A Picture is Worth 1000 Equations: The Art of Relativity

Gerald Feldman

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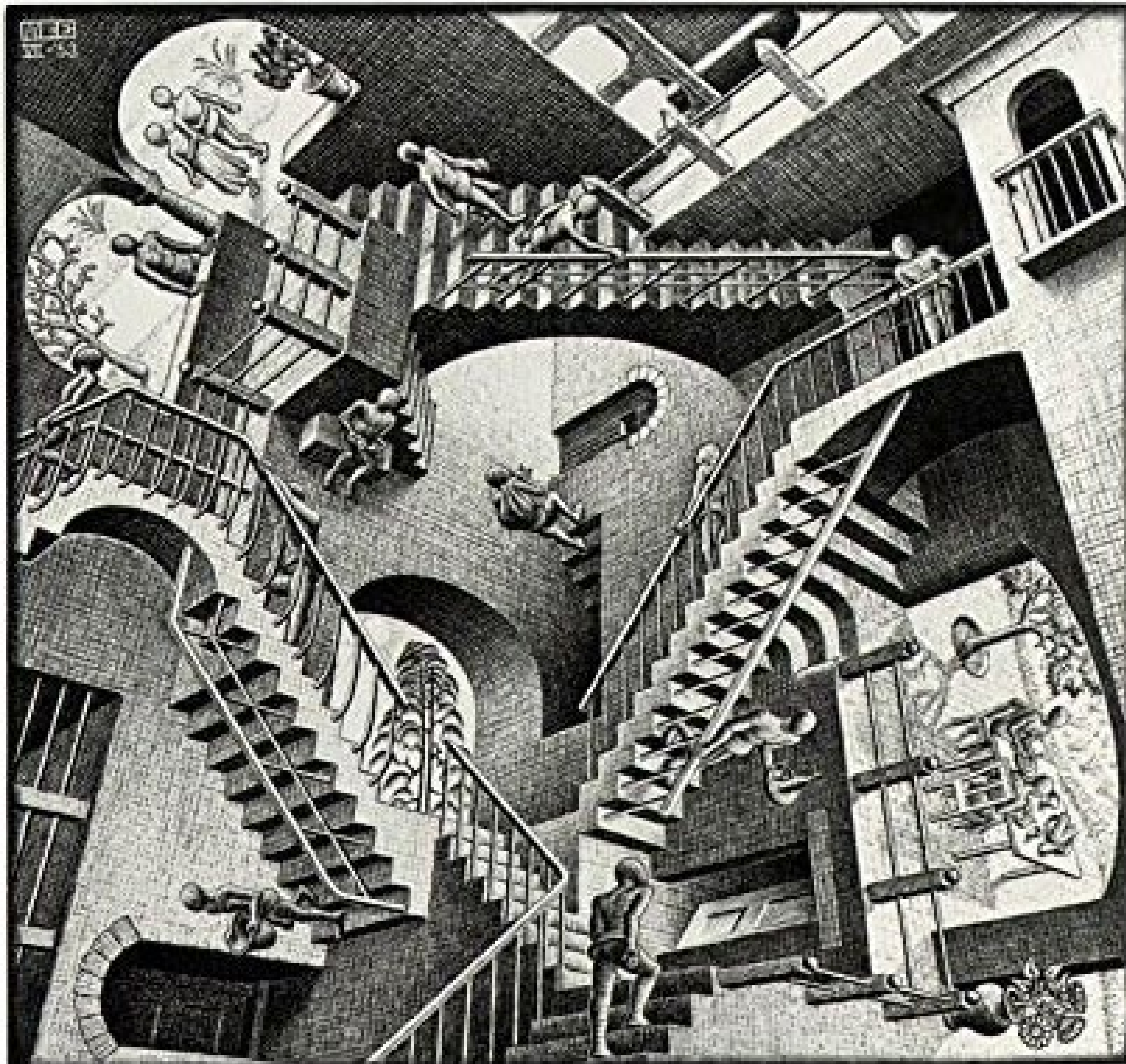
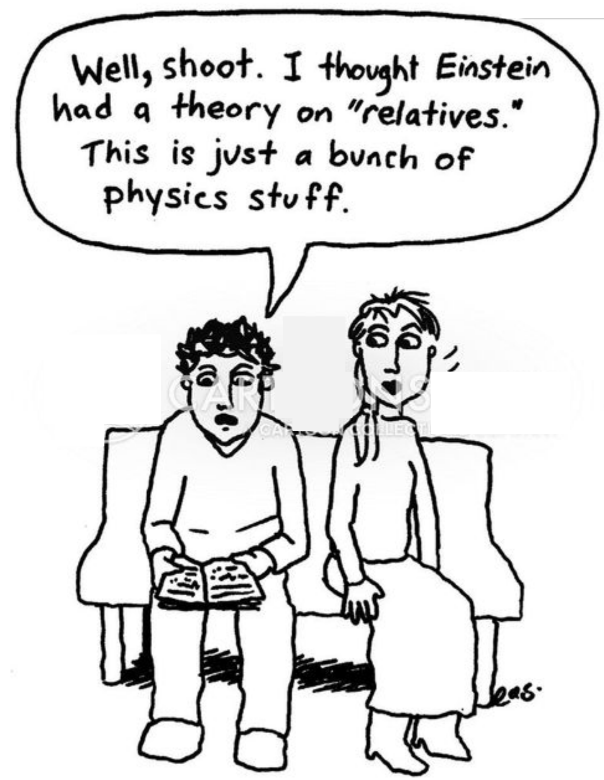
George Washington University
Washington, DC



Special Relativity in Modern Physics

- **Modern Physics**: usually in 2nd undergraduate year
  - ✓ Special Relativity
  - ✓ Quantum Physics
  - ✓ Atomic Physics
  - ✓ Selected topics (nuclear, solid state, molecular, astro, etc.)

- **Special Relativity**
  - ✓ first 1-2 chapters of MP textbook
  - ✓ relativistic kinematics
    - Lorentz Transformations
    - time dilation, length contraction
  - ✓ relativistic dynamics
    - energy and momentum



PHYS 2170 Foundations of Modern Physics (University of Colorado)

Covers **special relativity**, quantum mechanics and atomic structure. Completes the three-semester sequence of general physics for physics and engineering physics majors.

PHYSICS 264L Optics and Modern Physics (Duke University)

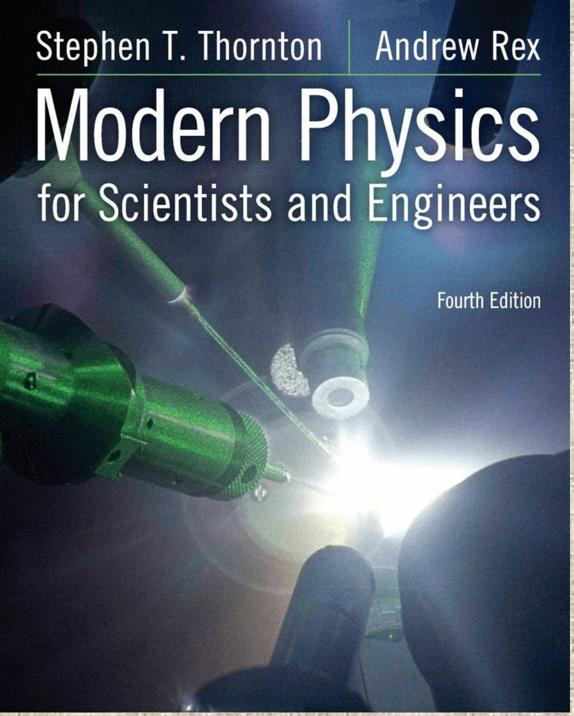
Third course in sequence for physics and biophysics majors. Introductory treatments of **special relativity** and quantum mechanics. Topics include: wave mechanics and interference; **relativistic kinematics, energy and momentum**; the Schrodinger equation and its interpretation; quantum particles in one-dimension; spin; fermions and bosons; the hydrogen spectrum. Applications to crystallography, semiconductors, atomic physics and optics, particle physics, and cosmology.

PHYS 2620 Modern Physics (University of Virginia)

Introduction to quantum physics and **relativity**, with application to atomic structure, nuclear and elementary particle physics, condensed matter physics, and cosmology. Three lecture hours, one problem hour.

PHYS 237 Introduction to Modern Physics (Penn State University)

Relativity and quantum theory with applications to selected topics in atomic, molecular, solid state, or nuclear physics. The course covers modern physics curriculum focusing on **special relativity**, concepts and formalism of quantum mechanics in one- and three-dimensional systems, and applications of quantum theory to modern topics such as atomic/molecular, nuclear, particle, condensed matter physics or astrophysics.


