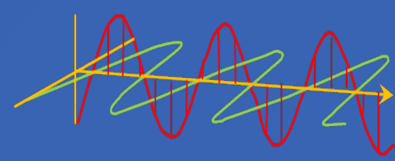
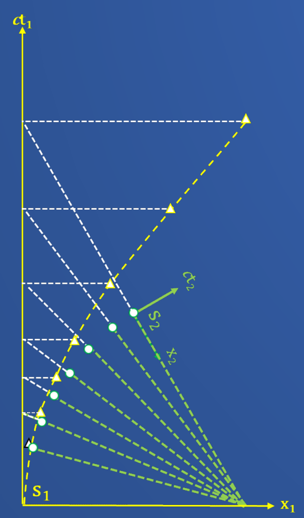
The Lorentz Transform
Axis of Symmetry





CS-AAPT Spring 2024

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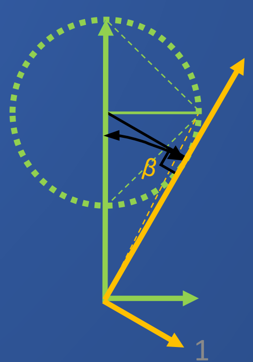
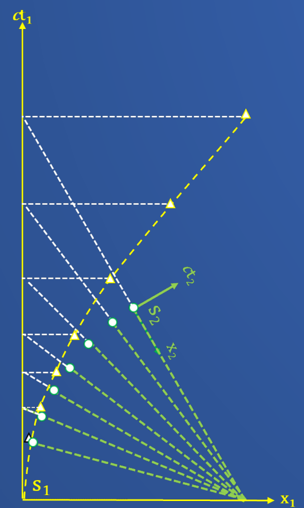


The Velocity Triangle and the Trigonometric Lorentz Transform



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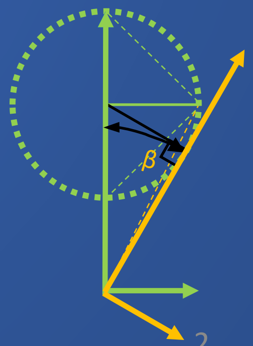


- HENDRICK LORENTZ DEVELOPED THE LORENTZ TRANSFORM IN 1899
- DERIVATION
  - Modifies the Galilean Transform $x_0 + vt_0$ with unknown weights α, β and γ:

$$x_1 = \alpha(x_0 + vt_0)$$
$$t_1 = \beta t_0 + \gamma x_0$$
$$y_1 = y_0$$
$$z_1 = t_0$$

  - With Constraint that Both Must Measure the Same Speed of Light.

$$c^2 t_0^2 = x_0^2 + y_0^2 + z_0^2$$
$$c^2 t_1^2 = x_1^2 + y_1^2 + z_1^2$$

  - Solving for Unknowns α, β and γ Yields the Lorentz Transform

$$x_1 = \frac{x_0 + ct_0 \cdot\ (v/c)}{\sqrt{1-(v/c)^2}}$$

$$ct_1 = \frac{ct_0 + x_0 \cdot\ (v/c)}{\sqrt{1-(v/c)^2}}$$

- VELOCITY TRIANGLE OAB:
  - S_1 in blue
  - S_0 in red
  - *β=arcsin (v/c)*

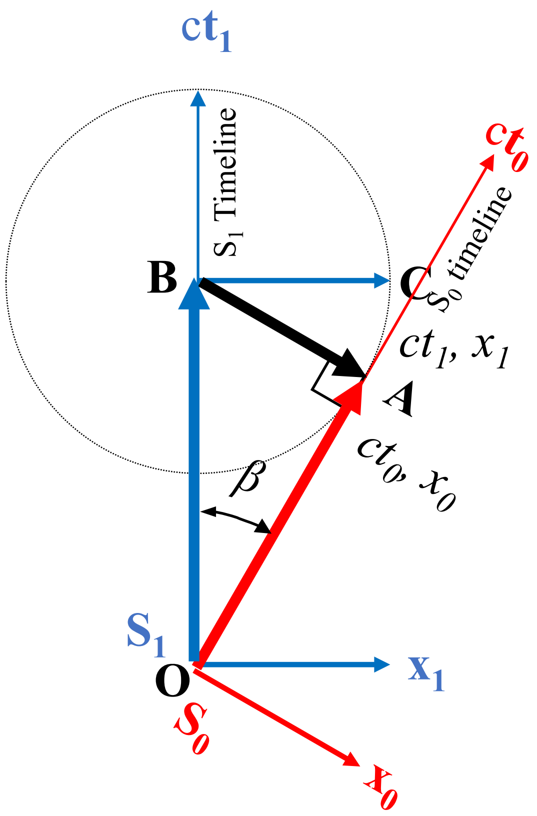
- TRIGONOMETRIC LORENTZ TRANSFORM

$$x_1 = \frac{x_0 + ct_0 \cdot \ (v/c)}{\sqrt{1-(v/c)^2}} = \frac{x_0 + ct_0 \cdot \sin(\beta)}{\cos(\beta)}$$

$$ct_1 = \frac{ct_0 + x_0 \cdot \ (v/c)}{\sqrt{1-(v/c)^2}} = \frac{ct_0 + x_2 \cdot \sin(\beta)}{\cos(\beta)}$$

- TRIGONOMETRIC DOPPLER SHIFT

$$c\Delta t_1 = c\Delta t_0 \frac{1 + \ (v/c)}{\sqrt{1-(v/c)^2}} = c\Delta t_0 \frac{1+\sin(\beta)}{\cos(\beta)}$$



Limitations of the Velocity Triangle

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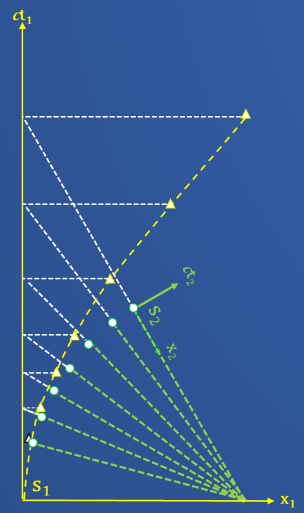
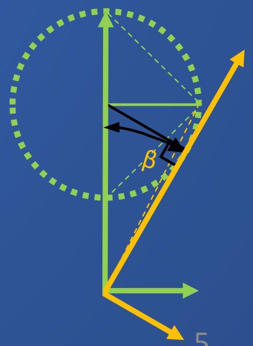
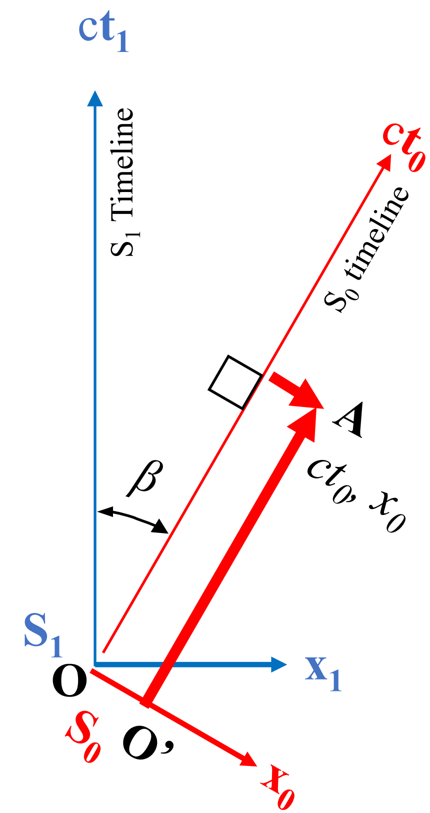




