Chesapeake Section of the
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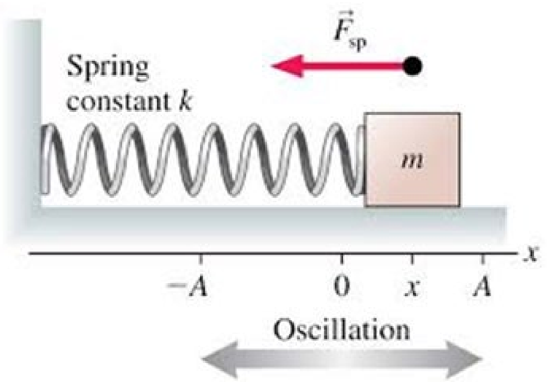
Mechanical Oscillations With and Without Damping - An Analog Computer Physics-Themed Simulation

Ryan Bischof
Advisor: Michael Cimorosi

Division of Physics, Engineering, Mathematics, and Computer Science (PEMaCS)
Delaware State University

DSU
College of Agriculture, Science and Technology

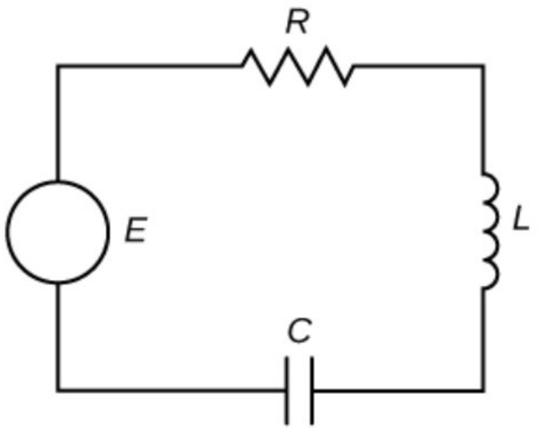
Simple Harmonic Motion



m - mass
b – damping coefficient
k – spring constant
$f(t)$ –external force

$$mx'' + bx' + kx = f(t)$$

RLC Circuit



L - inductance
R – resistance
C – capacitance
$E(t)$ – voltage source

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = E(t)$$

Both spring-mass systems and RLC circuits can be modeled by second-order constant-coefficient differential equations.

- **We constructed a breadboard analog computer**
  - **with currents following a second-order linear differential equation**
  - **to model the position of a sliding block attached to a massless spring**
  - **in two cases (1) with damping and (2) without damping.**
- **Solutions displayed on an oscilloscope**
- **An instructional approach for students to visualize classical physics through hands-on analog circuit design**

We began with the dynamics of the spring-mass system.

$\ddot{x} + c/M\,\dot{x} + k/M\,x = 0$ with initial conditions $\dot{x}(0)$ and $x(0)$, where

- $\ddot{x} = d^2x/dt^2 = a$ = acceleration (in cm/s^2)
- $\dot{x} = dx/dt = v$ = speed (in cm/s)
- x = position of block (in cm): – to the left, + to the right
- $\dot{x}(0)$ = initial speed of the block at time of release = 0 cm/s
- $x(0)$ = initial displacement of block at time of release = 3 cm
- t = time (in s)
- c = damping factor (in grams/s)
- M = mass of block (in grams)
- k = spring constant (in $grams/s^2$)

We then delved in the math and arrived at

With Damping

$\omega = 1 \rightarrow T = 2\pi$, $k/M = 82/81$, and $\alpha = \sqrt{(k/M - \omega^2)} = \sqrt{(82/81 - 1^2)} = 1/9$.