Stephen T. Thornton
Andrew Rex
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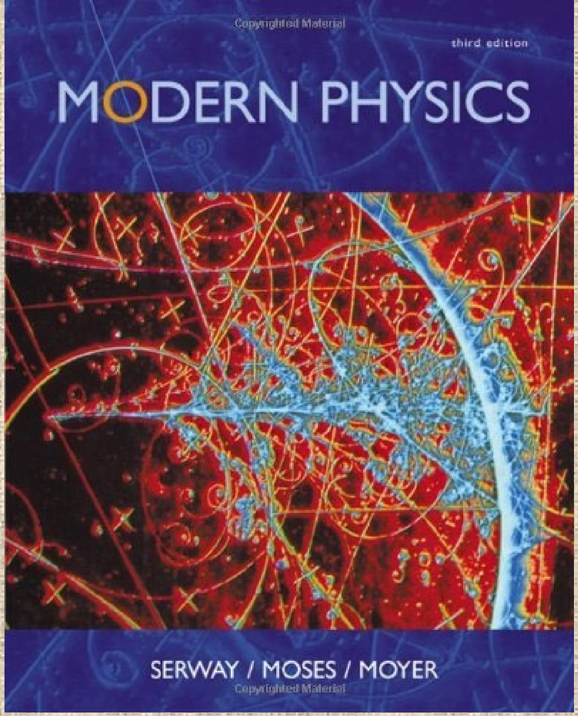
for Scientists and Engineers
Fourth Edition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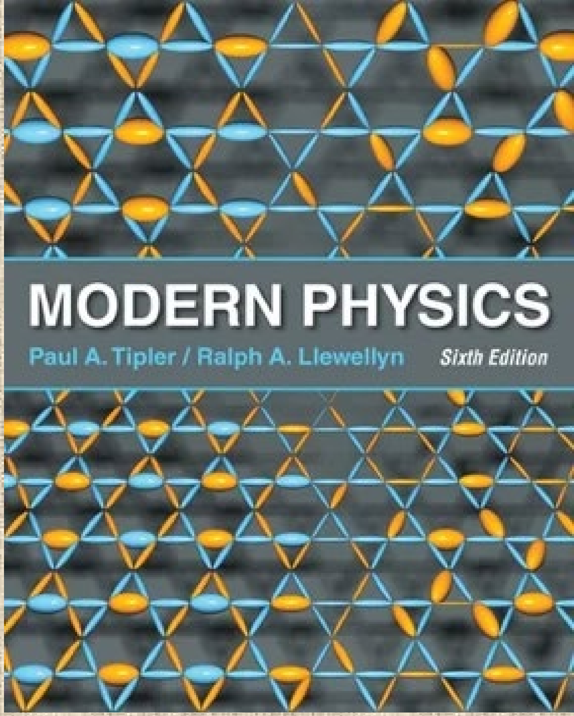
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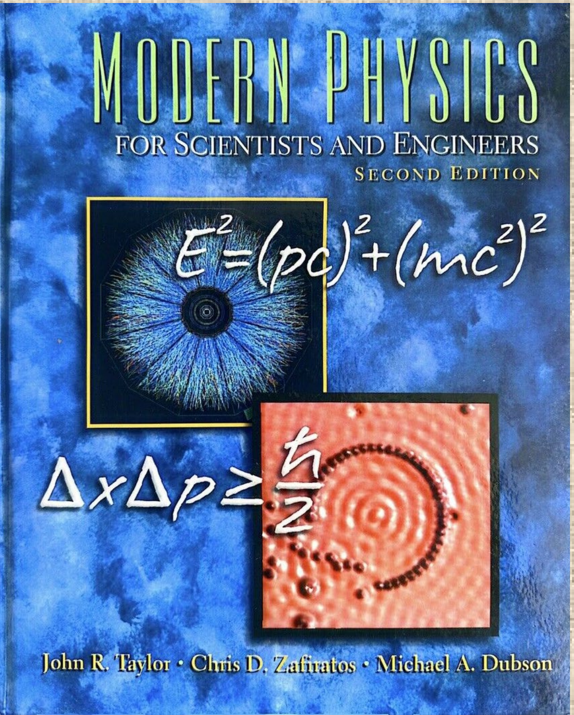
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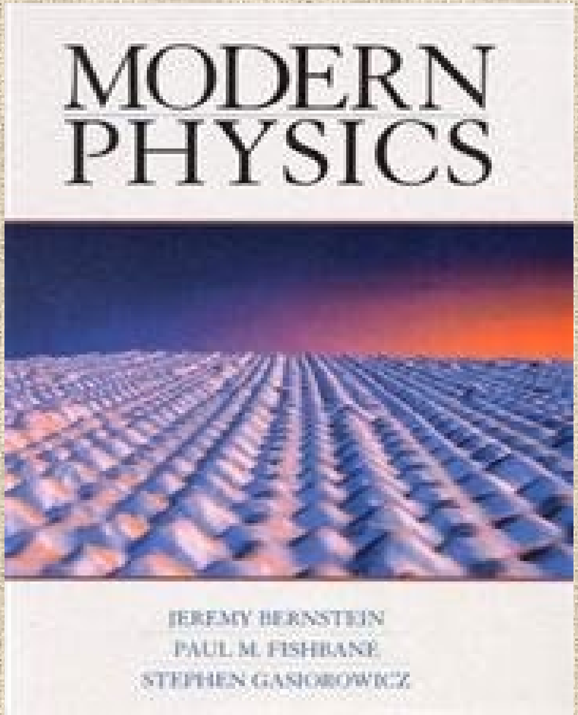
FOR SCIENTISTS AND ENGINEERS
SECOND EDITION
$E^2=(pc)^2+(mc^2)^2$
$\Delta x \Delta p \geq \frac{\hbar}{2}$
John R. Taylor · Chris D. Zafiratos · Michael A. Dubson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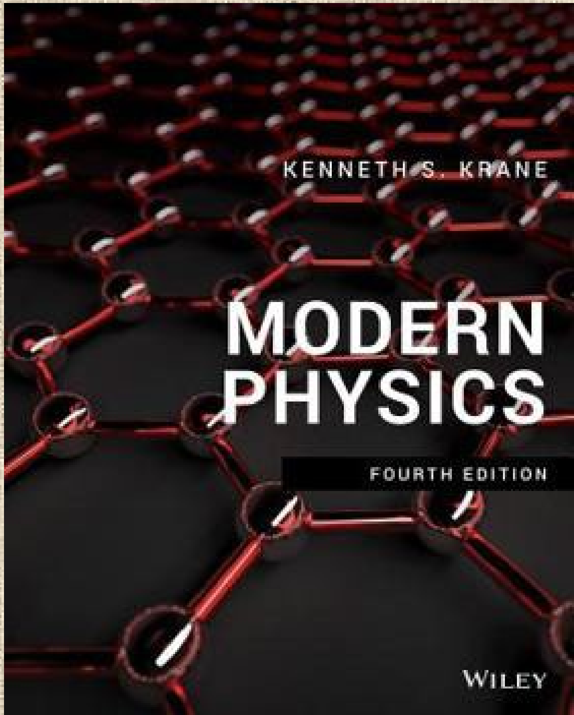
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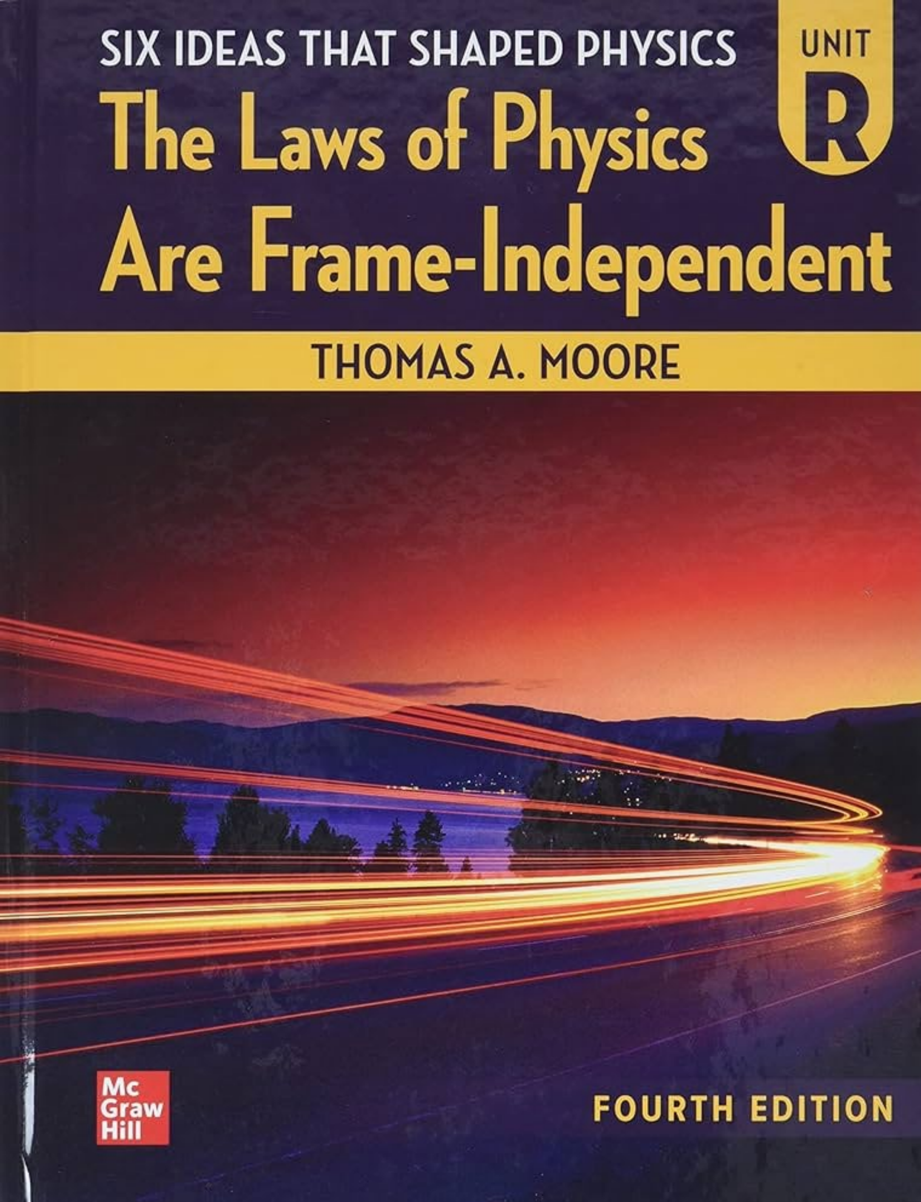
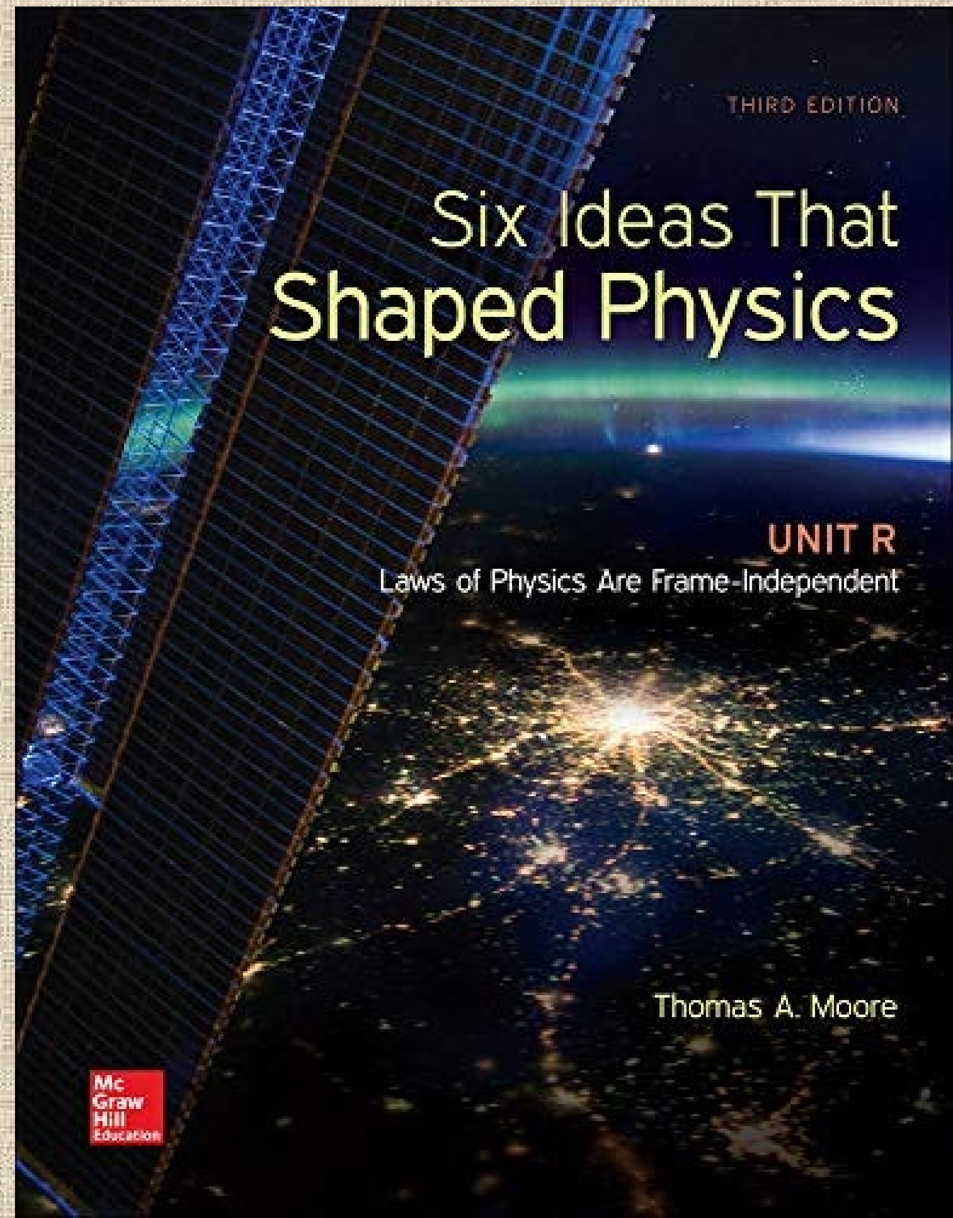
WILEY

