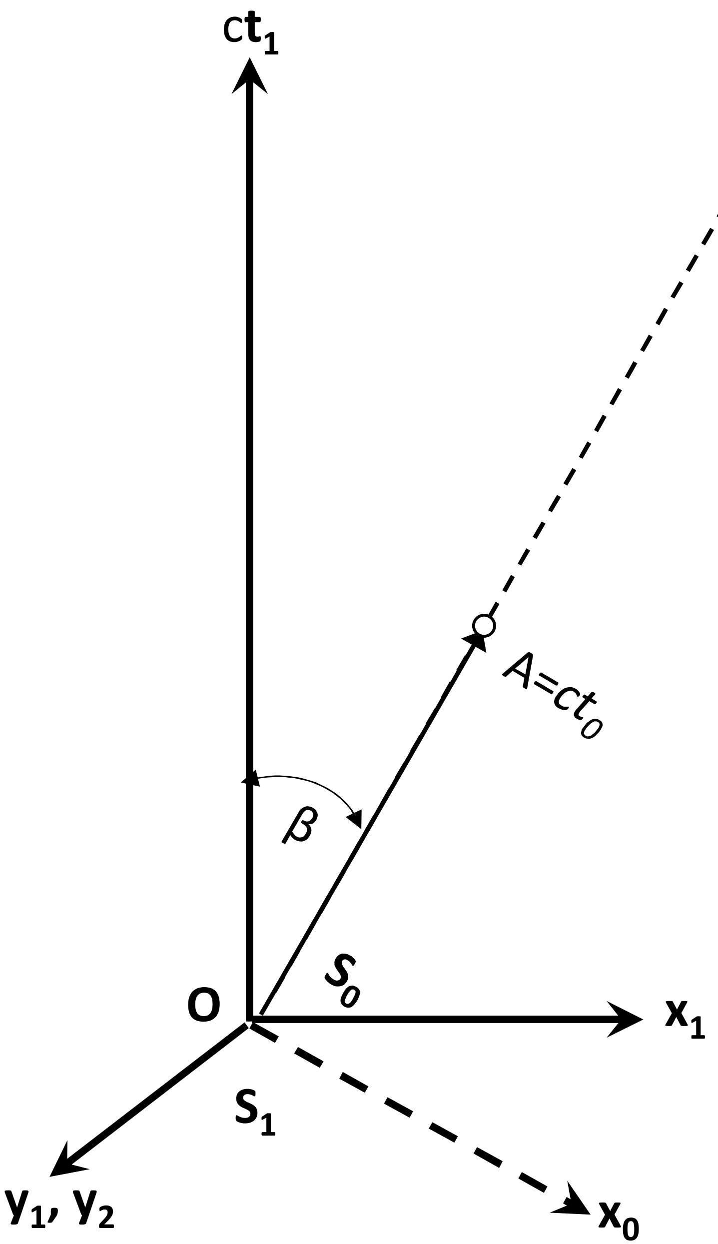


OUTLINE



- REVIEW OF VELOCITY TRIANGLE
- TIMELINE OF AN ACCELERATING BODY S_0
- UNACCELERATED S_1's MEASUREMENT OF ACCELERATED S_0's WORLDLINE
- SMALL ANGLE APPROXIMATION GIVES CLASSICAL LAWS FOR ACCELERATION
- S_1's MEASUREMENT OF S_0 CAUSES OBSERVED RELATIVISTIC MASS RISE

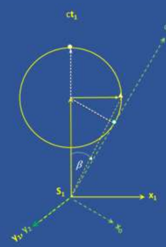
REVIEW OF VELOCITY TRIANGLE



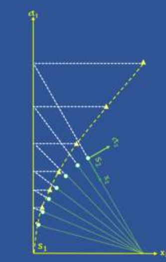
- $\beta = arcsin\,(v/c)$
- A= Time of Transmission ct_0