Substituting values into the expression for x(t),

$$x(t) = 3e^{(-t/9)}(\cos(t) + \sin(t)/9)$$

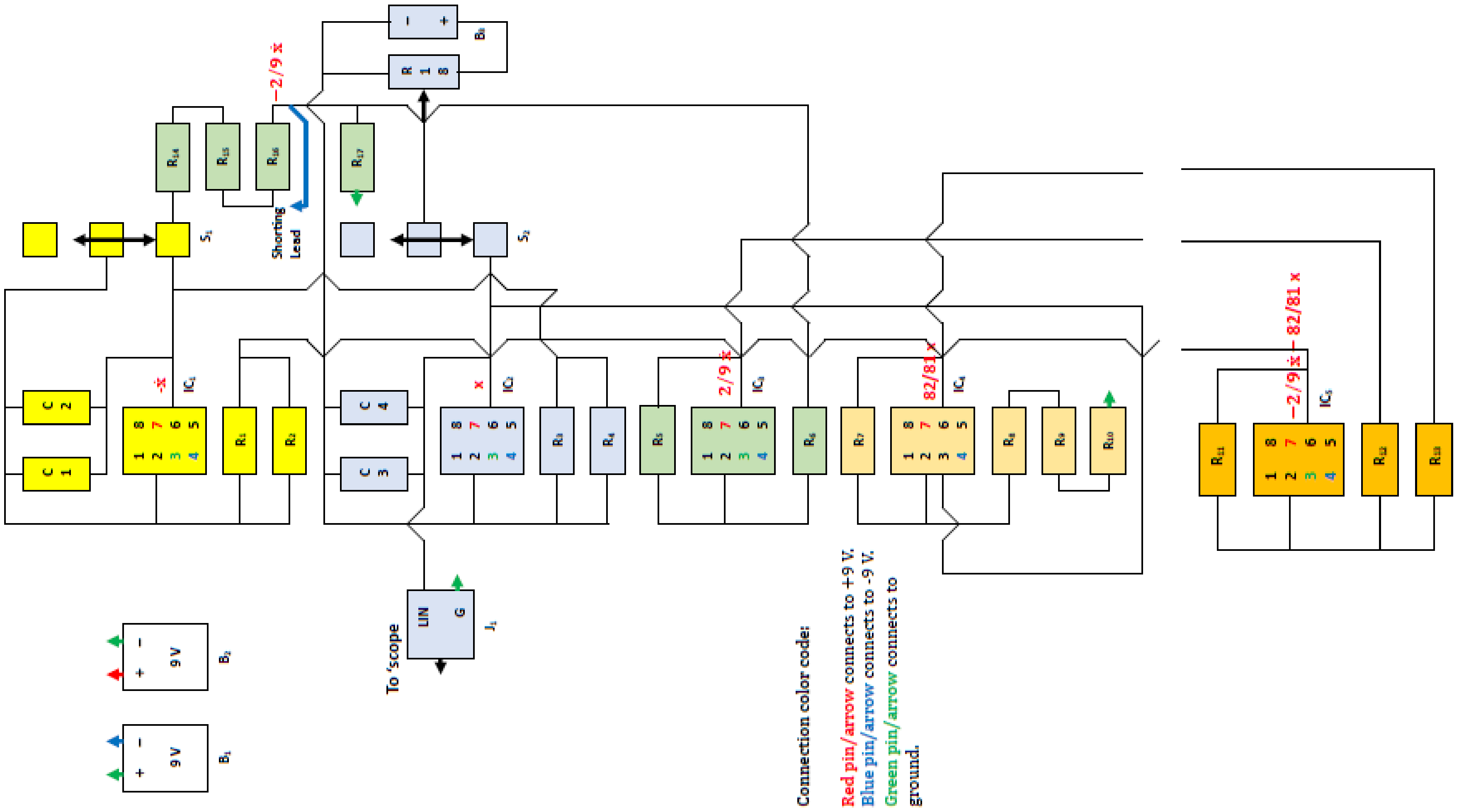
Without Damping

$\alpha = c/(2M) = 0$ and $\omega = \sqrt{(k/M - \alpha^2)} = \sqrt{(82/81 - 0^2)} = \sqrt{(82)}/9 \rightarrow T = 9\pi/41\sqrt{(82)}$.