Serway Moses Moyer

Chapter 2

Thornton and Rex


SIX IDEAS THAT SHAPED PHYSICS
UNIT R
The Laws of Physics Are Frame-Independent
THOMAS A. MOORE
Mc Graw Hill
FOURTH EDITION


THIRD EDITION
Six Ideas That Shaped Physics
UNIT R
Laws of Physics Are Frame-Independent
Thomas A. Moore
Mc Graw Hill Education

SIX IDEAS THAT SHAPED PHYSICS

UNIT N

The Laws of Physics Are Universal

SIX IDEAS THAT SHAPED PHYSICS

UNIT R

The Laws of Physics Are Frame-Independent

THOMAS A. MOORE

SIX IDEAS THAT SHAPED PHYSICS

UNIT C

Conservation Laws Constrain Interactions

SIX IDEAS THAT SHAPED PHYSICS

UNIT E

Electric and Magnetic Fields are Unified

SIX IDEAS THAT SHAPED PHYSICS

UNIT T

Some Processes Are Irreversible

SIX IDEAS THAT SHAPED PHYSICS

UNIT Q

Particles Behave Like Waves

Class	Date	Chapter	Topics
1	Tues., Jan. 17	R1, R2	Principle of Relativity, Coordinate Time
2	Thurs., Jan. 19	R3	The Spacetime Interval
3	Tues., Jan. 24	R4	Proper Time
4	Thurs., Jan. 26	R5	Coordinate Transformations
5	Tues., Jan. 31	R6	Lorentz Contraction
6	Thurs., Feb. 2	R7	The Cosmic Speed Limit
7	Tues., Feb. 7	R8	Four-Momentum
8	Thurs., Feb. 9	R9	Conservation of Four-Momentum
9	Tues., Feb. 14	3	Quantum Theory of Light
10	Thurs., Feb. 16	R1-R9	Review
$\pi^2 + \frac{1}{2}$	Fri., Feb. 17	R1-R9	Exam
11	Tues., Feb. 21	3	Quantum Theory of Light
12	Thurs., Feb. 23	4	Particle Nature of Matter
13	Tues., Feb. 28	4	Particle Nature of Matter
14	Thurs., Mar. 2	5	Matter Waves
15	Tues., Mar. 7	5	Matter Waves
16	Thurs., Mar. 9	6	Quantum Mechanics in 1D
17	Tues., Mar. 21	6	Quantum Mechanics in 1D
18	Thurs., Mar. 23	7	Tunneling Phenomena
19	Tues., Mar. 28	8	Quantum Mechanics in 3D
20	Thurs., Mar. 30	8	Quantum Mechanics in 3D
21	Tues., Apr. 4	9	Atomic Structure
22	Thurs., Apr. 6	9	Atomic Structure
23	Tues., Apr. 11	13	Nuclear Structure
24	Thurs., Apr. 13	3-9	Review
$\sqrt{600}$	Fri., Apr. 14	3-9	Exam
25	Tues., Apr. 18	13	Nuclear Structure
26	Thurs., Apr. 20	13/14	Nuclear Structure / Nuclear Applications
27	Tues., Apr. 25	14	Nuclear Physics Applications
28	Thurs., Apr. 27	14	Nuclear Physics Applications

George Washington University

PHYS 2023 Modern Physics

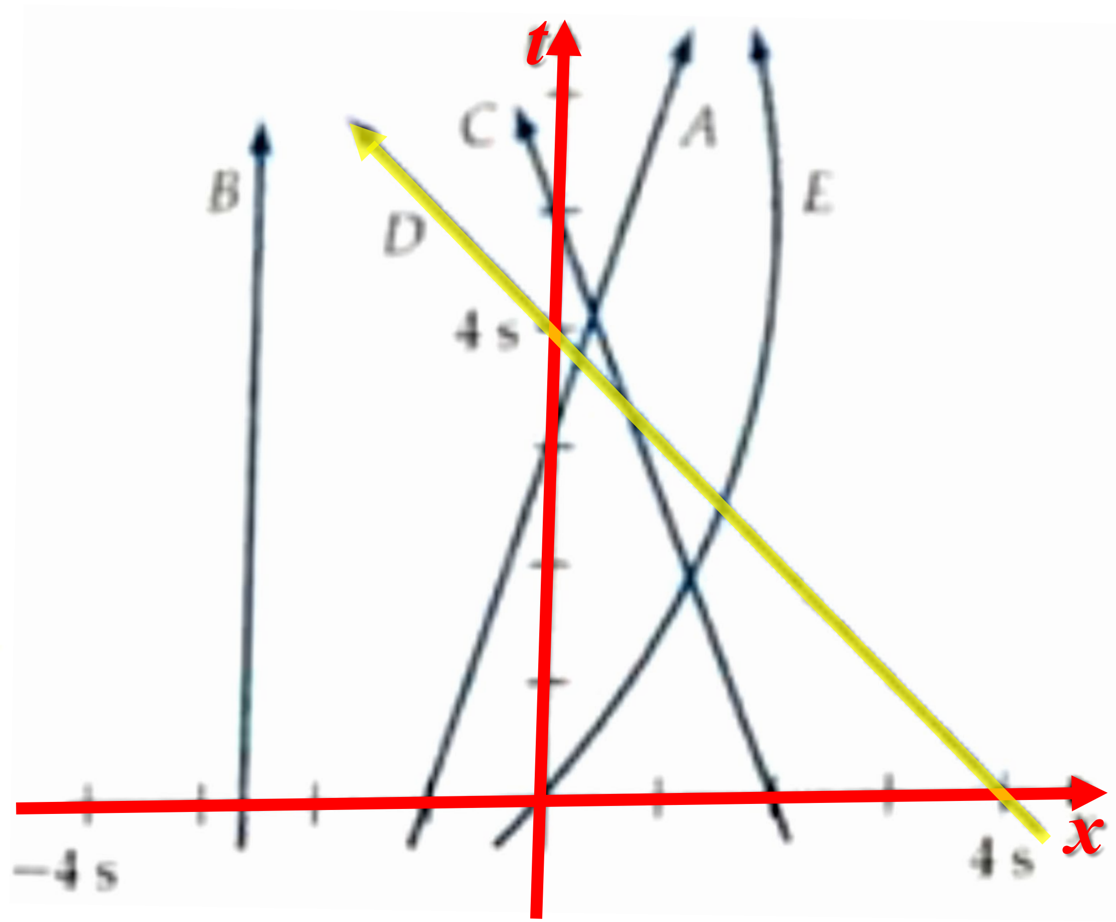
Chapter R1	The Principle of Relativity
R2	Coordinate Time
R3	The Spacetime Interval
R4	Proper Time
R5	Coordinate Transformations
R6	Lorentz Contraction
R7	The Cosmic Speed Limit
R8	Four-Momentum
R9	Conservation of Four-Momentum

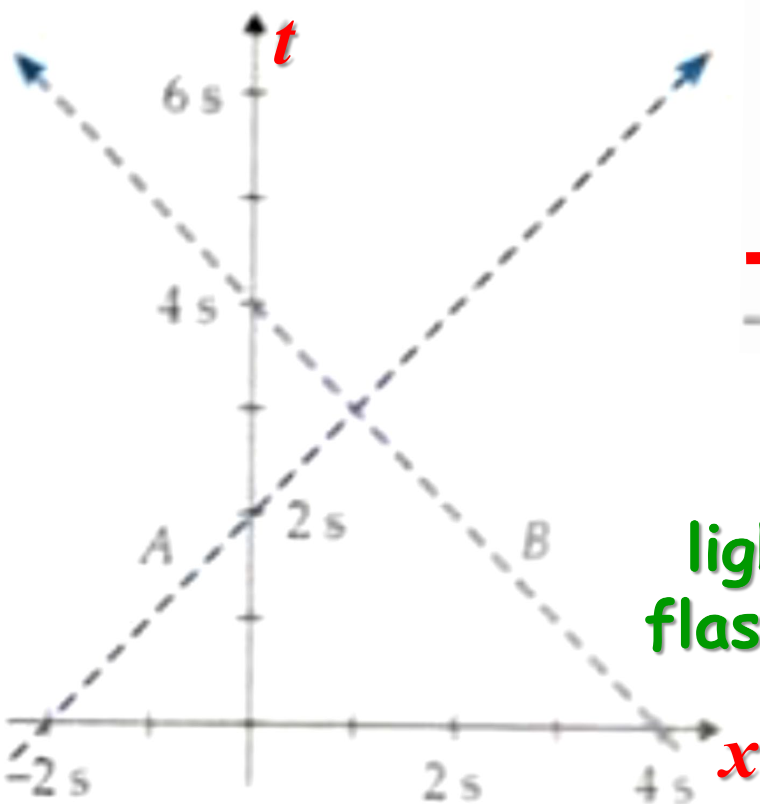
kinematics

dynamics

Spacetime Diagram

(worldlines)