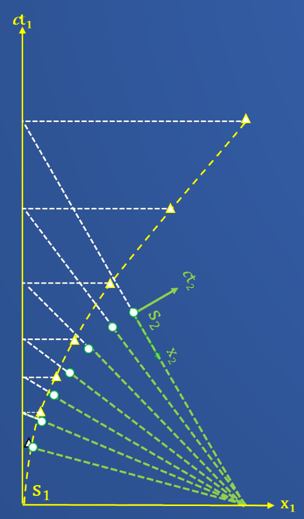
- THE VELOCITY TRIANGLE REQUIRES ONE POINT WITH COMMON TIME AND DISTANCE IN BOTH REFERENCE FRAMES
- O SATISFIES THIS WITH ZERO OFFSET

  $ct_0 = ct_1 = 0$

  $x_0 = x_1 = 0$
- O' DOES NOT MEET THIS CRITERIA DUE TO x OFFSET



The Axis of Symmetry of the Lorentz Transform



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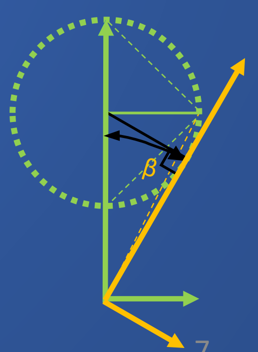
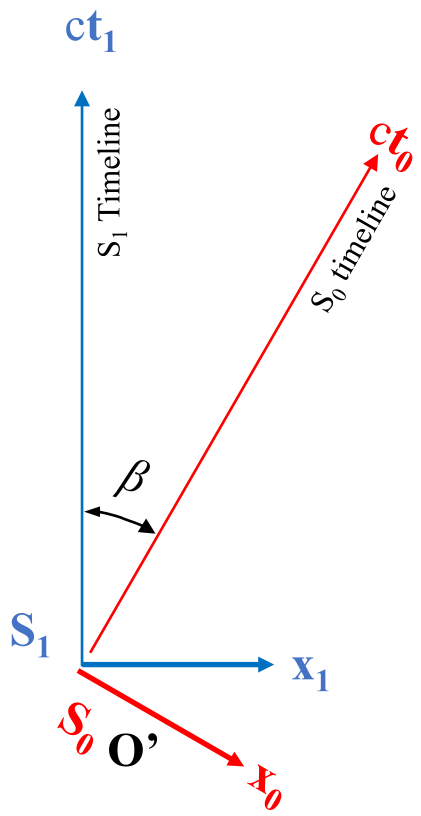


- LOCATE POINTS WITH EQUAL TIME IN BOTH REFERENCE FRAMES

$$ct_1 = ct_0 = \frac{ct_0 + x_0 \cdot (v/c)}{\sqrt{1-(v/c)^2}}$$

$$x_0 = -ct_0 \frac{1-\sqrt{1-(v/c)^2}}{v/c}$$



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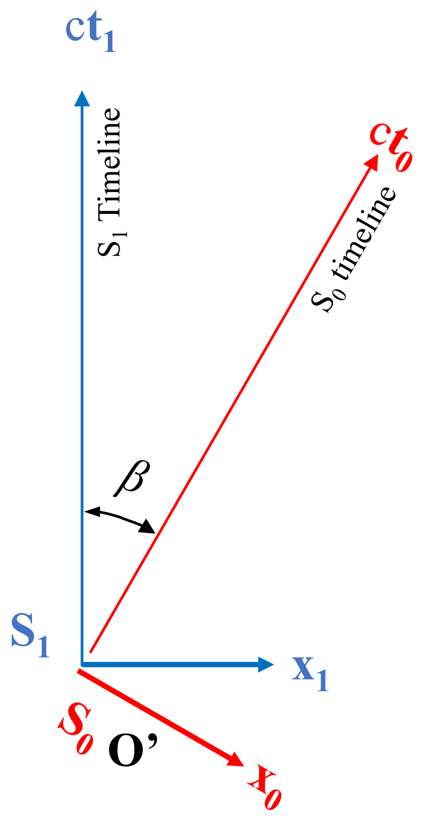
$$x_0 = -ct_0 \frac{1-\cos(\beta)}{\sin(\beta)}$$



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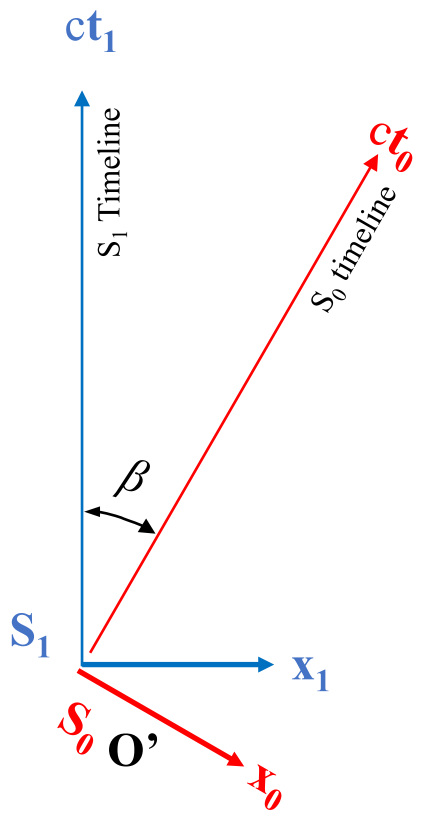
$$x_0 = -ct_0 \frac{1-\sqrt{1-(v/c)^2}}{v/c}$$