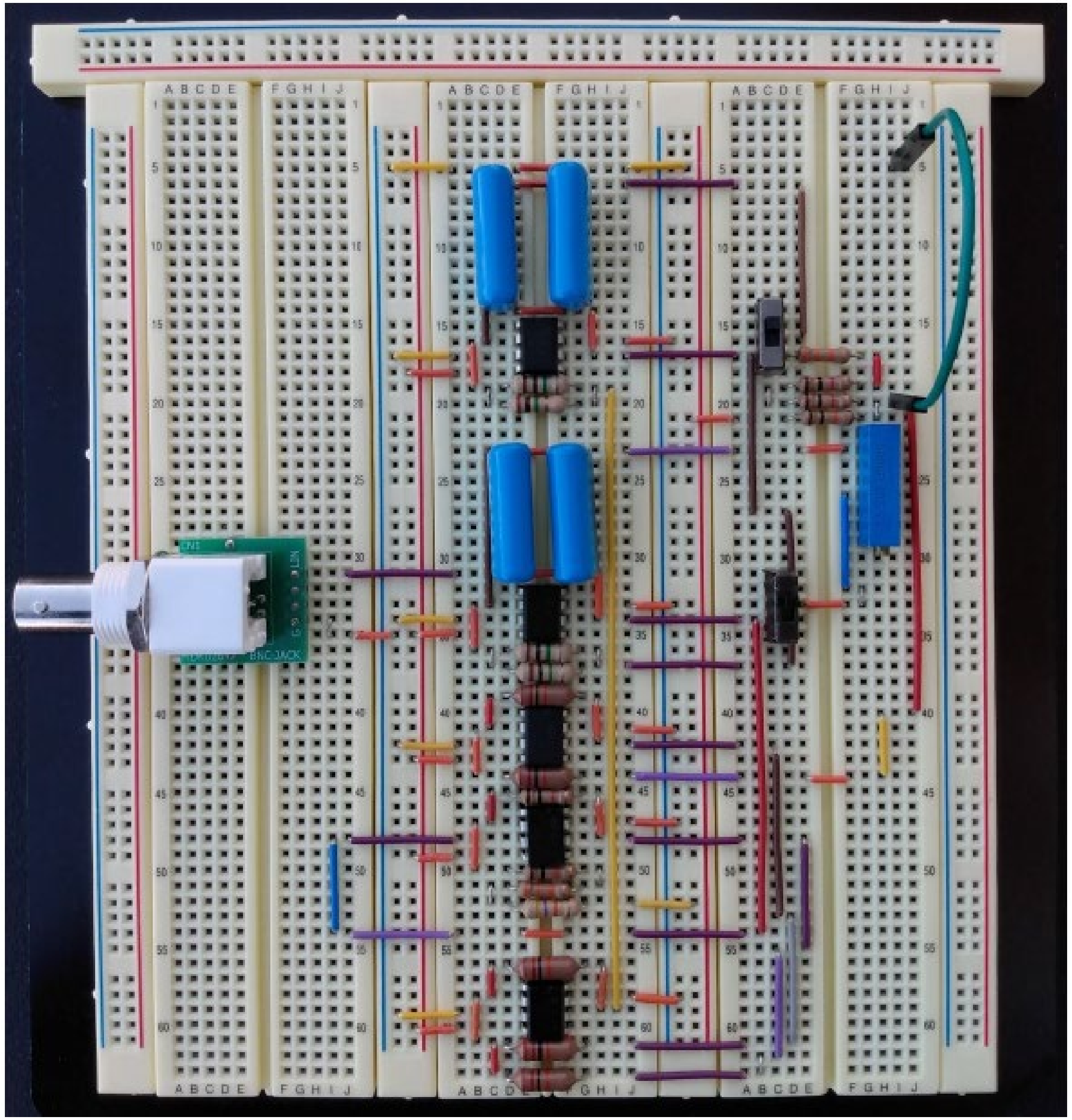
Substituting values into the expression for x(t),