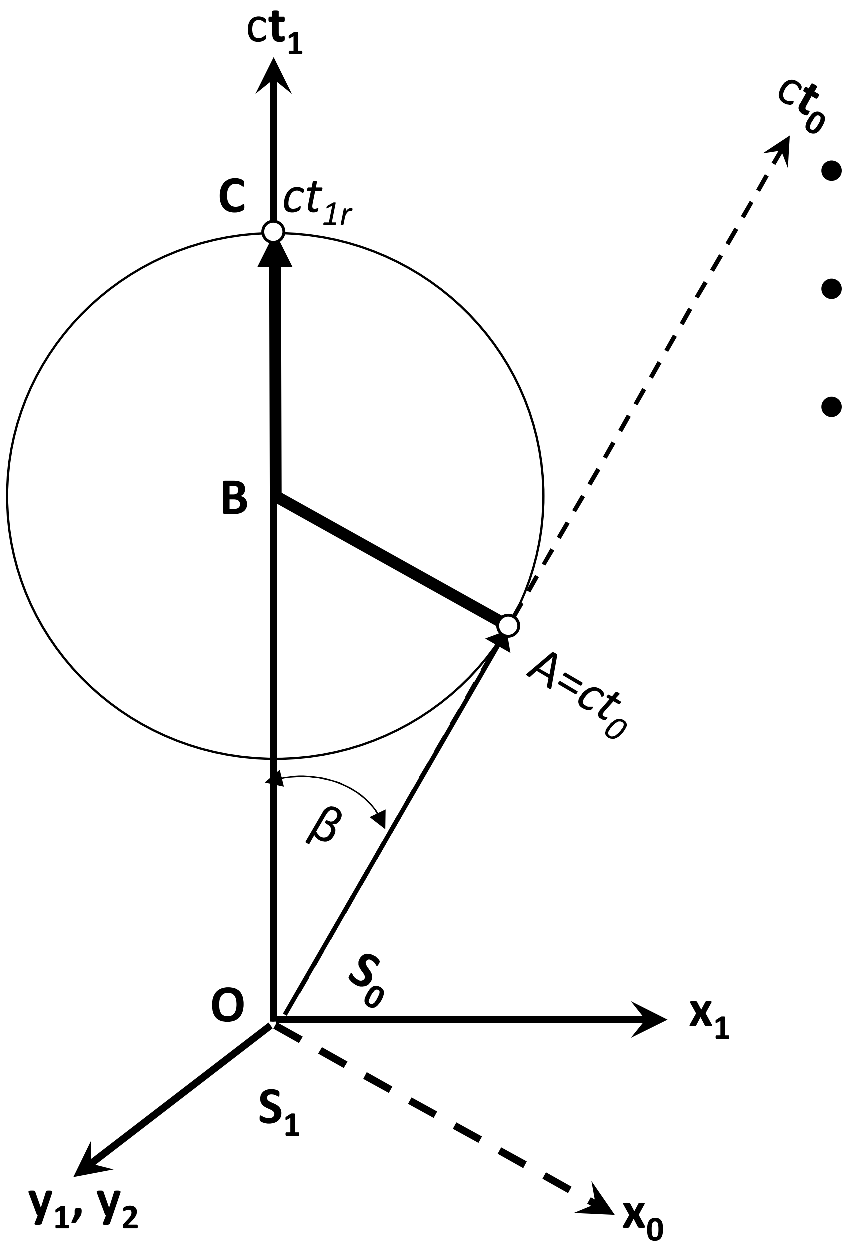
$$ct_1 = \frac{ct_0 + x_0 \cdot sin(\beta)}{cos(\beta)}$$