$$x_0 = -ct_0 \frac{1-\cos(\beta)}{\sin(\beta)} = -ct_0 \cdot \tan(\beta/2)$$



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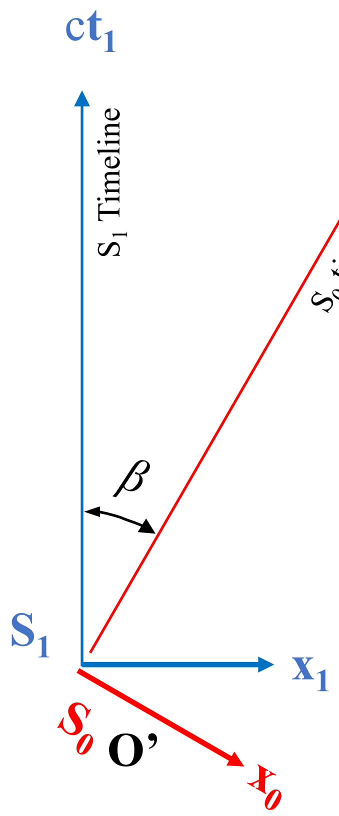
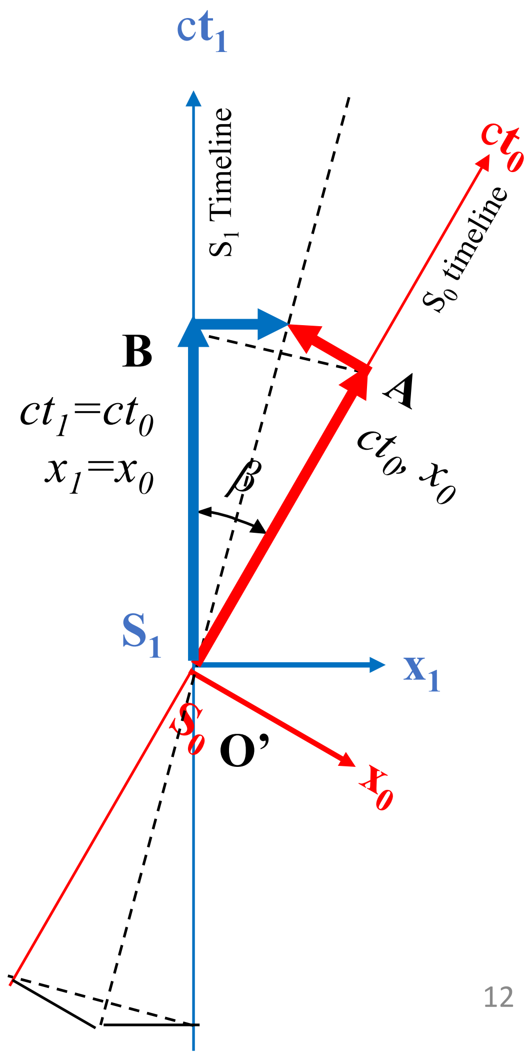
$$x_0 = -ct_0 \frac{1-\cos(\beta)}{\sin(\beta)} = -ct_0 \cdot \tan(\beta/2)$$

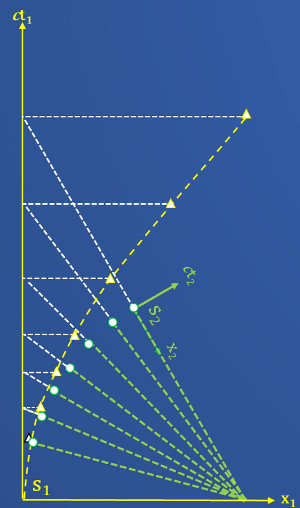
$$x_1 = \frac{x_0 + ct_0 \cdot \sin(\beta)}{\cos(\beta)} = ct_0 \cdot \tan(\beta/2) = -x_0$$

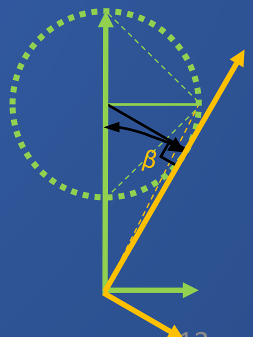


- THE BISECTOR OF THE VELOCITY ANGLE IS THE AXIS OF SYMMETRY FOR THE LORENTZ TRANSFORM
  - Locate Equal Times In S_0 and S_1
  - The *x* coordinate is $ct \cdot tan(\beta/2)$, equal and opposite.