$$x(t) = 3(\cos(\sqrt{(82)}t/9)$$

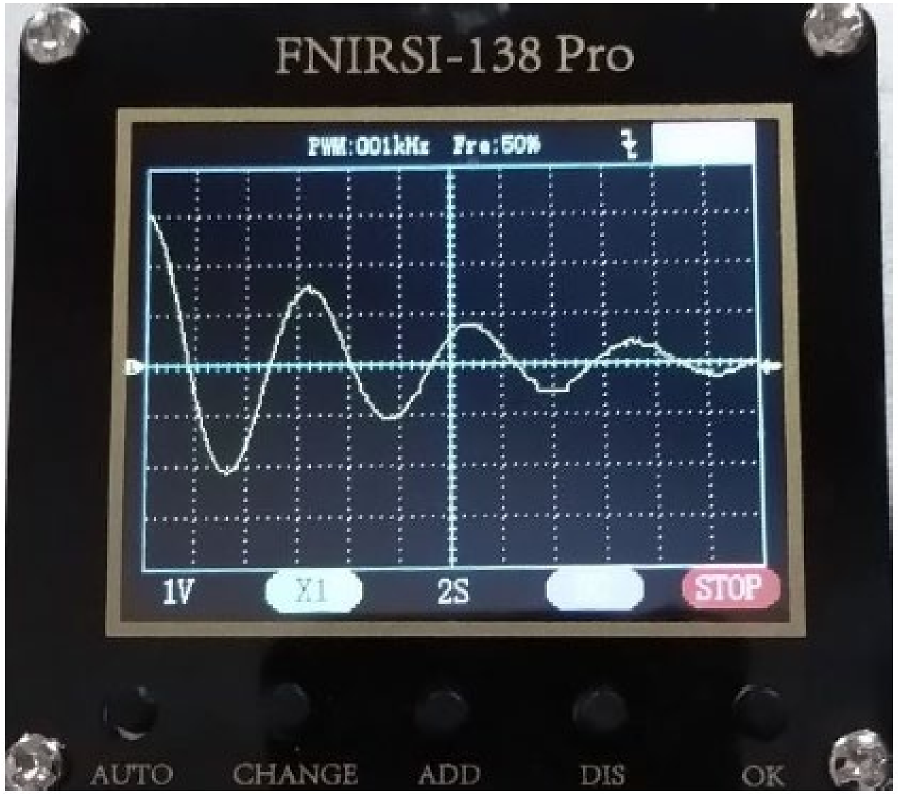
We drew the diagram of analog computer breadboard connection and mathematics flow.

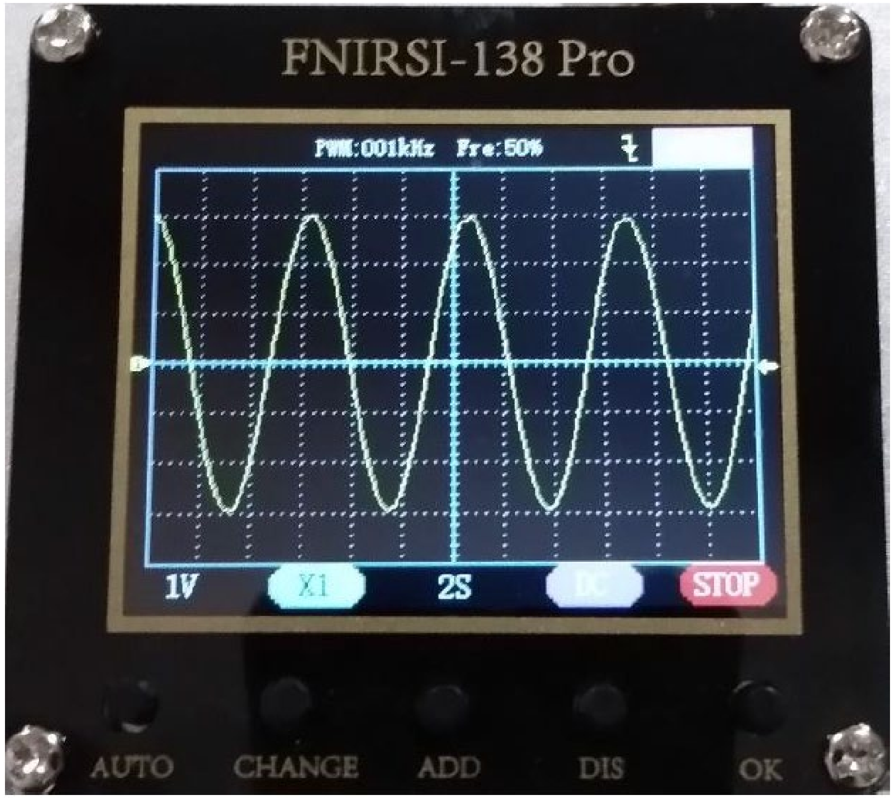


Finally, we built the breadboard circuit according to the diagram.