$$x_1 = \frac{x_0 + ct_0 \cdot sin(\beta)}{cos(\beta)}$$



REVIEW OF VELOCITY TRIANGLE

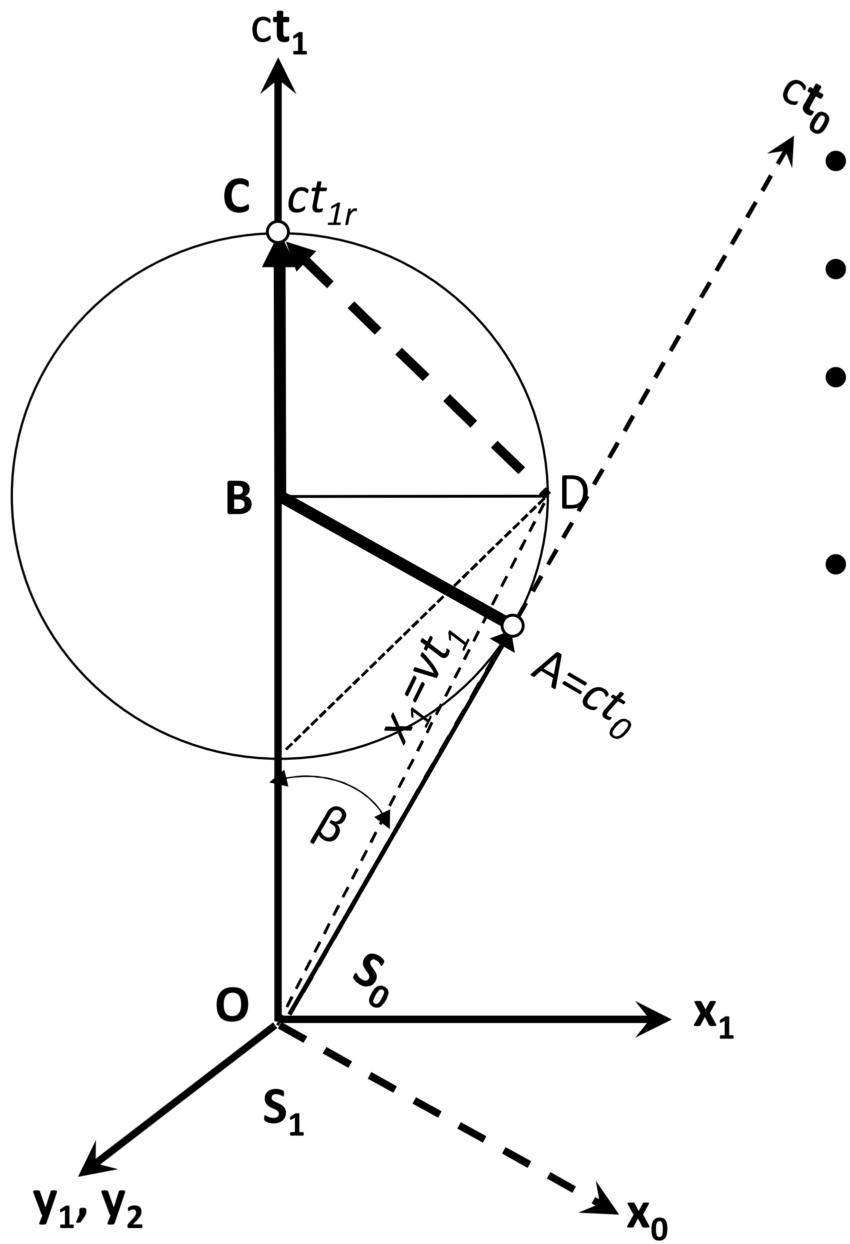




- *β=arcsin(v/c)*
- A= Time of Transmission *ct_0*
- Doppler Shift A along ABC, to C=Time of Receipt

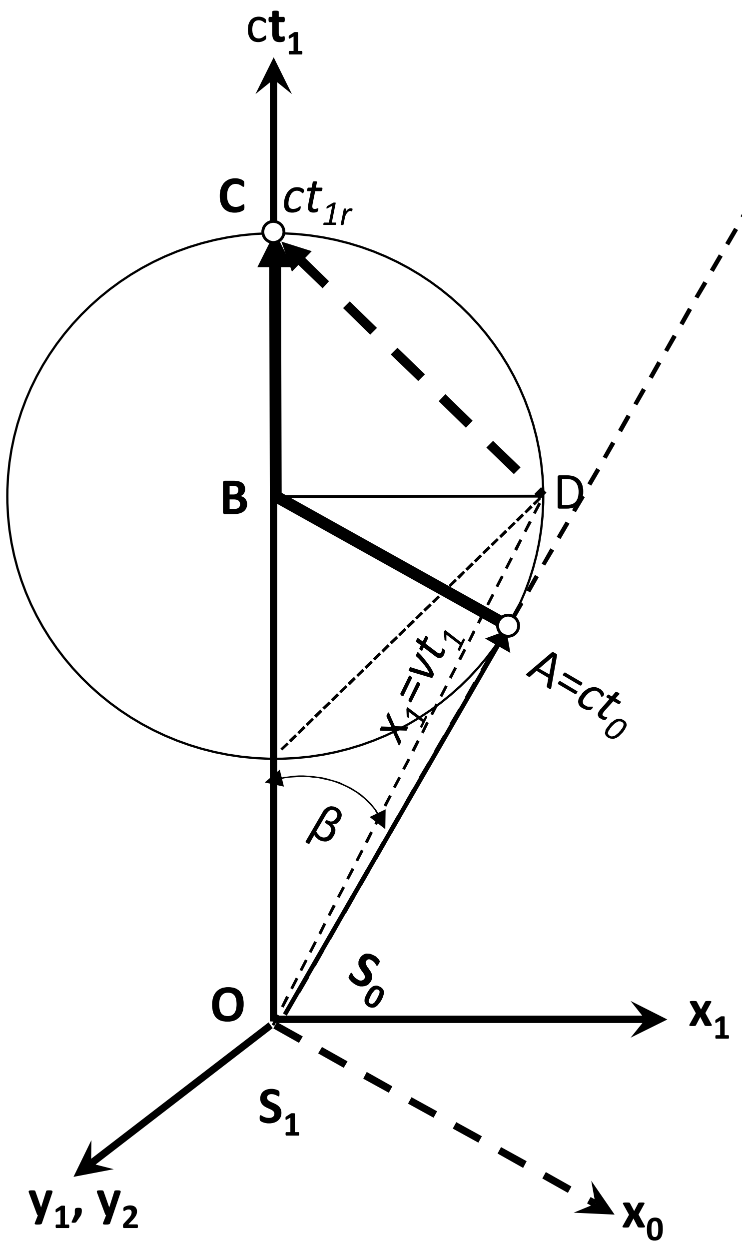
$$ct_{1r} = ct_0 \frac{1 + sin(\beta)}{cos(\beta)}$$