Using the Axis of Symmetry to Solve the Generalized Lorentz Transform





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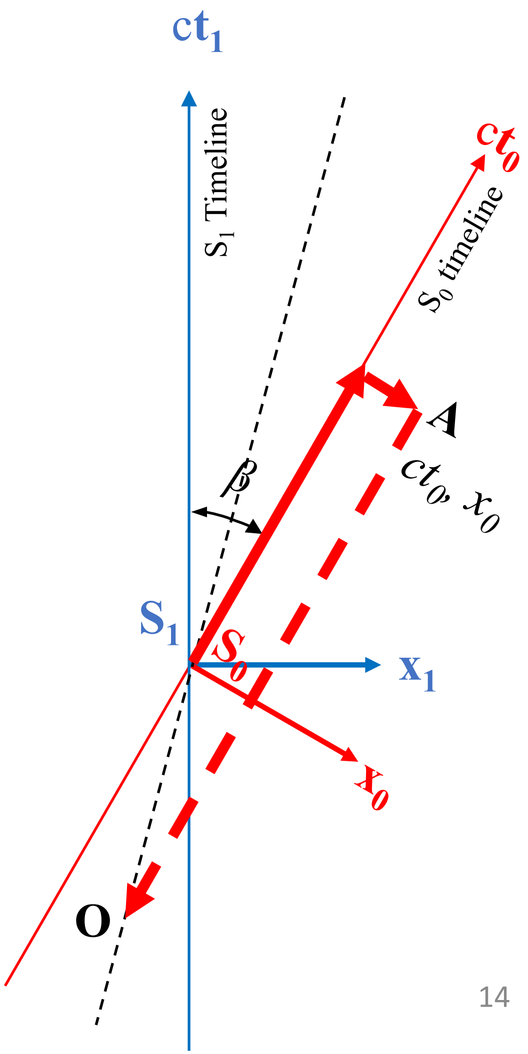
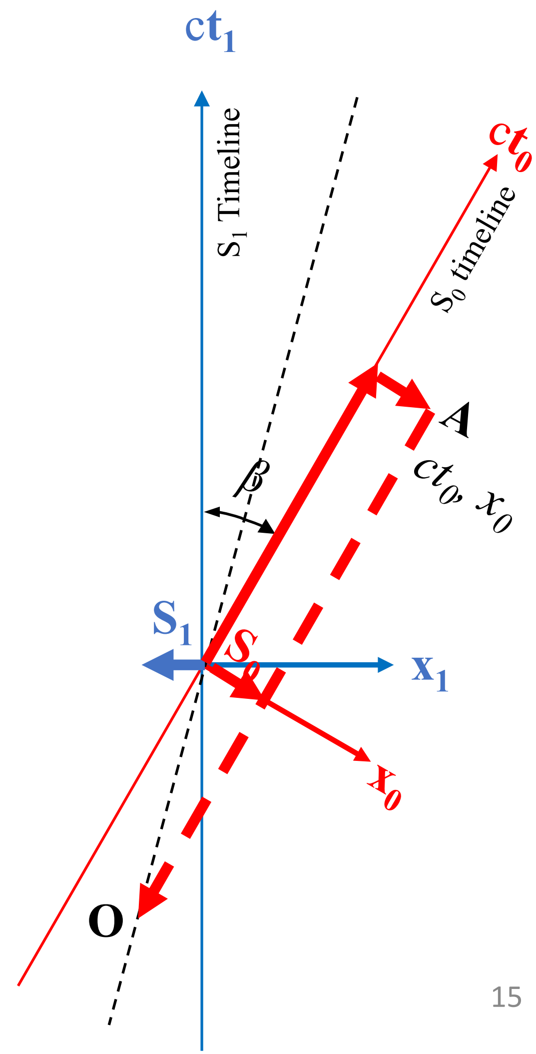
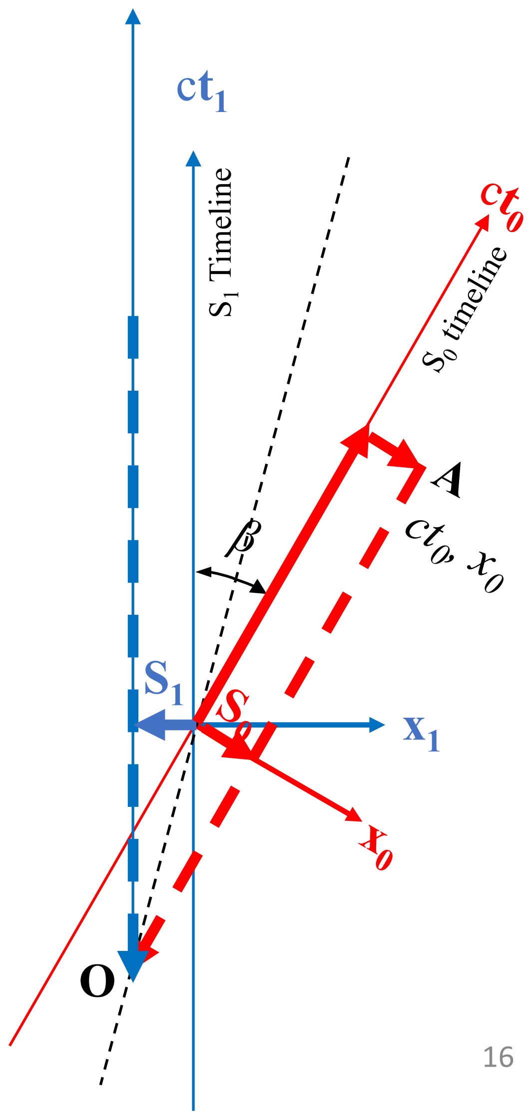
- EXTEND S_0 TIMELINE THROUGH A PARALLEL TO ct_0 TO INTERSECT AXIS OF SYMMMETRY AT O



- EXTEND S_0 TIMELINE THROUGH A PARALLEL TO ct_0 TO INTERSECT AXIS OF SYMMMETRY AT O
- ADD x_1 OFFSET TO S_1 EQUAL TO $-x_0$



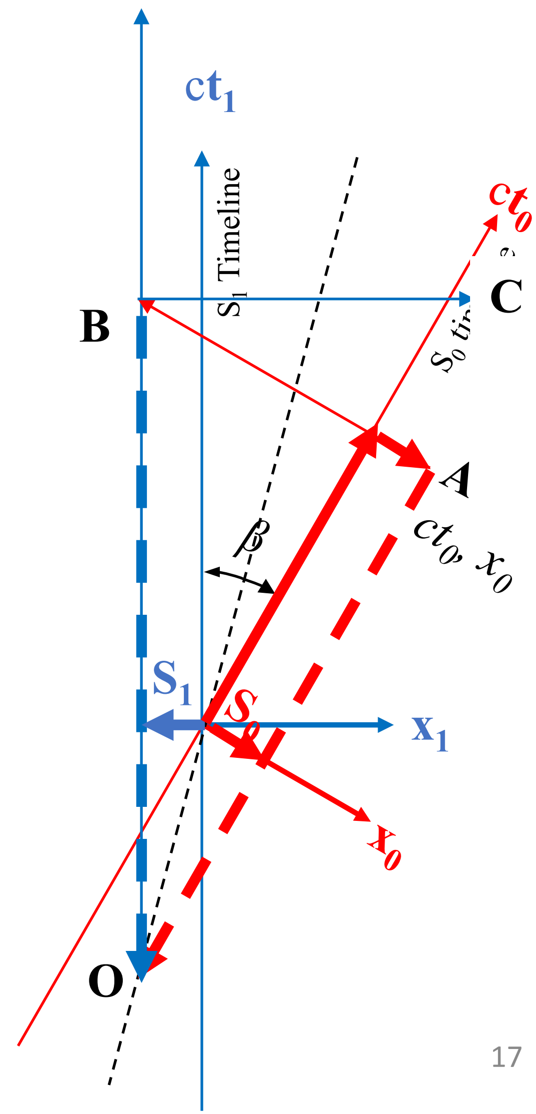
- EXTEND S_0 TIMELINE THROUGH A PARALLEL TO ct_0 TO INTERSECT AXIS OF SYMMMETRY AT O
- ADD x_1 OFFSET TO S_1 EQUAL TO $-x_0$
- EXTEND OFFSET TIMELINE TO INTERSECT AXIS OF SYMMETRY ALSO AT O



- EXTEND S_0 TIMELINE THROUGH A PARALLEL TO ct_0 TO INTERSECT AXIS OF SYMMMETRY AT O
- ADD x_1 OFFSET TO S_1 EQUAL TO $-x_0$
- EXTEND OFFSET TIMELINE TO INTERSECT AXIS OF SYMMETRY ALSO AT O
- OAB NOW FORMS VELOCITY TRIANGLE WITH COMMON POINT O

