Chesapeake Section of AAPT

Delaware State University, Dover

PHYSICS OF AN ACCELERATING UNICYCLE

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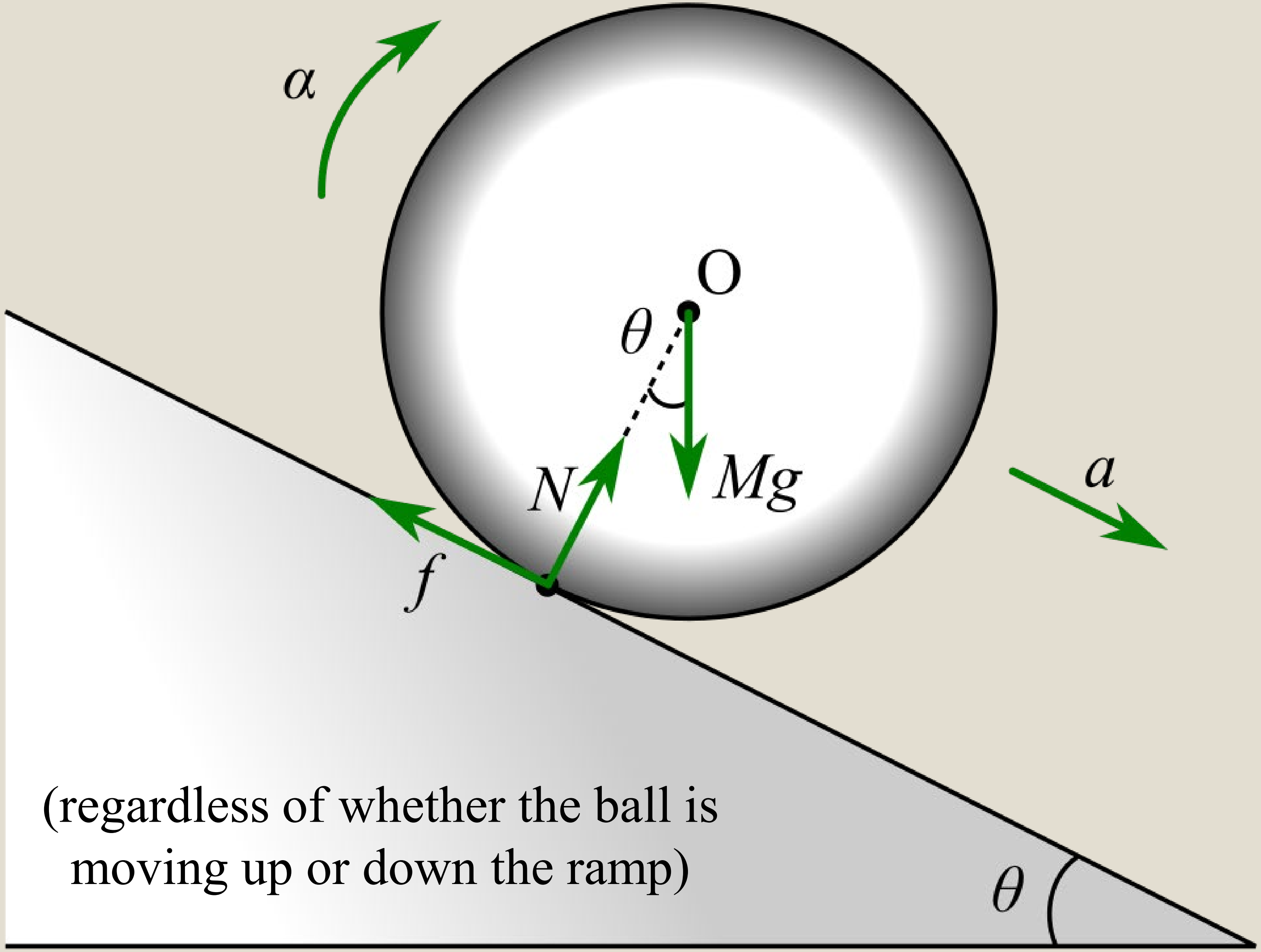
A ball rolls along a ramp without slipping.
(Neglect air drag and rolling friction throughout this talk.)