REVIEW OF VELOCITY TRIANGLE



- $\beta = arcsin(v/c)$
- A= Time of Transmission ct_0
- Doppler Shift A along ABC, to C=Time of Observation
- D=Measurement
  - Simultaneous with Event A
  - $OB = ct_1 = ct_0/cos(\beta)$
  - $BD = x_1 = ct_0 \cdot sin(\beta)/cos(\beta)$
  - $v = x_1/ct_1 = c \cdot sin(\beta)$

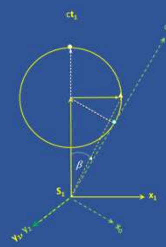
REVIEW OF VELOCITY TRIANGLE



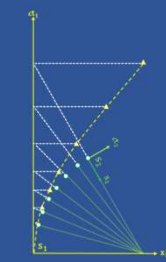
- $\beta = arcsin(v/c)$
- A= Time of Transmission ct_0
- Doppler Shift A along ABC, to C=Time of Observation
- D is Lorentz Transform of A

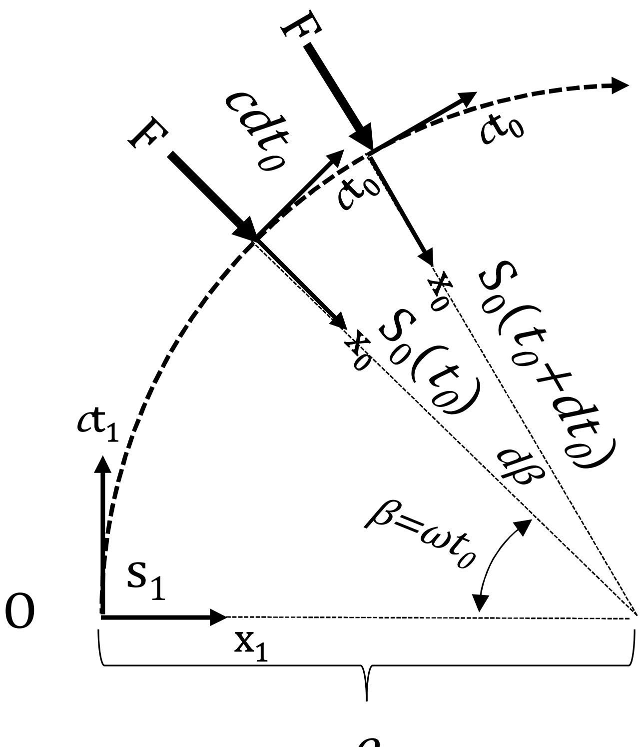
$$ct_1 = \frac{ct_0 + x_0 \cdot sin(\beta)}{cos(\beta)}$$

$$x_1 = \frac{x_0 + ct_0 \cdot sin(\beta)}{cos(\beta)}$$



TIMELINE OF ACCELERATING BODY





- CONSTANT CENTRIPETAL FORCE *F* APPLIED NORMAL TO S_o TIMELINE in $+x_0$ DIRECTION
- S_0 OBSERVES CONSTANT ACCELERATION
- CONSTANT VELOCITY ANGLE CHANGE WITH RESPECT TO ITSELF