  $ct_0=ct_1=-x_0/tan(\beta/2)$

  $x_1=-x_0$



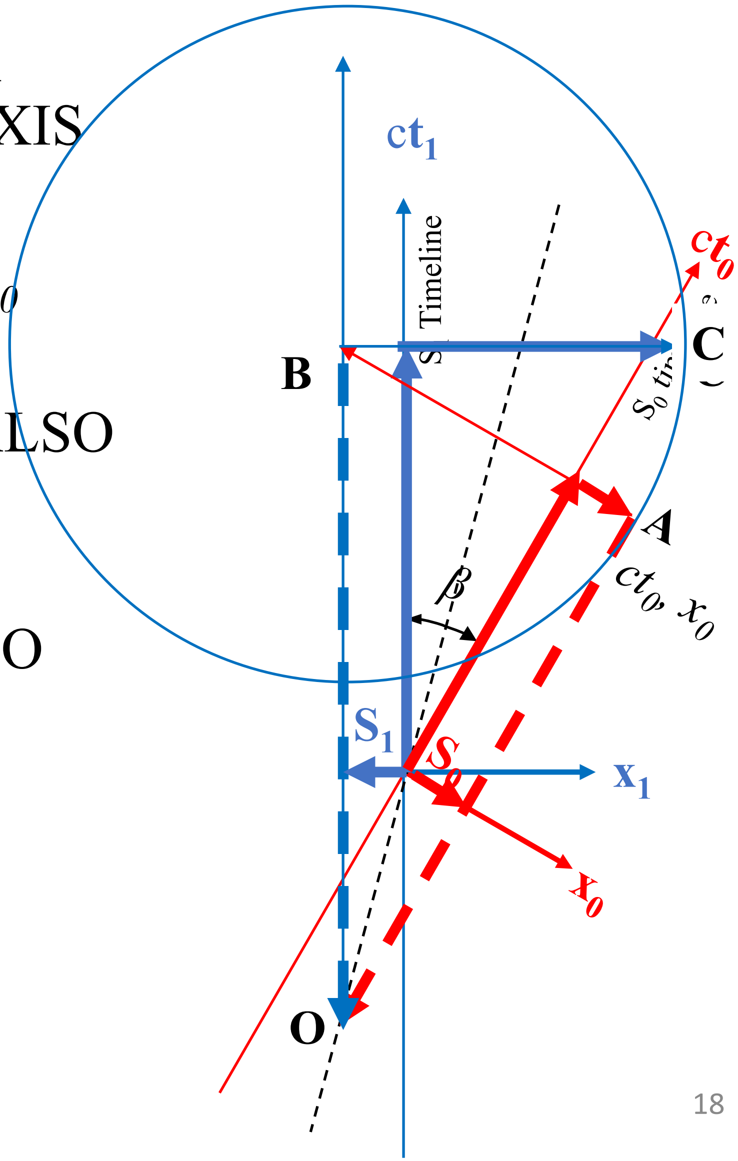
- EXTEND S_0 TIMELINE THROUGH A PARALLEL TO ct_0 TO INTERSECT AXIS OF SYMMMETRY AT O
- ADD x_1 OFFSET TO S_1 EQUAL TO $-x_0$
- EXTEND OFFSET TIMELINE TO INTERSECT AXIS OF SYMMETRY ALSO AT O
- OAB NOW FORMS VELOCITY TRIANGLE WITH COMMON POINT O

  $ct_0 = ct_1 = -x_0/\tan(\beta/2)$

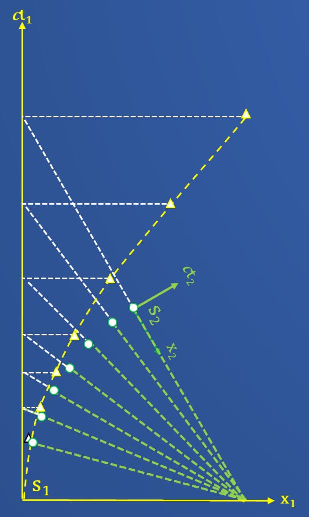
  $x_1 = -x_0$
- SUBTRACT ct_1 AND x_1 OFFSETS TO OBTAIN LORENTZ TRANSFORM

  $OB = [ct_0 + x_o \cdot \sin(\beta)]/\cos(\beta) + x_0/\tan(\beta/2)$

  $BC = BA = [x_0 + ct_o \cdot \sin(\beta)]/\cos(\beta) + x_0$



A Shortcut Solution



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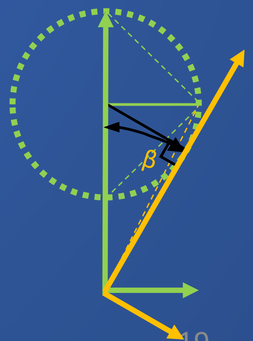
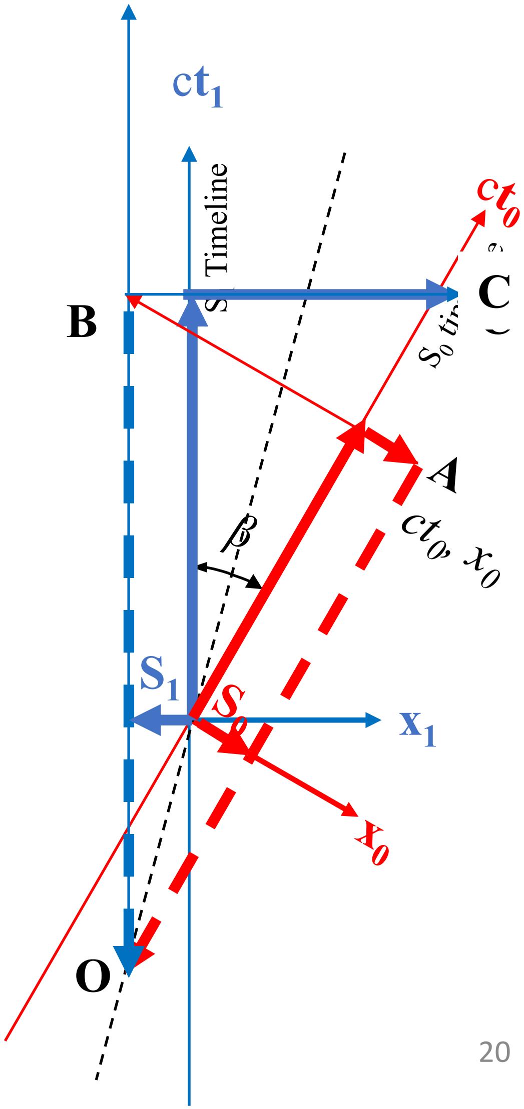
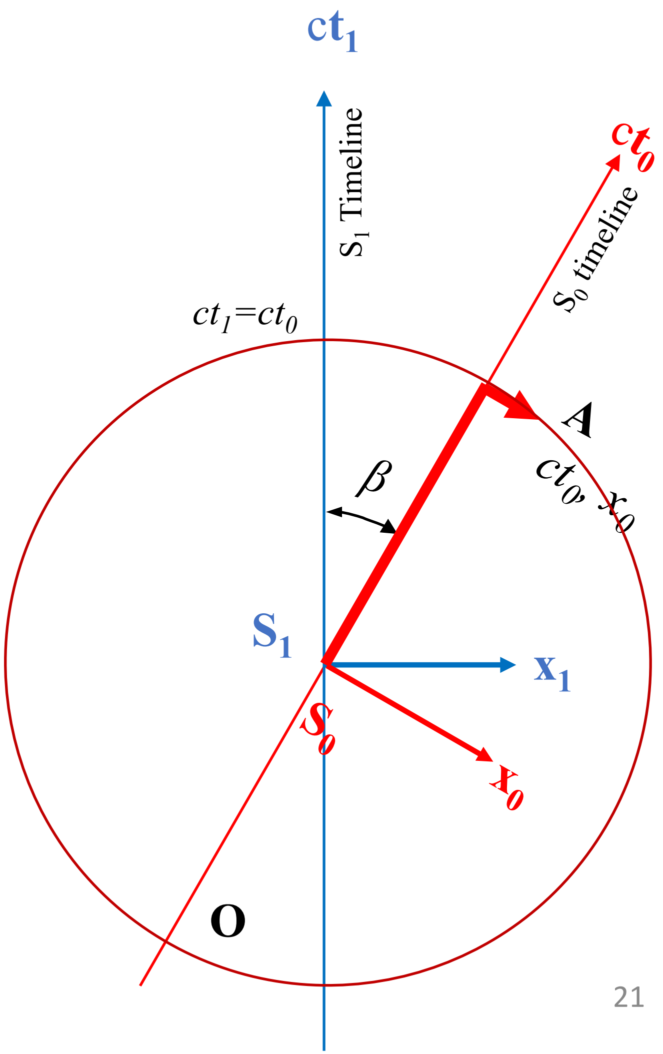