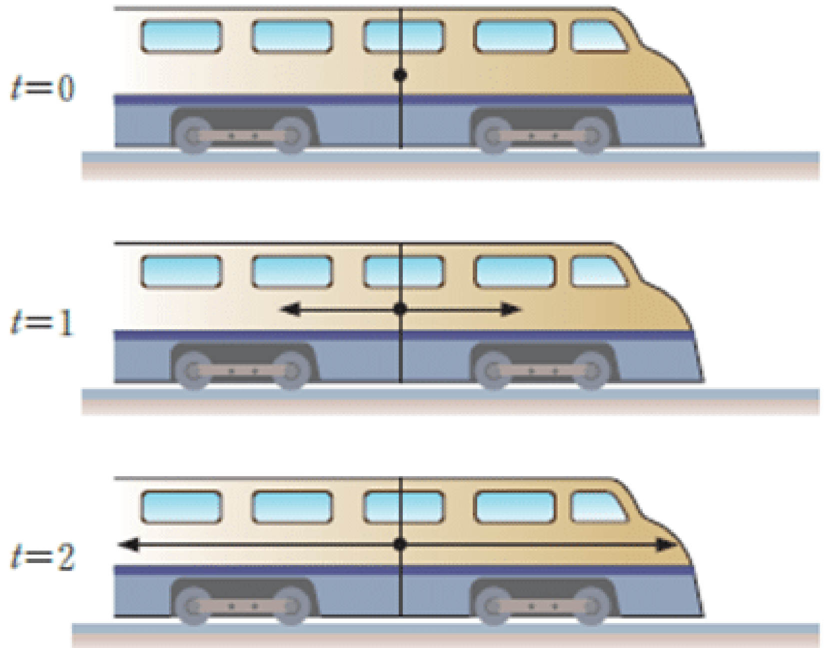




light flashes

simultaneity

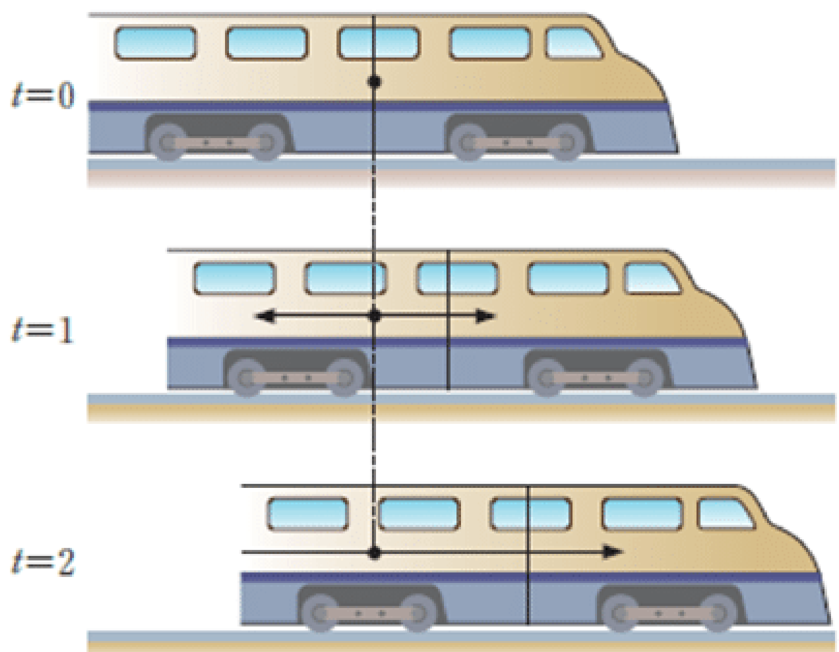
observer in the train



HIT left

HIT right

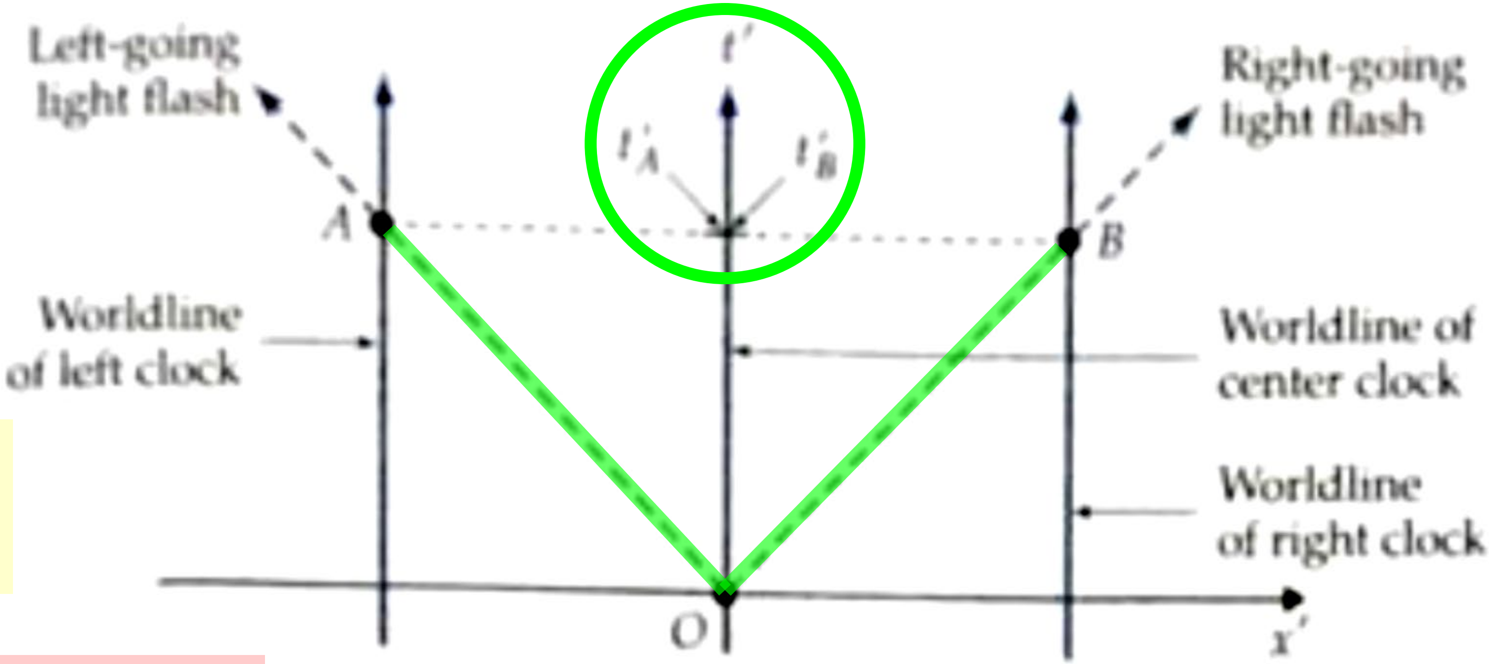
observer on the ground



HIT left

not yet...

train frame



simultaneity

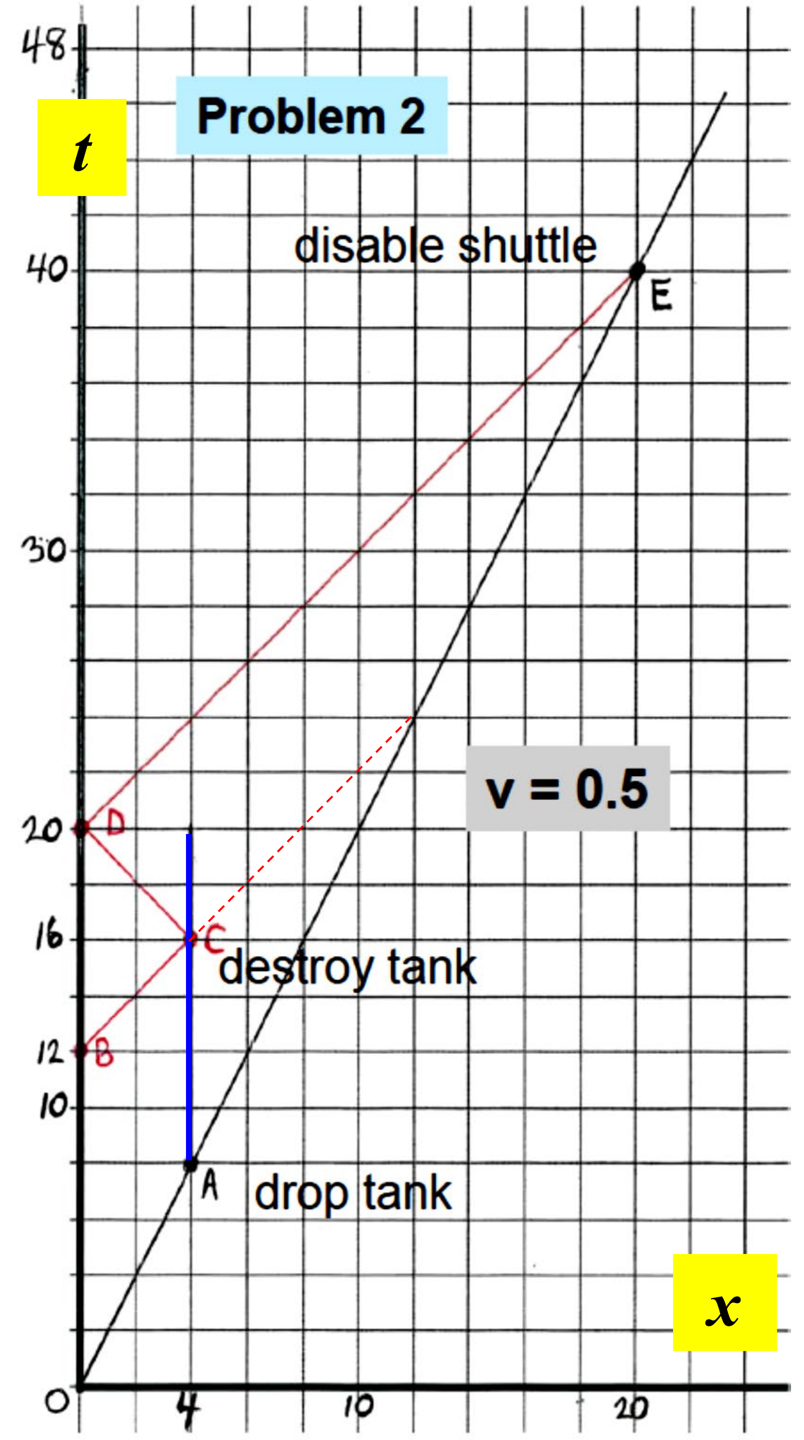
ground frame

At midnight (t = 0 hr), while the space station is asleep, renegades steal a shuttle and escape. The ship is slow, only capable of v = 0.5, so the renegades fear that they will be caught. When they reach x = 4 hr from the station, they come up with the brilliant idea of dropping a fuel tank behind them (event A), leaving it at rest to block laser blasts coming from the station.