Why doesn’t the ball slide instead of rolling?

<u>Answer</u>: Because of static friction.
Let’s see how that works by drawing a FBD.



(regardless of whether the ball is moving up or down the ramp)

Does $f = \mu N$

where μ is the coefficient of static friction?

Hopefully not,

because otherwise the ball will be about to slip.

Specifically suppose θ is small enough and μ large enough that $\tan\theta < \mu$ to avoid that.

In that case, how do we find an expression for f?

Answer: We need it to get α so use N2L.

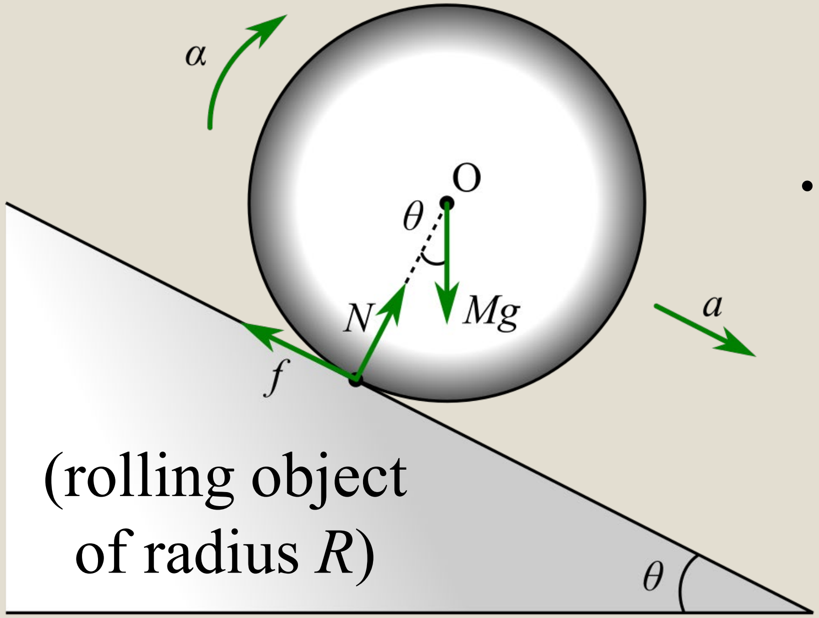
translations of O: $Mg\sin\theta - f = Ma$

rotations about O: $fR = \gamma MR^2 \dfrac{a}{R} \Rightarrow f = \gamma Ma$

add the two equations: $\cancel{M}\, g\sin\theta = (1+\gamma)\,\cancel{M}\, a$

$\therefore\ a = \dfrac{g\sin\theta}{1+\gamma}$ and $f = \dfrac{\gamma Mg\sin\theta}{1+\gamma}$

e.g. $\gamma = \frac{2}{5}$ for solid sphere or $\frac{1}{2}$ for uniform disk



(rolling object of radius *R*)

$$\boxed{a = \frac{g\sin\theta}{1+\gamma} \quad \text{and} \quad f = \frac{\gamma M g \sin\theta}{1+\gamma}}$$

Now let $\theta = 0$ so we're on a horizontal surface.

Then $f = 0$! Does that mean it <u>cannot</u> roll without slipping?

<u>Answer</u>: No, but $a = 0$ and $\alpha = 0$ so it must be rolling at constant translational speed υ and angular speed ω with $\upsilon = R\omega$.

But what if we want to speed up or slow down the rolling?