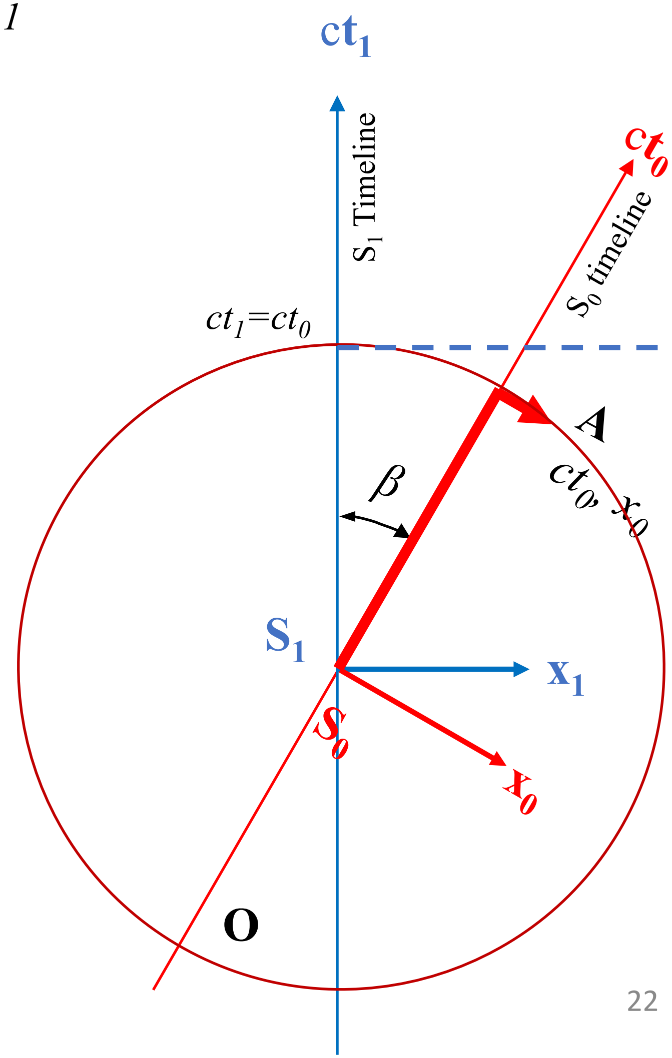
- A LOT OF GRAPHIC OVERHEAD!



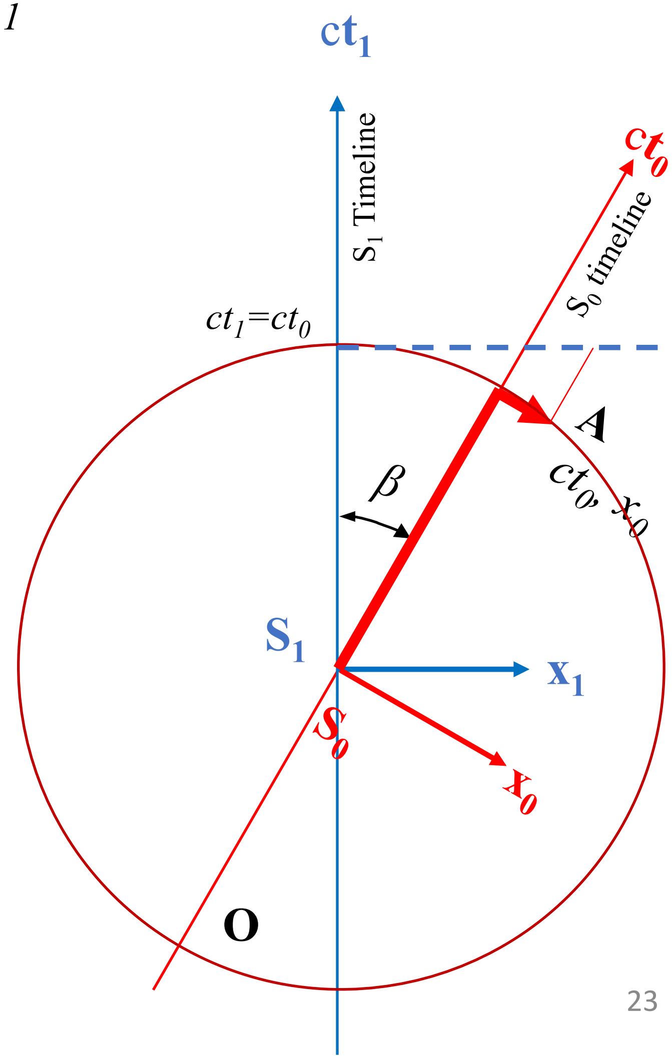
- LOCATE $ct_1=ct_0$ ON THE S_1 TIMELINE