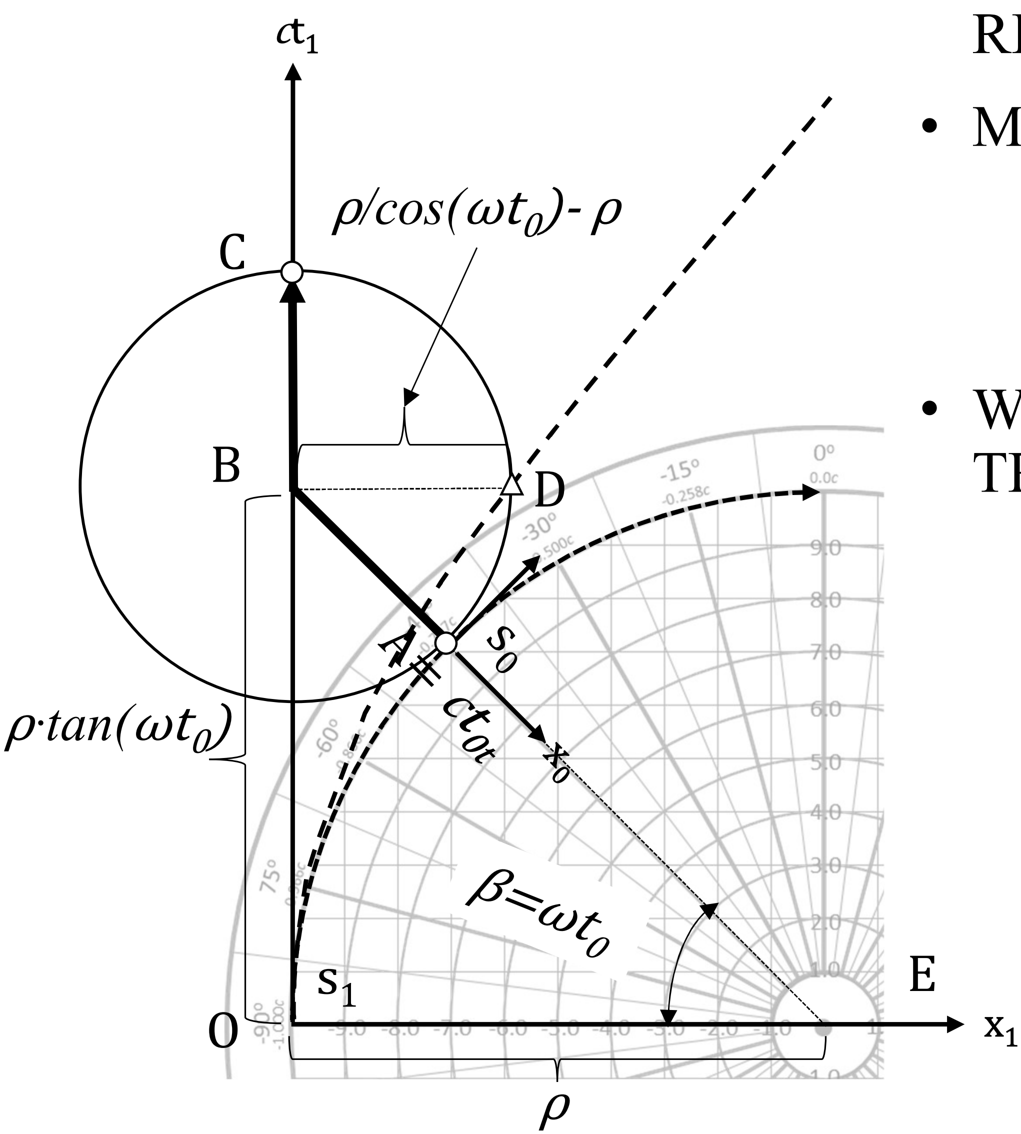
$$\beta = \omega \cdot t_0$$
$$d\beta = \arcsin(a \cdot dt_0/c) \approx a \cdot dt_0/c$$
$$d\beta/dt_0 = \omega = a/c$$

- S_0 FEELS RESULTING INERTIAL CENTRIFUGAL FORCE IN $-x_0$ DIRECTION
- FOR SMALL dt_0, $dv \ll c$: CLASSICAL MECHANICS APPLIES:

$$\omega = a/c = c/\rho$$
$$\rho = c^2/a \quad (At\ 1g,\ \rho = 0.97 \text{ lightyears})$$