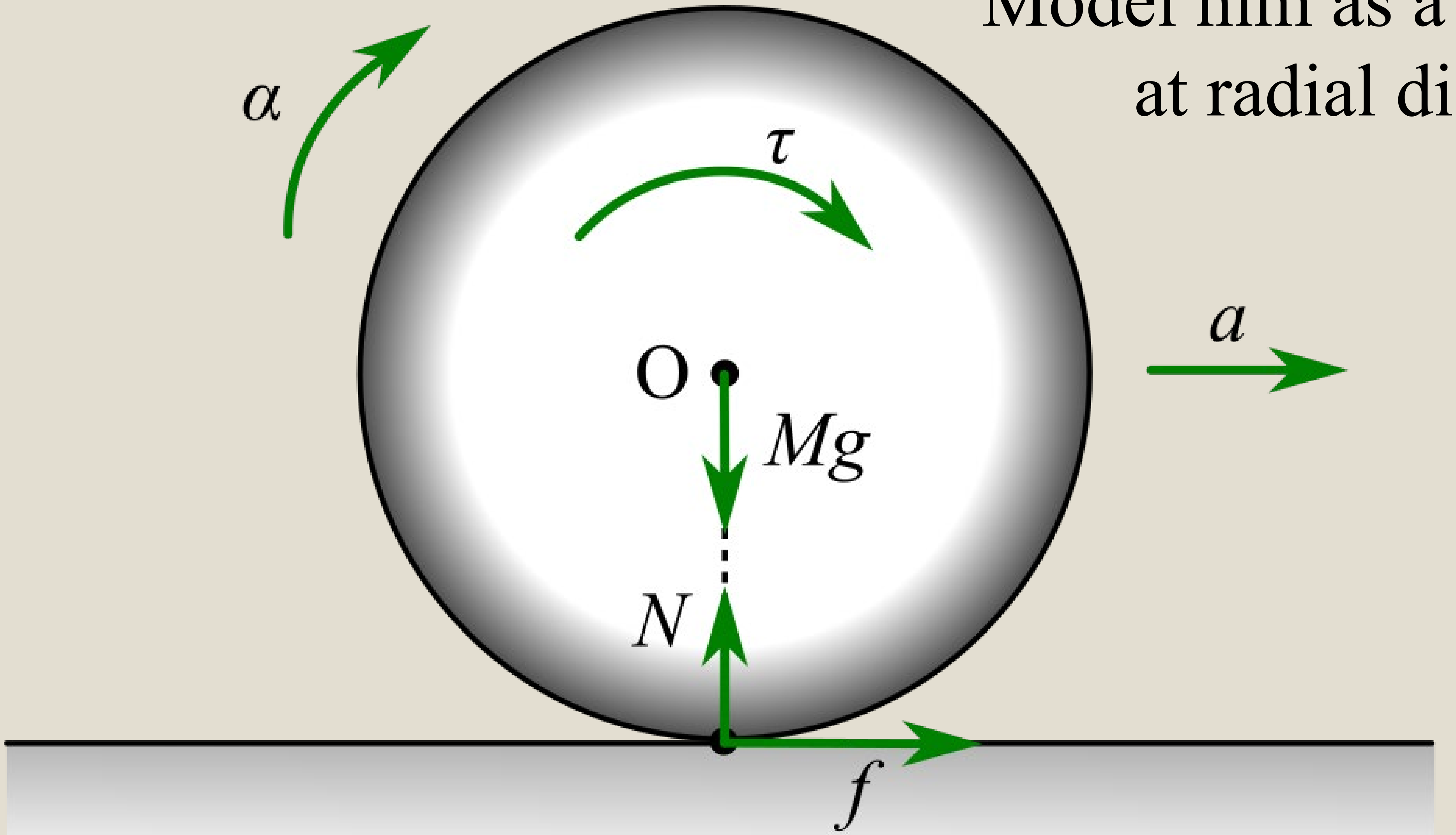
Good question! As an example of a single-wheeled object that can increase in speed, consider a unicycle.