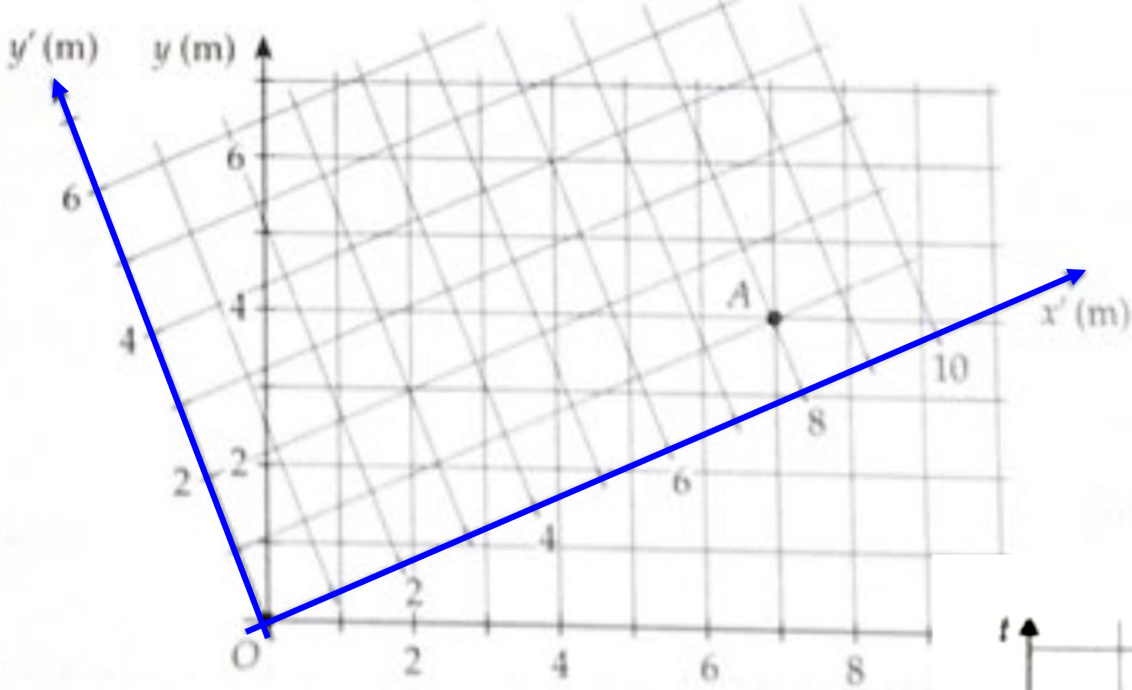
The space station crew has overslept and finally wakes up at 12:00 noon (t = 12 hr). When they realize what has happened, they immediately fire on the stolen shuttle (event B), but the shot is blocked by the fuel tank, which is destroyed (event C). When the light from the explosion reaches the space station (event D), the station crew realizes that their first shot was unsuccessful, so they immediately fire a second laser blast at the shuttle, which does eventually hit it (event E), disabling the ship and foiling the renegades' escape.

Draw a spacetime diagram of this story.
Identify the (x,t) spacetime coordinates of these events.