$$F = m\rho\omega^2 = m\frac{c^2}{a}\left(\frac{a}{c}\right)^2 = ma$$

S_1's MEASUREMENT OF ACCELERATING BODY S_0



- TIME OF TRANSMISSION TO TIME OF RECEIPT: ABC
- MEASUREMENT AT D

  $x_1 = \rho / \cos(\omega t_0) - \rho$

  $ct_1 = \rho \cdot \sin(\omega t_0) / \cos(\omega t_0)$

- WORLDLINE IS HYPERBOLIC: TRIANGLE OBE

  $$(x_1 + \rho)^2 - c^2 t_1^2 = \rho^2 = c^4 / a_0^2$$

cf Gravitation, p. 166

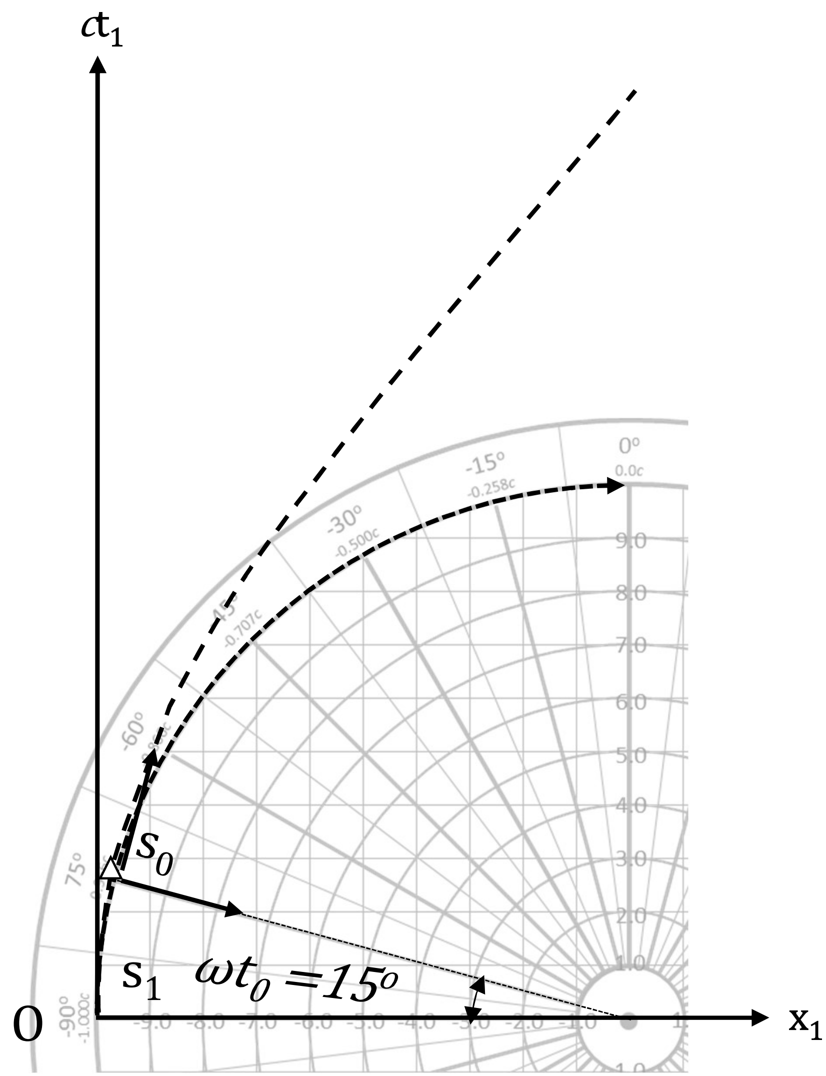
(Misner, Thorne and Wheeler)

$x^2 - t^2 = 1/a^2$

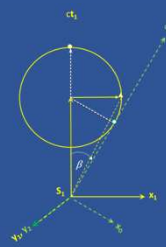
(*c=1* and omitted, initial offset)

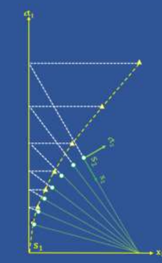
Done with tensors vs. trigonometry

S_1's MEASUREMENT OF ACCELERATING BODY S_0



SMALL ANGLE APPROX CLASSICAL SOLUTION





- SMALL ANGLE APPROXIMATIONS

  $a_1 = a_0 \cdot cos(\omega t_0) \cong a_0$

  $ct_1 = \rho \cdot tan\ (\omega t_0) \cong ct_0$

  $v = c \cdot sin(\omega t_0) \cong a_0 t_0$

- TRIG IDENTITY: $sin^2(\alpha) = (1 - cos(2\alpha))/2$

$$x_1 = \rho \frac{1 - cos(\omega t_0)}{cos(\omega t_0)} = \rho \frac{2 \cdot sin^2\left(\omega t_0/_2\right)}{cos(\omega t_0)} \cong \frac{a_0 {t_0}^2}{2}$$