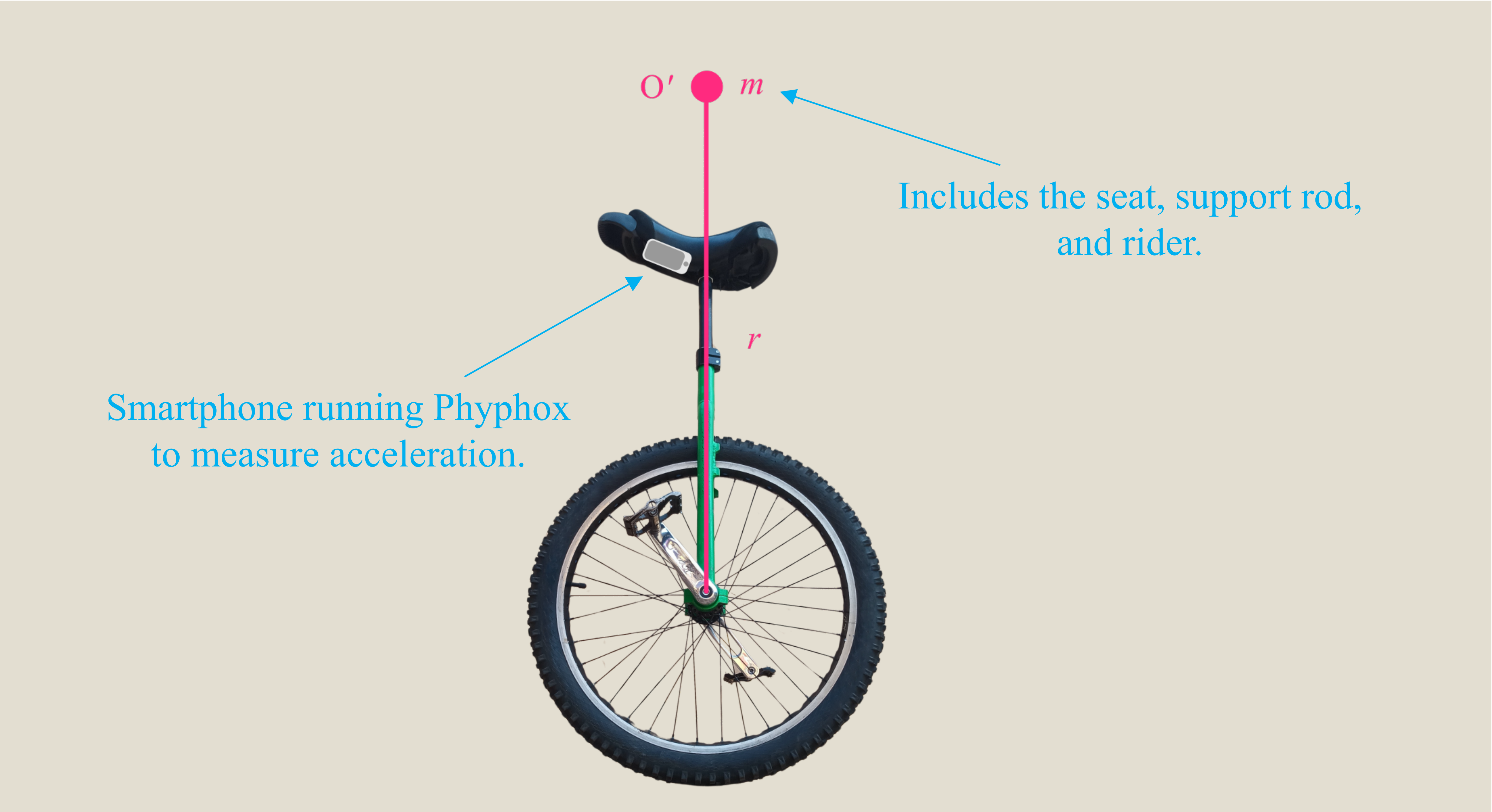
There must now be a forward static frictional force to produce the forward translational acceleration.

But the frictional torque then tries to angularly decelerate the wheel. Thus the driving torque τ applied by the rider must be opposite in direction and larger in magnitude than it.

partial FBD:

We're missing the rider!
Model him as a point mass m
at radial distance r.