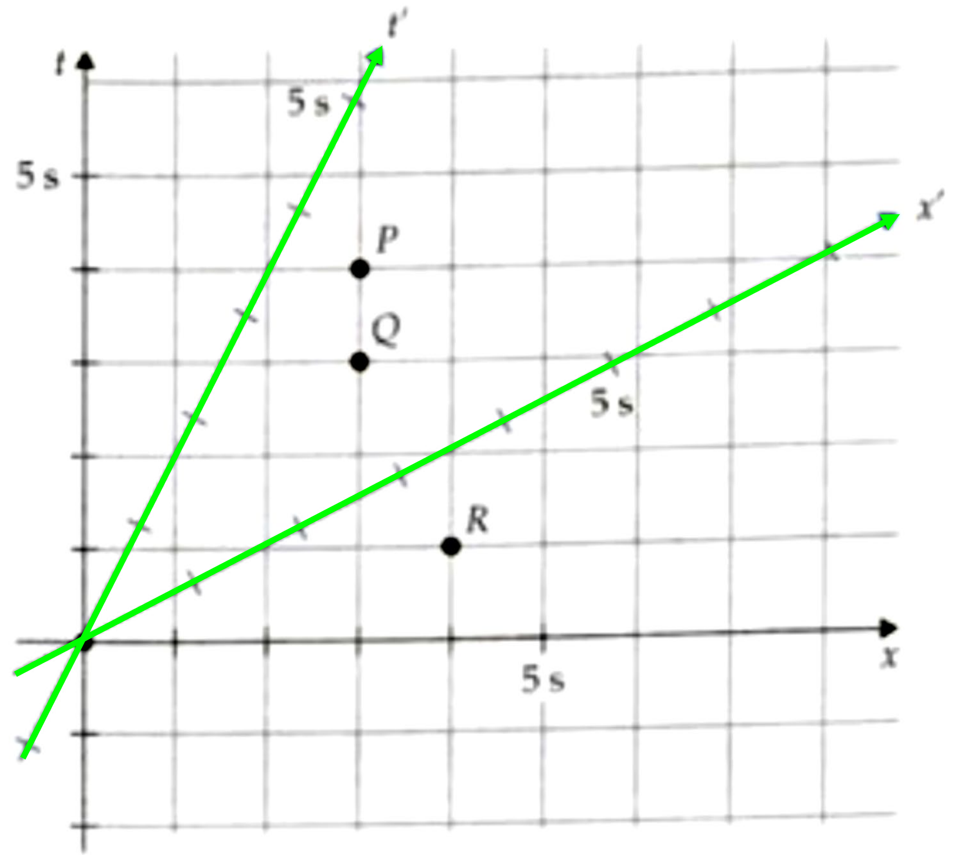
Quiz Question



Coordinate Transformations



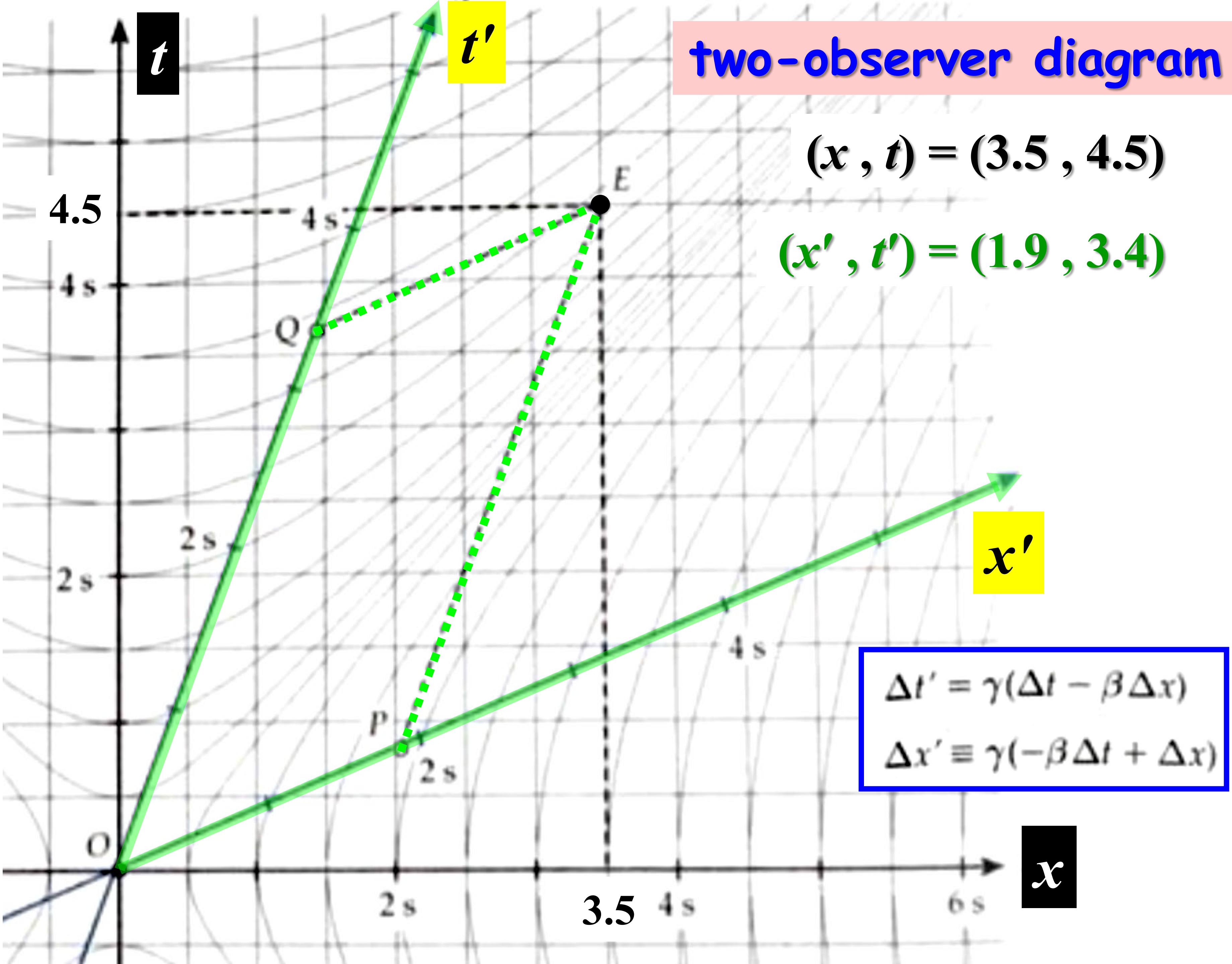
Two-observer Diagram



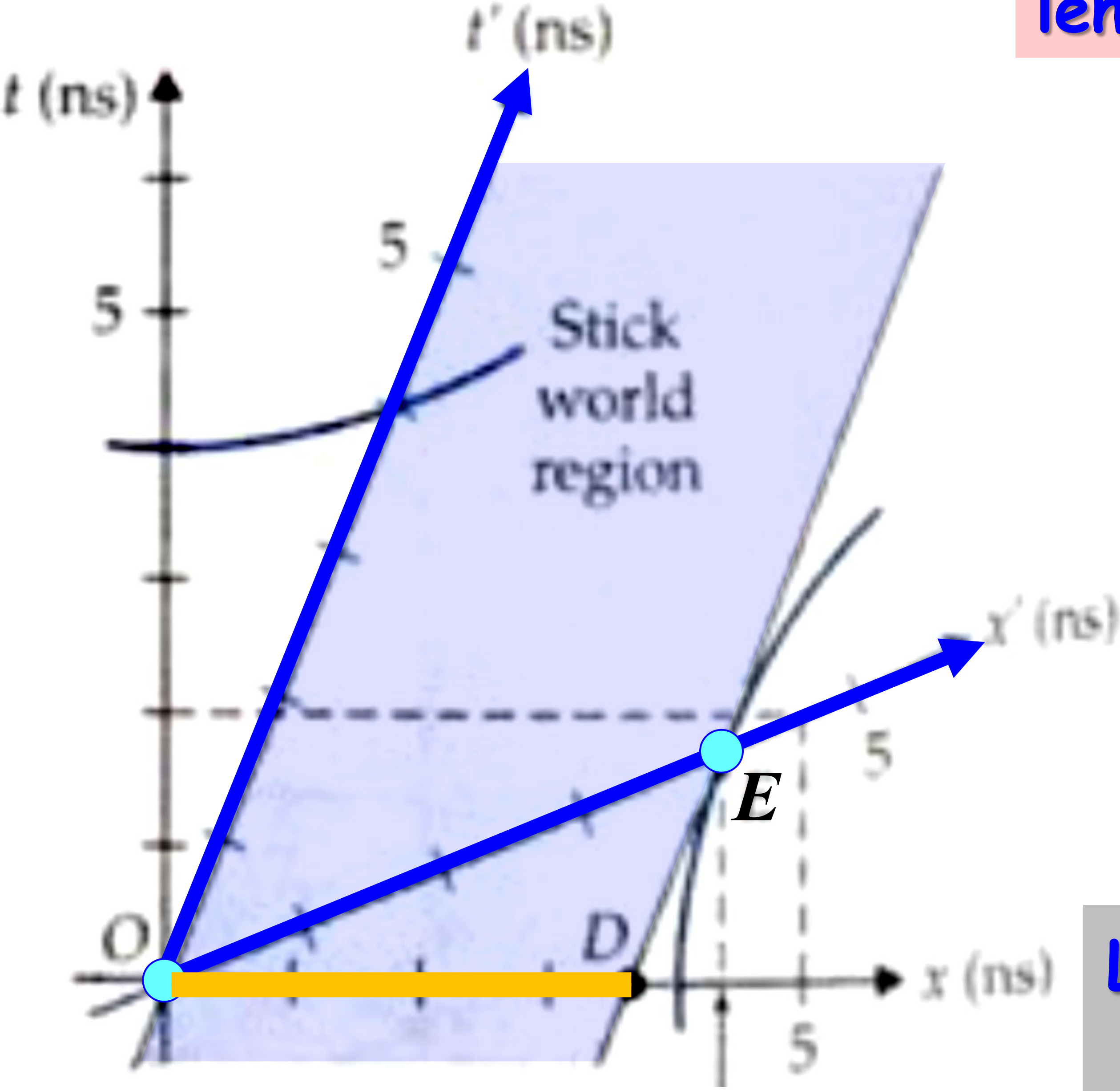
two-observer diagram

$(x, t) = (3.5, 4.5)$

$(x', t') = (1.9, 3.4)$



length contraction



$v = 0.4$

$\gamma = 1.09$

rest length

$L_0 = 4\,\text{ns}$

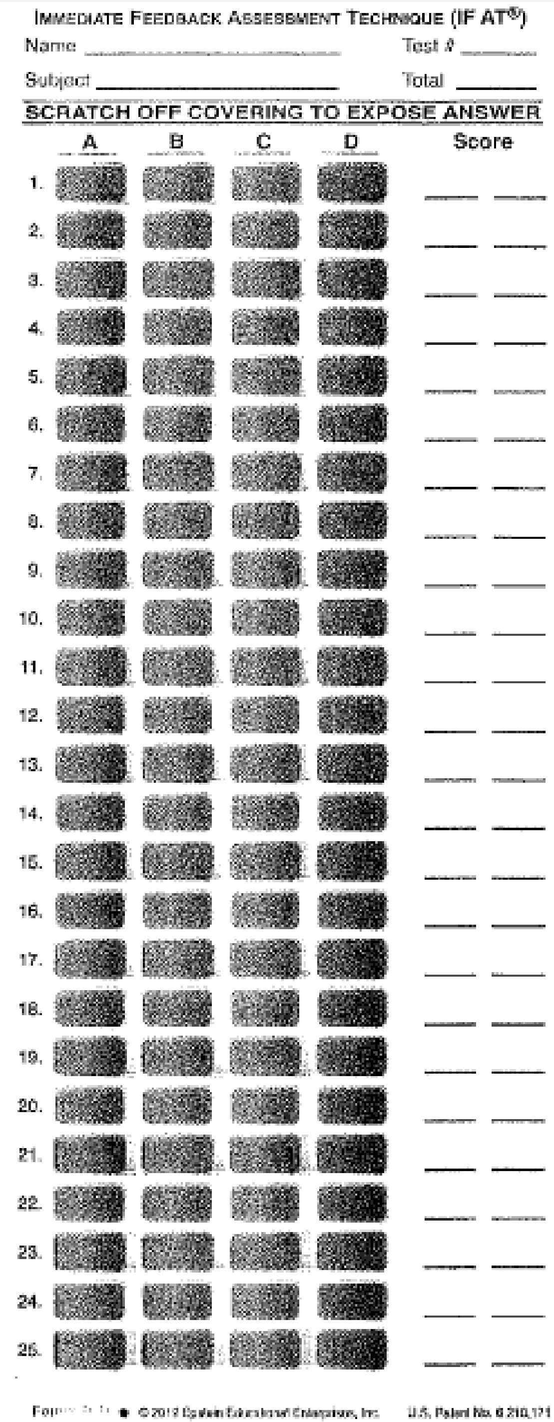
(OE)

$L = L_0/\gamma = 3.67\,\text{ns}$

(OD)

IF-AT Forms

Immediate Feedback Assessment Technique


IMMEDIATE FEEDBACK ASSESSMENT TECHNIQUE (IF AT®)
Name ______ Test # ______
Subject ______ Total ______
SCRATCH OFF COVERING TO EXPOSE ANSWER

	A	B	C	D	Score
1.					
2.					
3.					
4.					
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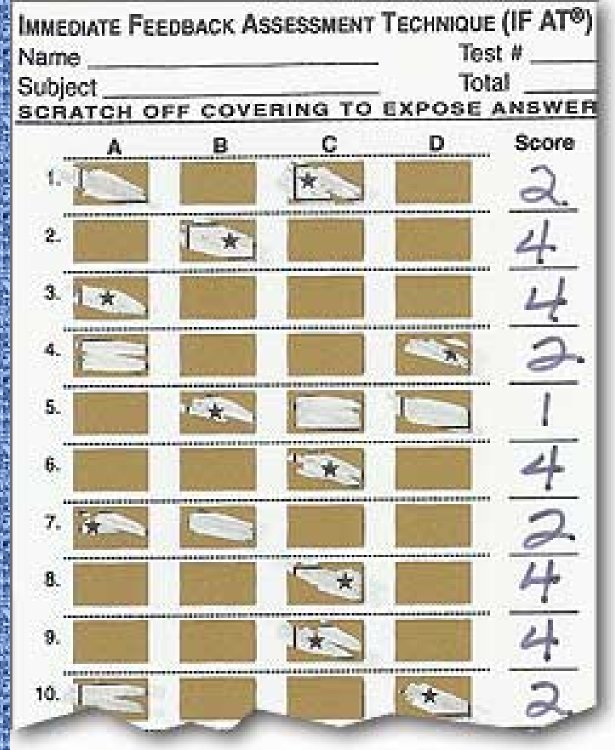
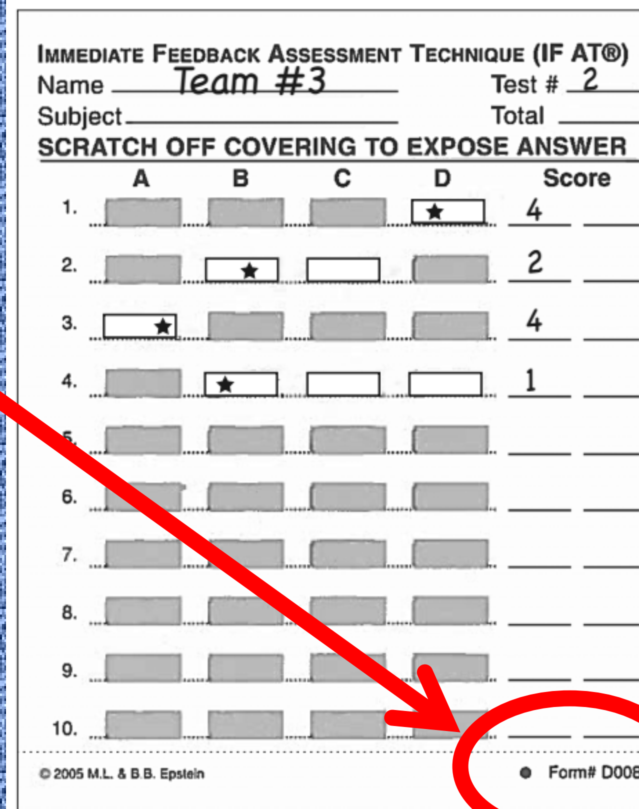

IMMEDIATE FEEDBACK ASSESSMENT TECHNIQUE (IF AT®)
Name ______ Test # ______
Subject ______ Total ______
SCRATCH OFF COVERING TO EXPOSE ANSWER

	A	B	C	D	Score
1.			★		2
2.		★			4
3.	★				4
4.				★	2
5.		★			1
6.			★		4
7.	★				2
8.			★		4
9.			★		4
10.				★	2

unique code for each form for answer key


IMMEDIATE FEEDBACK ASSESSMENT TECHNIQUE (IF AT®)
Name Team #3 Test # 2
Subject ______ Total ______
SCRATCH OFF COVERING TO EXPOSE ANSWER

	A	B	C	D	Score
1.				★	4
2.		★			2
3.	★				4
4.		★			1
5.					
6.					
7.					
8.					
9.					
10.					