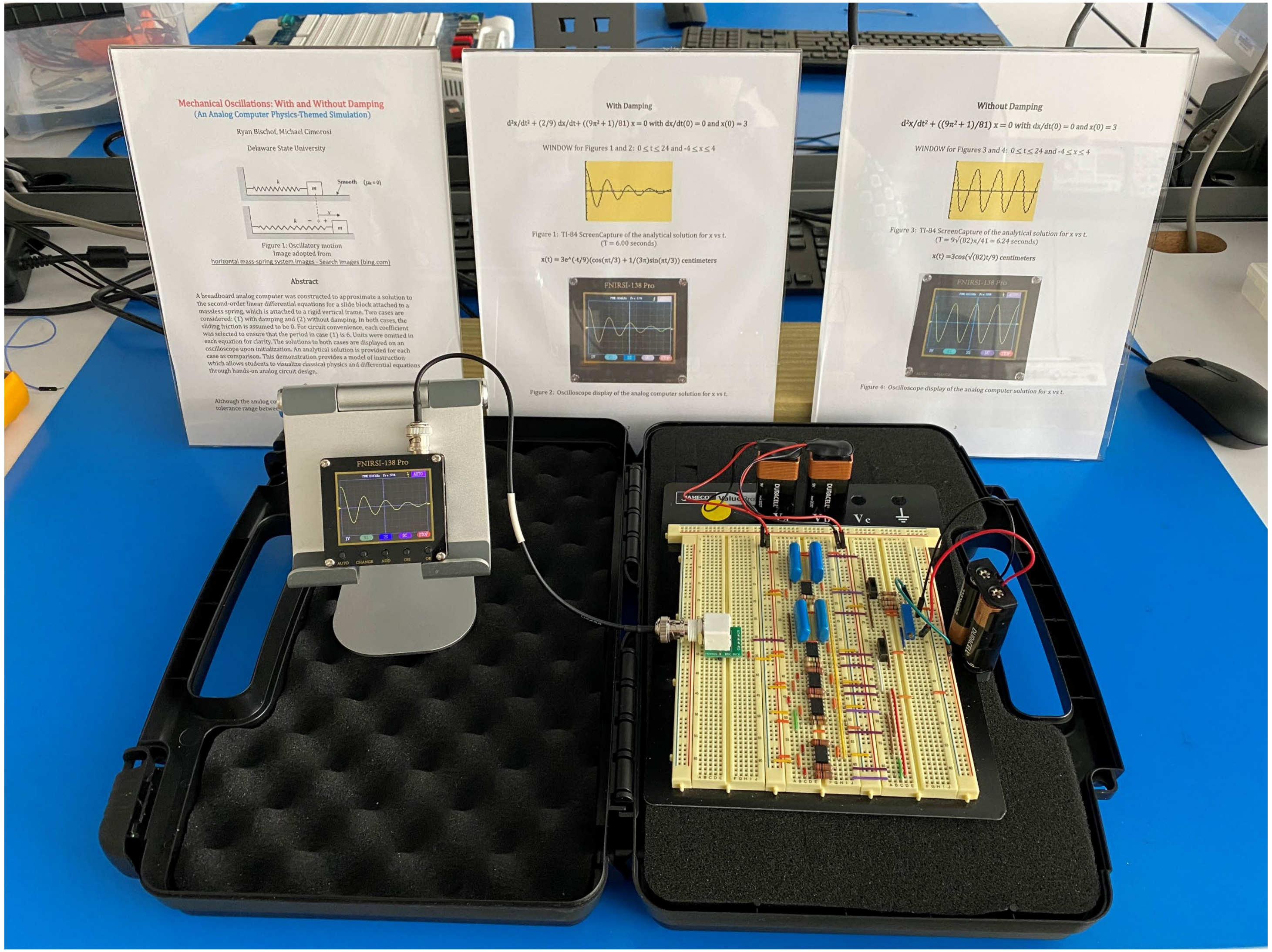
When we connect the batteries to the circuit and have the initial conditions set right, we see these displayed on the oscilloscope.