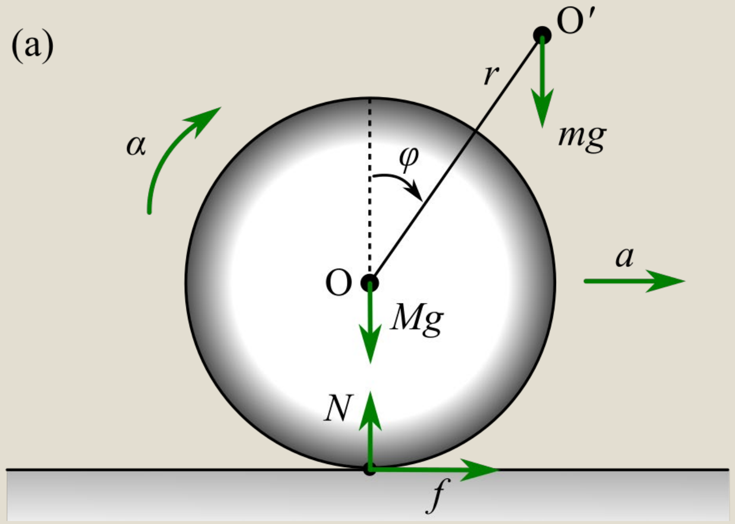


Includes the seat, support rod, and rider.

Smartphone running Phyphox to measure acceleration.

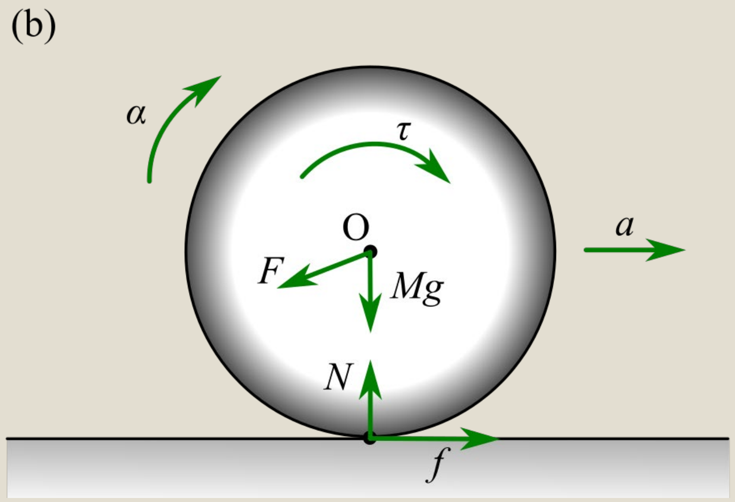
FBD:

for whole system



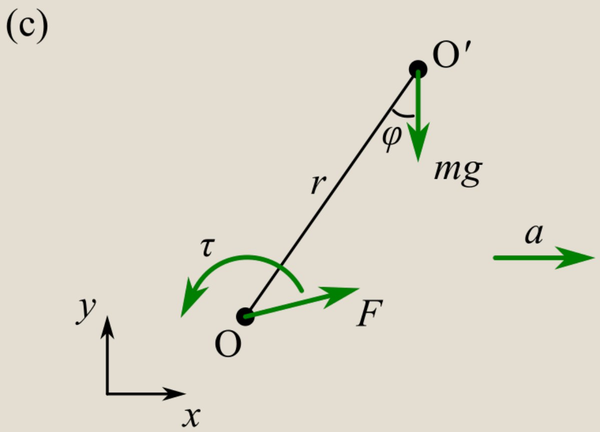
Rider must
tilt forward by φ
to accelerate forward
as panel (c) shows.

for wheel alone



Rider applies
driving torque τ
and reaction force F
to the wheel.

for rider alone



Equal & opposite F
and τ as on the wheel
by N3L.
Rider is accelerated
forward by F_x and
must tilt to counter τ.

Apply N2L for translations and rotations
both to the wheel alone and to the rider alone.
Use small-angle approximation for φ.
(Details in upcoming paper in *Physics Education*.)

acceleration of unicycle: $$a = \frac{\tau / R}{m + (1+\gamma)M}$$

tilt angle to balance torques on rider: $$\varphi = \frac{\tau}{g}\left[\frac{1}{mr} + \frac{1}{mR + (1+\gamma)MR}\right]$$

frictional coefficient to avoid slipping: $$\mu_{\min} = \frac{\tau / Rg}{m + (1+\gamma)M}$$

(all three are proportional to the applied torque)