Form# D008

Group Quiz

(31) Which of the following quantities are dependent on the particular inertial reference frame that you are in?
A. position
B. velocity
C. acceleration

a) B and C
b) A, B and C
c) B only
d) A and B
e) A only

(32) Which of the following worldlines *cannot* exist?
A. straight vertical line
B. straight horizontal line
C. straight line making a 30° angle with the x-axis

a) A and C
b) B and C
c) A and B
d) A only
e) B only

(33) What is the natural unit for ***force*** in SR units?

a) $1/s^2$ (s^{-2})
b) $kg \cdot m/s^2$
c) kg/s
d) $1/s$ (s^{-1})
e) kg/s^2

(34) Two events that are 60 ns of distance apart in the Home Frame are also simultaneous in that frame. If the events are separated by 80 ns of time in the Other Frame, what is their spatial separation in the Other Frame?

a) –53 ns
b) 20 ns
c) 100 ns
d) 53 ns
e) the Other Frame event is impossible

(35) In the Home Frame, two events separated in distance by 90 m occur within a time interval of 0.6 μs. What is the constant speed of a moving frame that measures a ***proper time*** between these events?

a) 0.50
b) 0.35
c) 1.0
d) 0.25
e) 0.75

(36) Imagine that a certain worldline connecting two events is represented by a line segment of fixed length. What orientation of that line segment on a spacetime diagram would give a spacetime interval of the ***smallest*** magnitude between those two events?

(36) continued . . .

a) oriented vertically
b) oriented at 60° from the x-axis
c) all orientations will yield the same spacetime interval
d) oriented horizontally
e) oriented at 45° from the x-axis

(37) Imagine that an electron in an accelerator ring is traveling around in a circular path at a constant speed of v = 0.995. If the time for one full revolution around the ring is Δt_{lab} as measured by a clock on the ground, then what is the time Δt_{elec} that would be measured for one revolution by a clock riding with the electron?

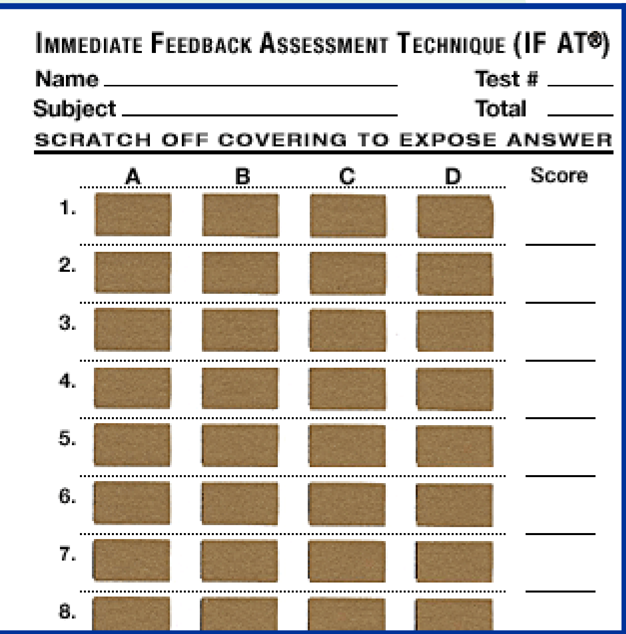
a) $\Delta t_{elec} = 14.0\ \Delta t_{lab}$
b) $\Delta t_{elec} = 0.10\ \Delta t_{lab}$
c) $\Delta t_{elec} = 0.43\ \Delta t_{lab}$
d) $\Delta t_{elec} = 0.22\ \Delta t_{lab}$
e) $\Delta t_{elec} = 10.0\ \Delta t_{lab}$

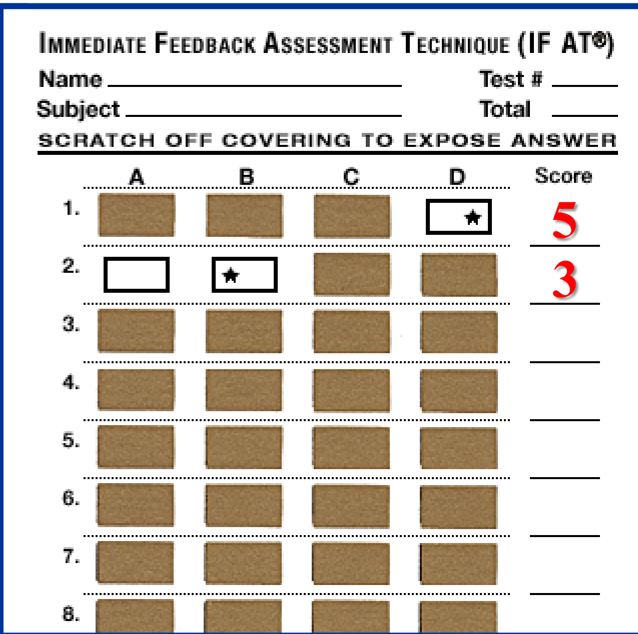
(38) If you spent your entire lifetime (100 yrs) orbiting the Earth in the Space Shuttle moving at a constant speed of 30,000 km/hr, roughly how much longer would you live compared to an Earth-bound observer?

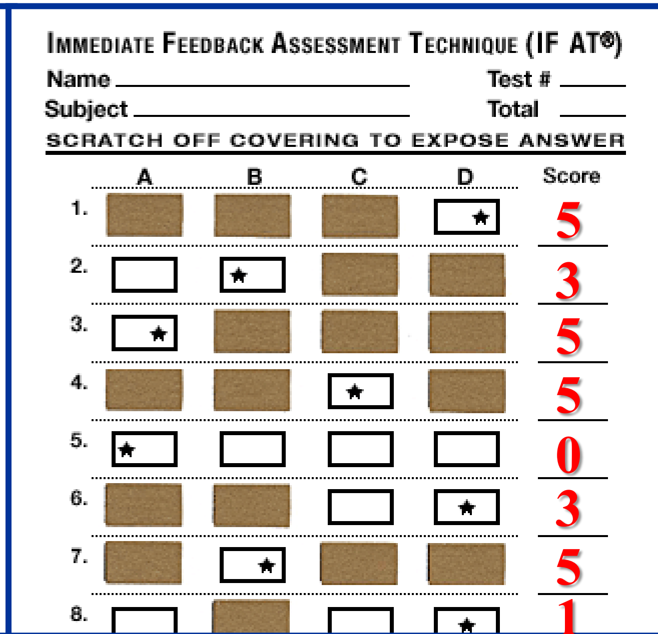
a) ~1 μs
b) ~1 s
c) ~10 s
d) ~1 day
e) ~1000 s

Delivering Group Quizzes

- quiz consists of 10 multiple-choice questions (5 pts each)
- **part 1** – students work on quiz individually (50 pts)
- **part 2** – rework quiz in groups using scratch-off (50 pts)
  - decreasing point values (5,3,1) for multiple attempts
- each part is about 10-15 minutes



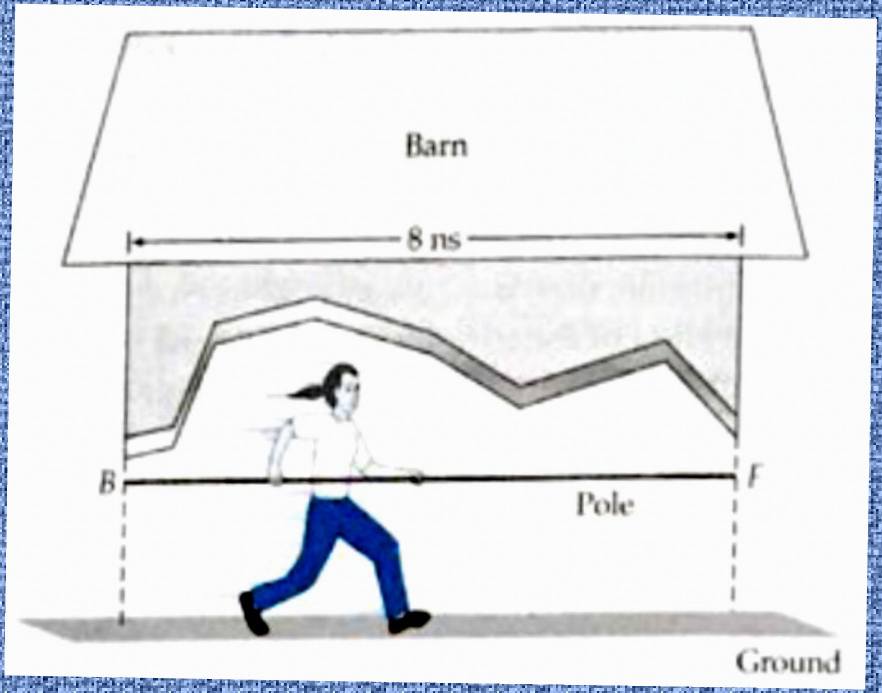




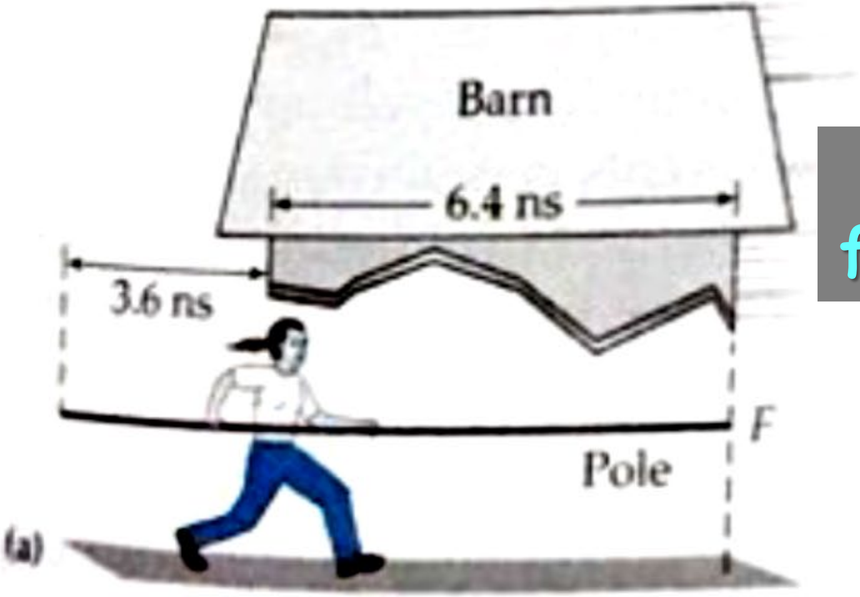
Name		Individual	Group	Total
Group 1		35	48	83
Group 1		25	48	73
Group 1		35	48	83
Group 2		25	46	71
Group 2		25	46	71
Group 2		40	46	86
Group 3		20	44	64
Group 3		45	44	89
Group 3		30	44	74
Group 4		40	48	88
Group 4		30	48	78
Group 4		30	48	78
Group 5		30	45	75
Group 5		30	45	75
Group 5		35	45	80
Group 6		35	44	79
Group 6		30	44	74
Group 6		5	44	49
Group 7		40	46	86
Group 7		30	46	76
Average:		**30.8**	**45.9**	**76.7**