- FOR SMALL VALUES OF ωt_0, $v \ll c$

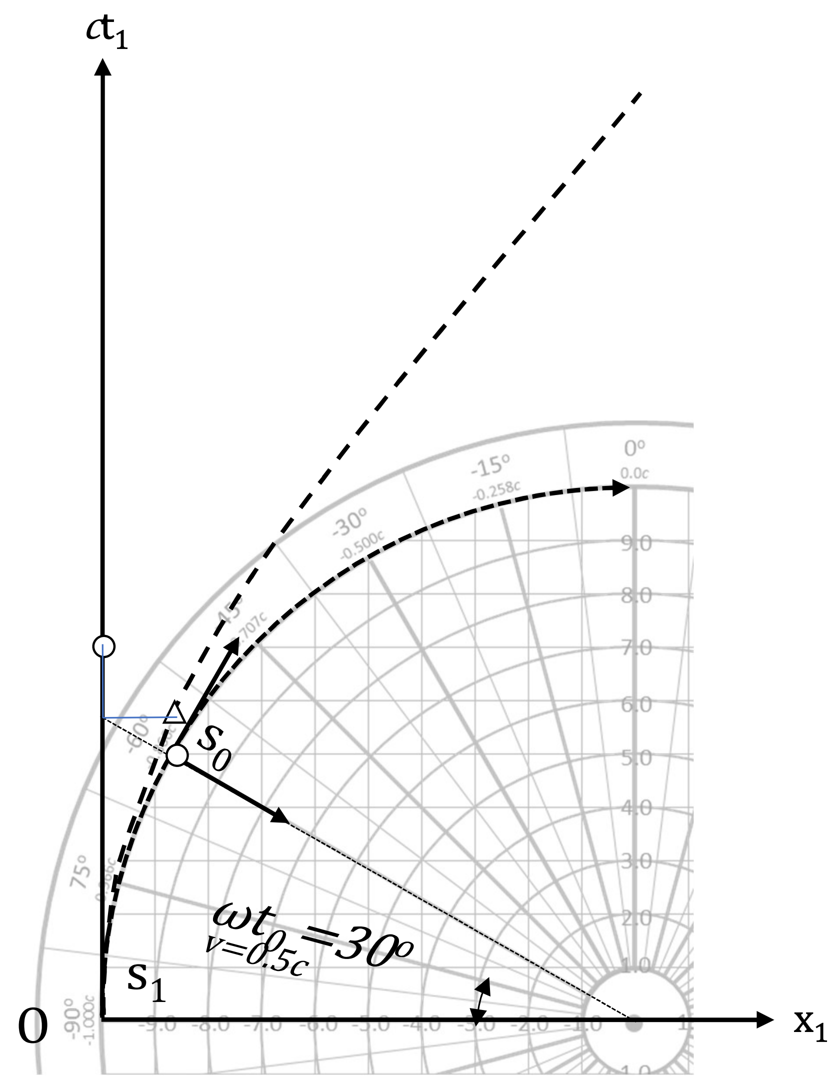
  $a_1 \cong a_0$

  $ct_1 \cong ct_0$

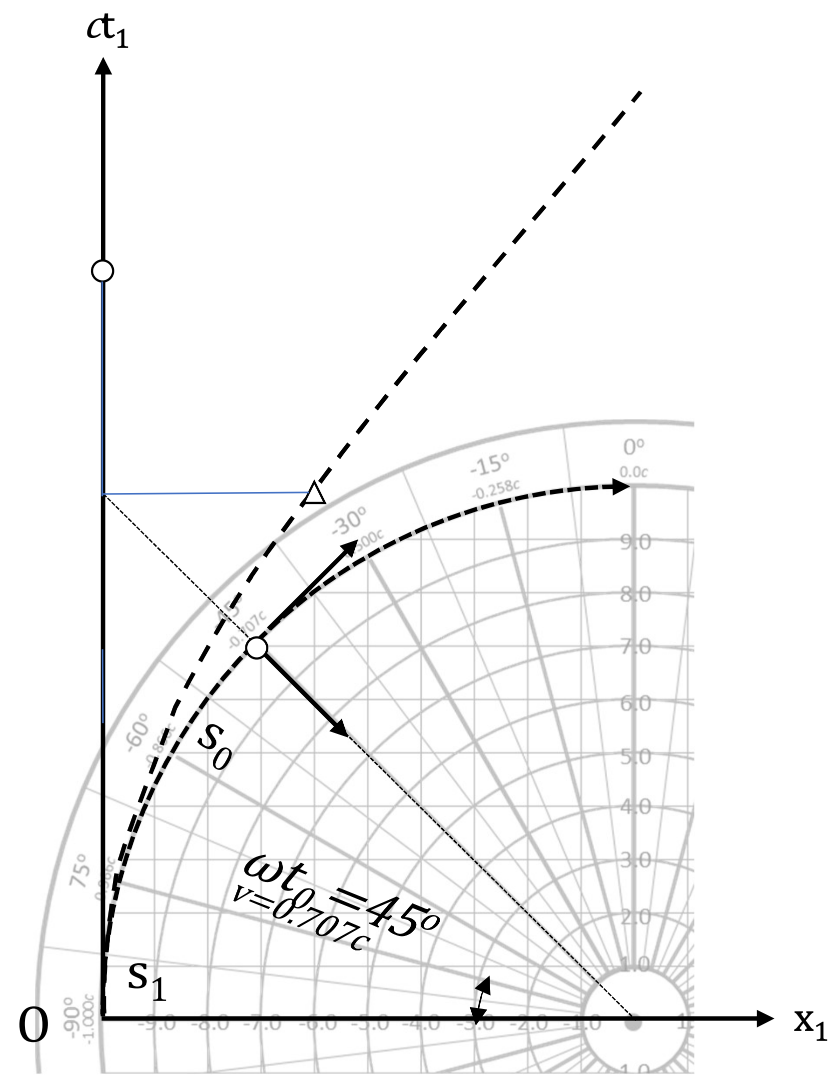
  $v \cong a_0 t_0$

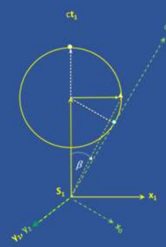
  $x_1 \cong a_0 t_0^2/2$

S_1's MEASUREMENT ACCELERATING BODY S_0

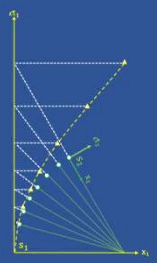


S_1's MEASUREMENT OF ACCELERATING BODY S_0





S_1's MEASUREMENT OF RELATIVISTIC MASS



- S_1's MEASUREMENT OF S_0's VELOCITY AND ACCELERATION

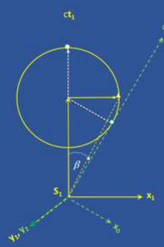
$$v/_c = \frac{dx_1}{dt_1} = \frac{dx_1}{dt_0} \cdot \frac{dt_0}{dt_1} = \frac{\omega\rho \cdot sec(\omega t_0) \cdot tan(\omega t_0)}{\omega\rho \cdot sec^2(\omega t_0)} = sin(\omega t_0)$$

$$a_1 = \frac{d}{dt} \cdot c \cdot sin(\omega t_0) = a_0 \cdot cos(\omega t_0)$$

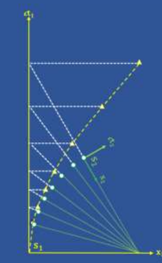
  - As $\omega \cdot t_0 \rightarrow 90^o$, Velocity$\rightarrow$ c, Acceleration $\rightarrow 0$