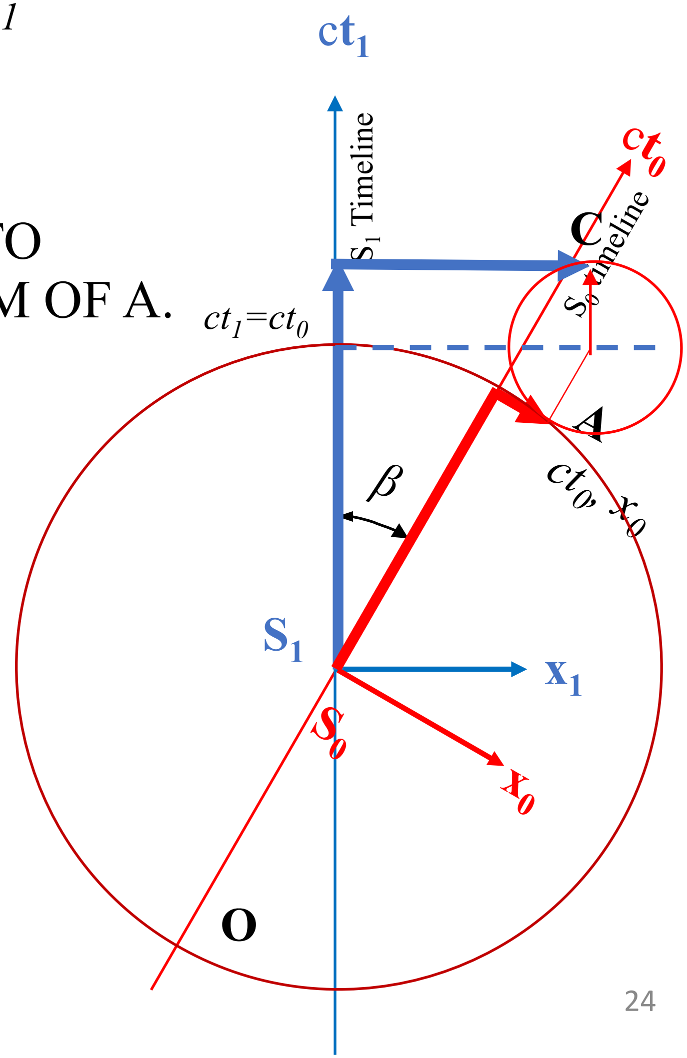
- LOCATE $ct_1 = ct_0$ ON THE S_1 TIMELINE
- ERECT A TEMPORARY x_1 AXIS AT ct_1



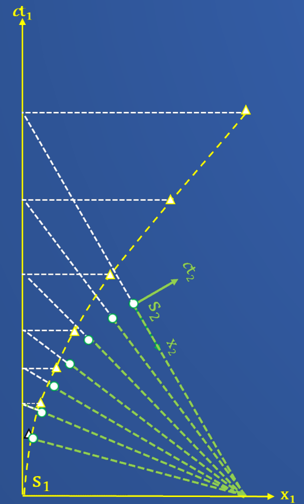
- LOCATE $ct_1 = ct_0$ ON THE S_1 TIMELINE
- ERECT A TEMPORARY x_1 AXIS AT ct_1
- FROM A, PARALLEL TO ct_0 TO INTERCEPT x_1 AXIS



- LOCATE $ct_1=ct_0$ ON THE S_1 TIMELINE
- ERECT A TEMPORARY x_1 AXIS AT ct_1
- FROM A, PARALLEL TO ct_0 TO INTERCEPT x_1 AXIS
- FROM x_1 AXIS AN EQUAL DISTANCE TO LOCATE C, THE LORENTZ TRANSFORM OF A.



Simultaneous Points in S_0



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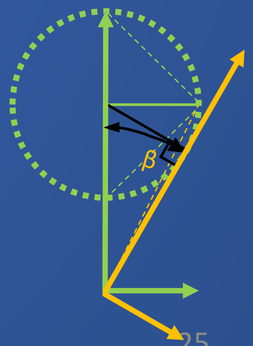
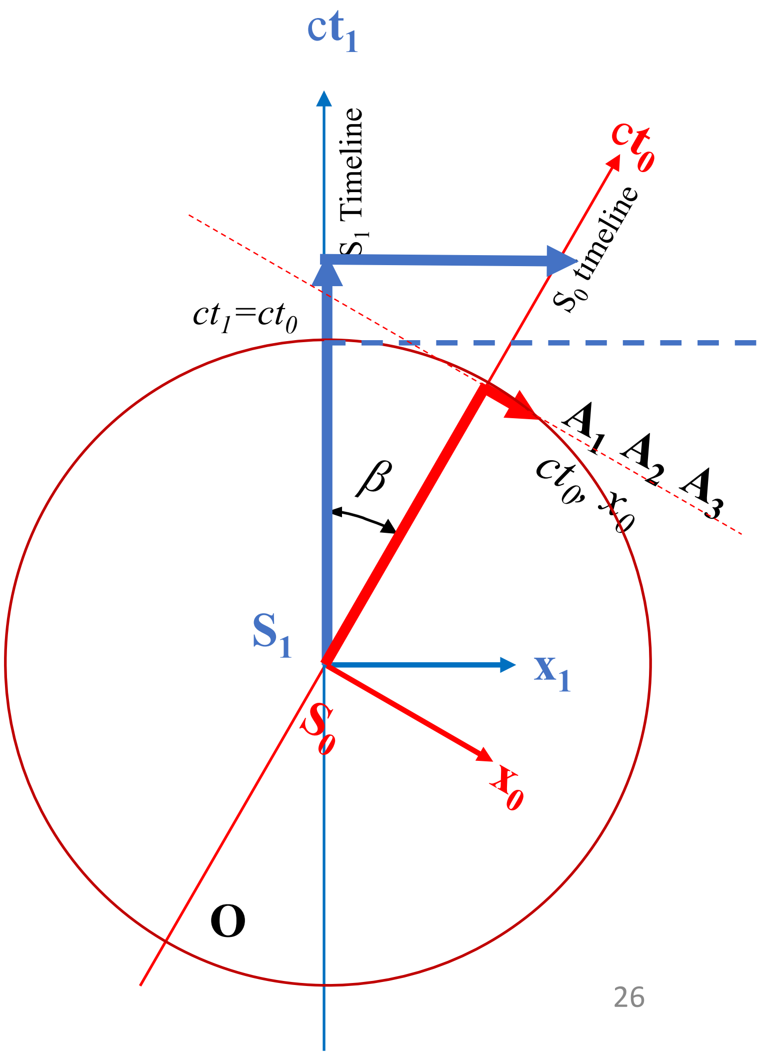
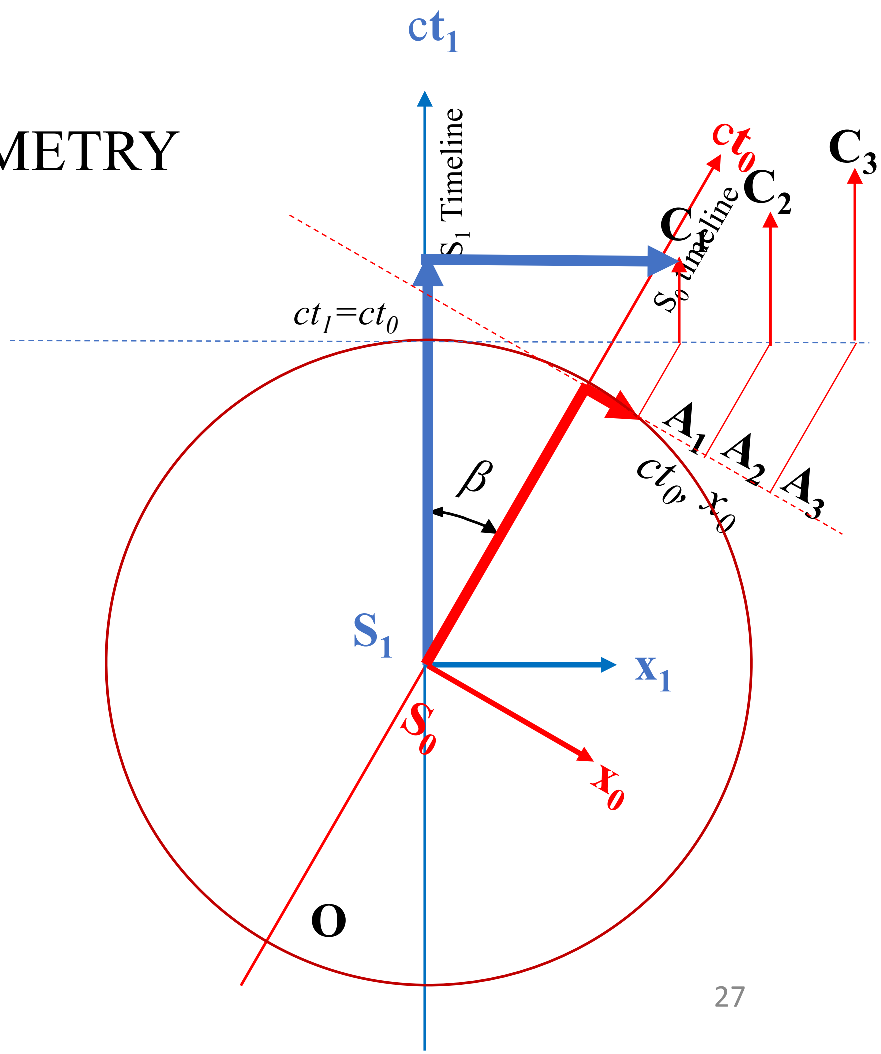