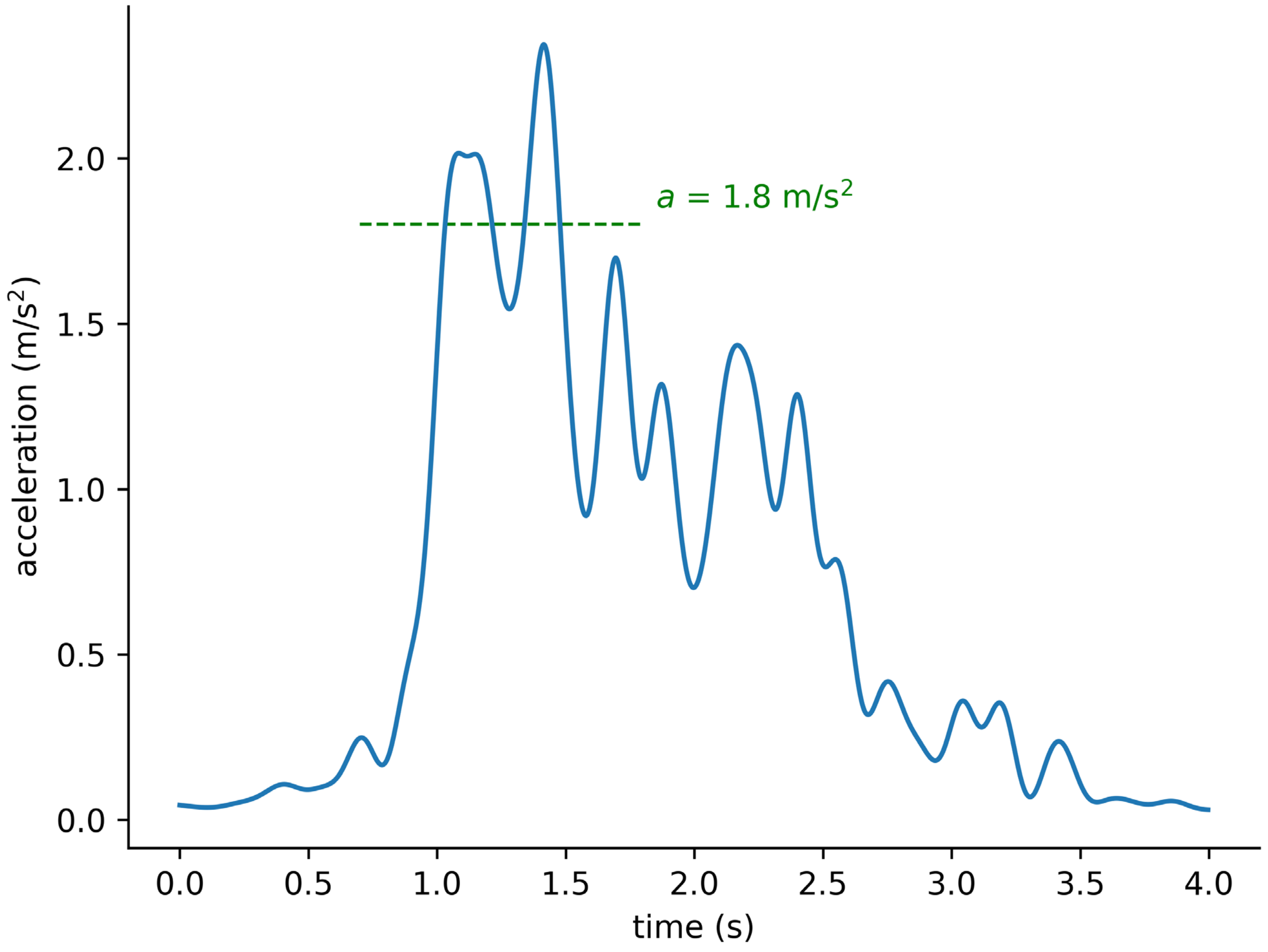
Compare theory to experiment

Variable	**Value**
M	6.0 kg
m	65 kg
γ	0.90
R	0.38 m
r	1.1 m
g	9.8 N/kg

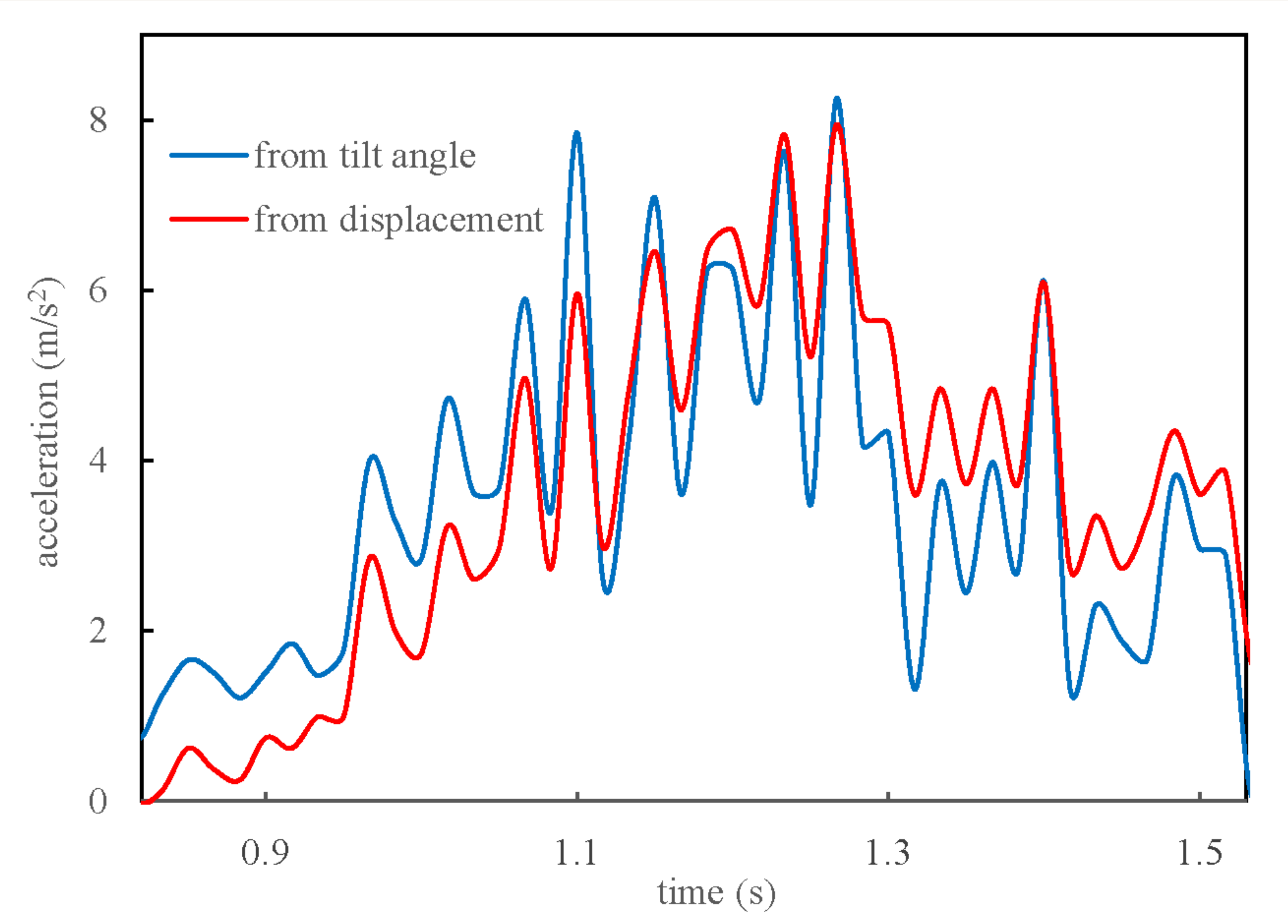
Accelerating by tilting 15° and then returning to constant velocity:

(Ph.D. student
Heiko Kabutz at
Univ. of CO Boulder)

Measured acceleration in blue versus theoretical peak acceleration in green:



More detailed comparison from video analysis of tilt and displacement:



(used Tracker to measure coordinates of axle and of seat)

Comments or questions?

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