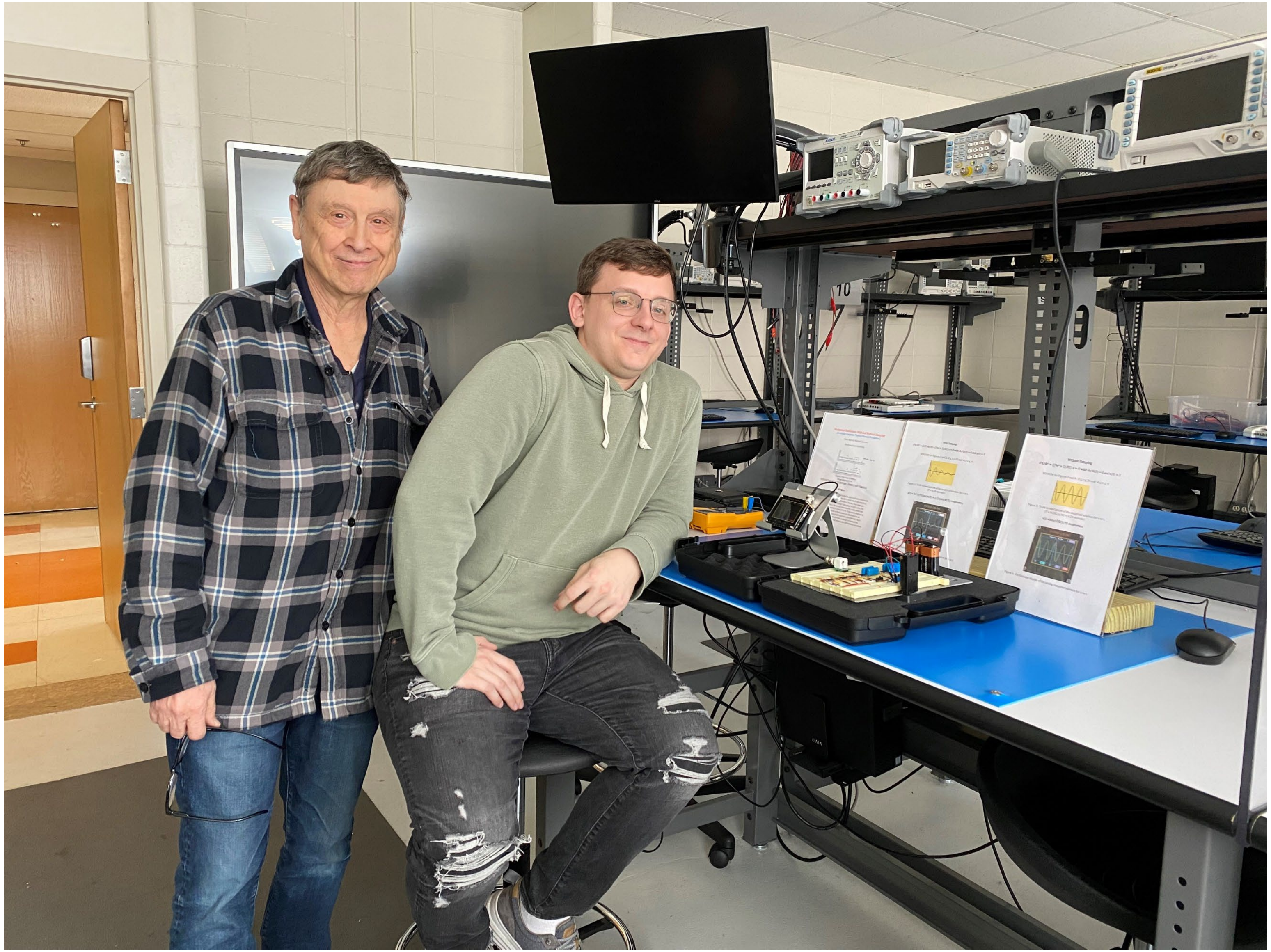


They behave exactly the same as the oscillation of a spring-mass system with or without damping.