- MULTIPLE MEASUREMENTS A_1-A_3 SIMULTANEOUS IN S_0 …



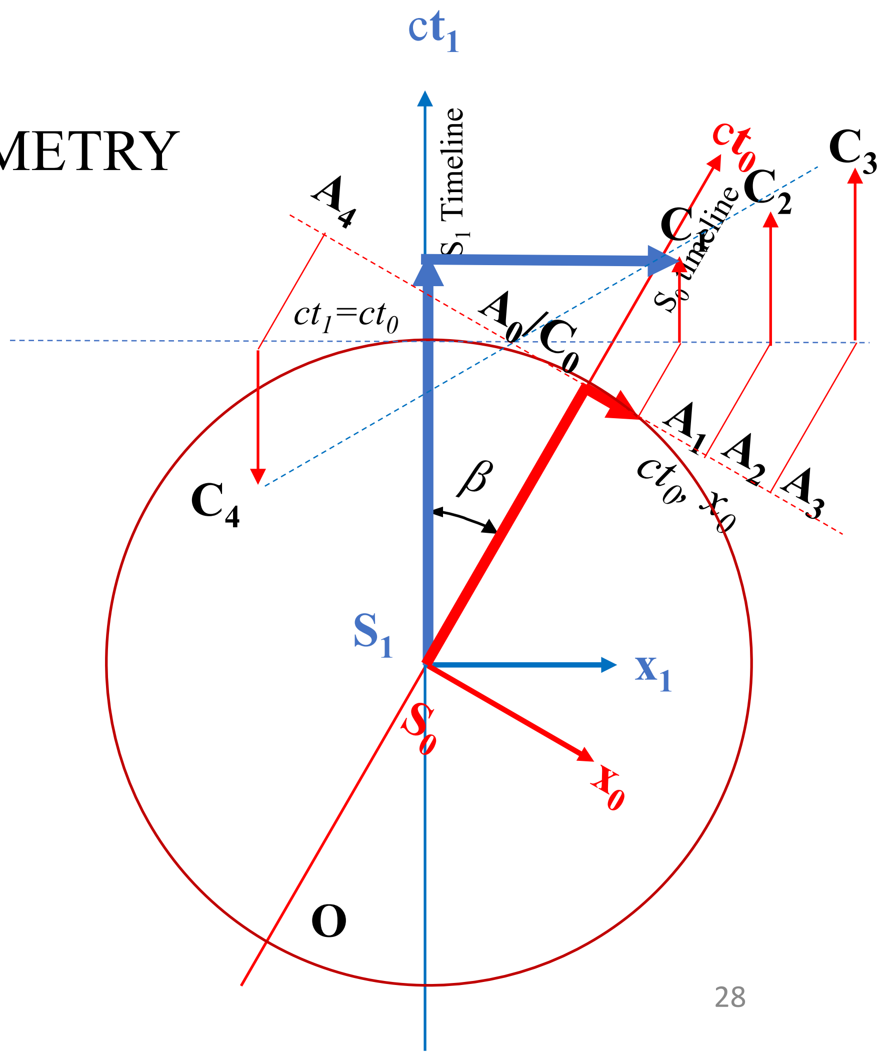
THE POINT ON THE AXIS OF SYMMETRY HAS THE SAME TIME IN S_0 AND S_1



THE POINT ON THE AXIS OF SYMMETRY HAS THE SAME TIME IN S_0 AND S_1

TRANSFORMATION POINTS C_0-C_4 LIE ON SLOPE IN S_1

$dct_1/dx_1 = (dct_1/dct_0)/(dx_1/dct_0) = sin(\beta)$



SUMMARY

- AXIS OF SYMMETRY
  - Previously Unknown Property of the Lorentz Transform
  - Allows the solution of all problems with simple graphics

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