- MEASURE MASS:
  - Divide Constant Force F_0=$m_0 \cdot a_0$ by Observed Acceleration

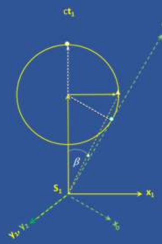
$$m_1 = \frac{F_0}{a_1} = \frac{m_0 \cdot a_0}{a_0 \cdot cos(\omega t_0)} = \frac{m_0 \cdot}{\sqrt{1 - v^2/_{c^2}}}$$



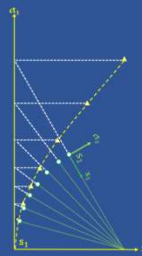
SUMMARY

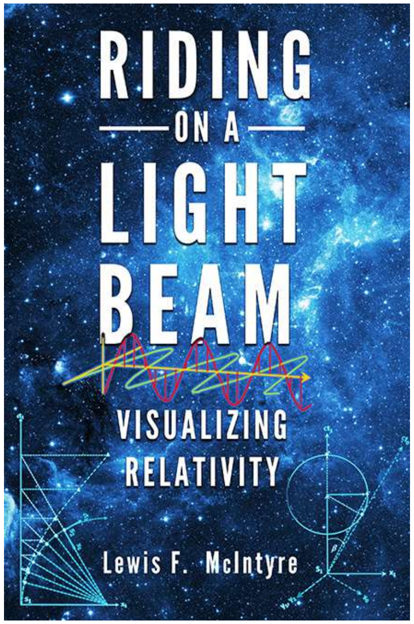


- CONSTANT LINEAR ACCELERATION IS A CIRCULAR TRAJECTORY IN 4-DIMENSIONAL SPACE
- UNACCELERATED OBSERVER WILL SEE A HYPERBOLIC WORLDLINE ASYMPTOTIC TO *c* AS $\omega t_0 \rightarrow 90^o$
- PROOF
  - Small Angle Approximation For $v \ll c$ Gives Classical Equation for Accelerated Motion
  - Diminishing Acceleration as $\omega t_0 \rightarrow 90^o$ Results in Measured Mass Rise
  - Validated By *Gravitation* (Misner, Thorne and Wheeler)



Q.E.D.





QUOD ERAT DEMONSTRANDUM!

(Which was to be shown!)