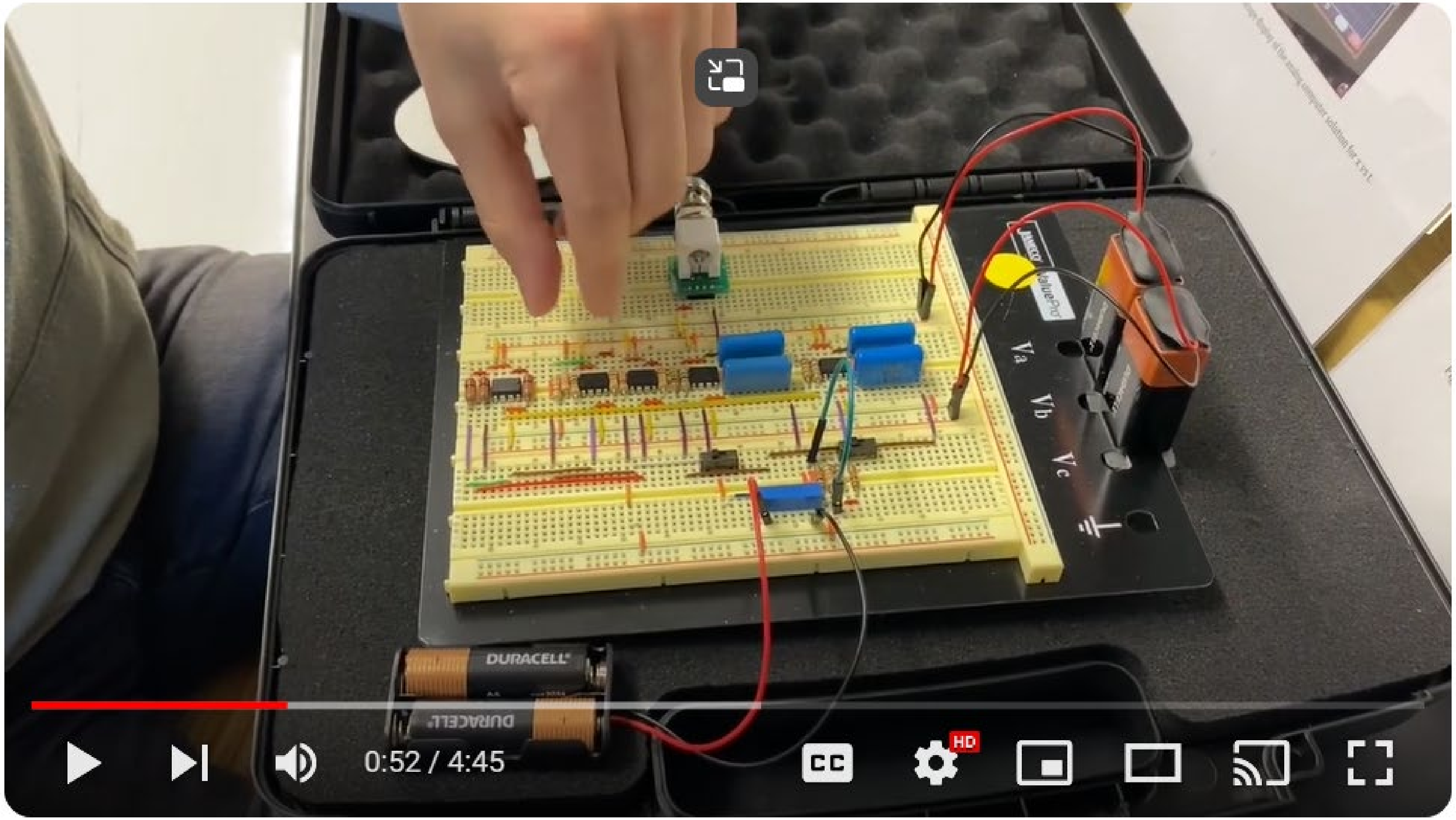
The complete demonstration set



Me with my advisor Mr. C



Enjoy the demonstration!



Analog Computer Simulation of Spring-Mass Oscillator

https://www.youtube.com/watch?v=syRmXpcJmA8