Group Quiz

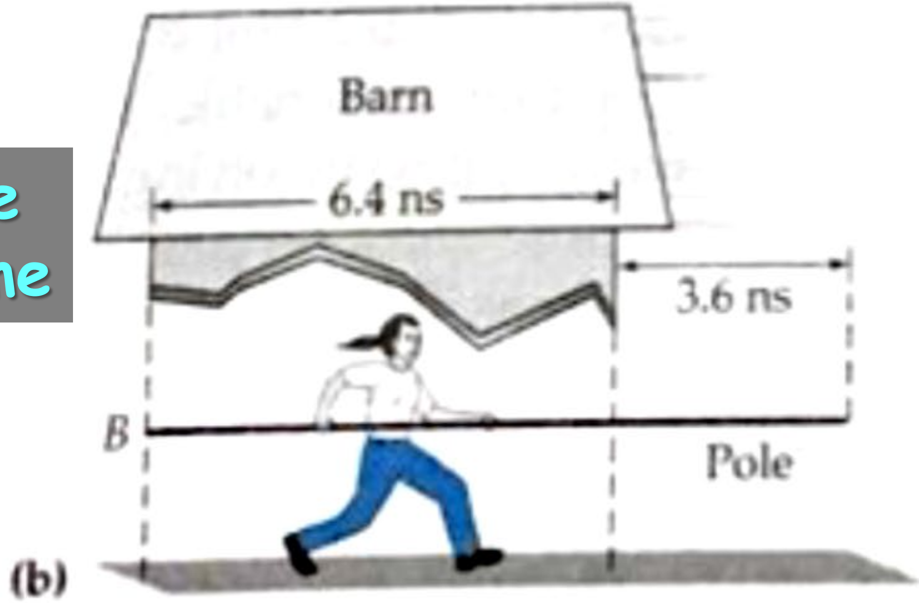
barn and pole paradox

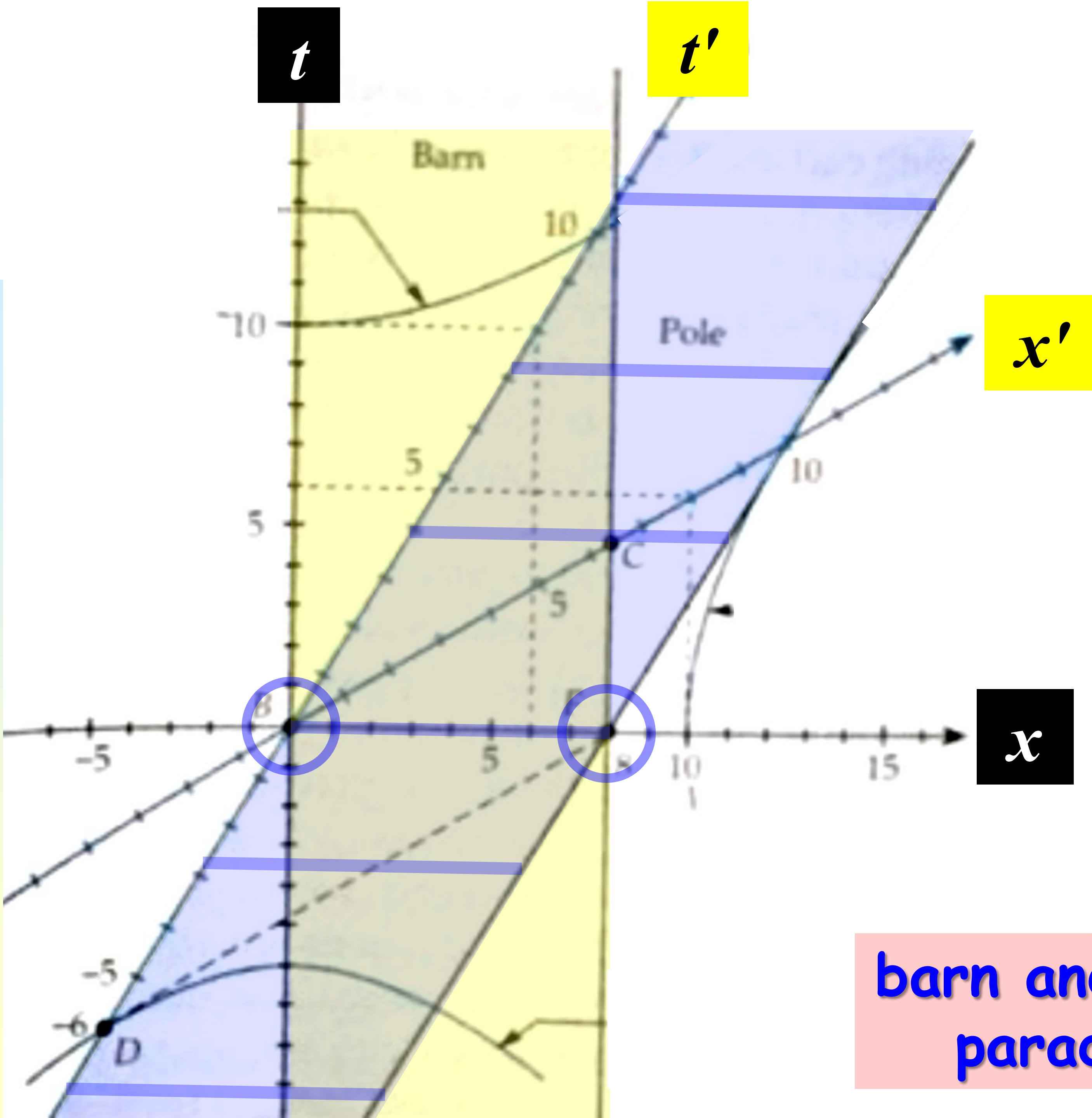


barn frame

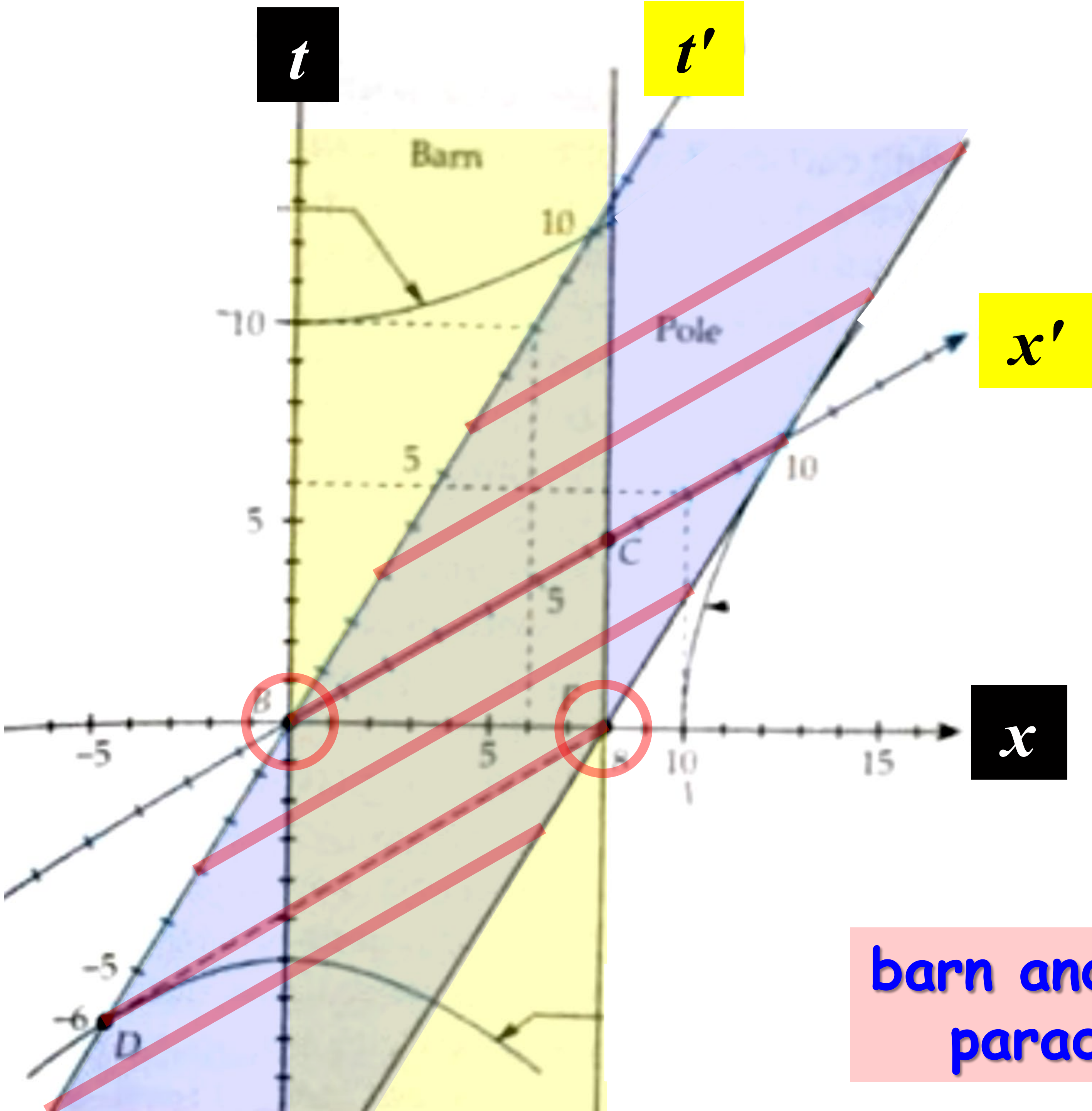


pole frame





barn and pole paradox



barn and pole paradox

Given that some traditional mainstays of introductory physics curricula like fluids and geometrical optics have been jettisoned, it may seem bizarre that Moore spends the equivalent of one-third of a semester on special relativity, and devotes a separate *Unit R* to it. His rationale is that special relativity gives students the chance to see a complete development of one topic in theoretical physics without the usual, difficult mathematical appurtenances. *Unit R* is a masterpiece of pedagogy. Its language is precise and clear, its methodology thorough and modern. Students get extensive experience in the use of space-time diagrams, coordinate transformations, and four-momentum. My own students were so delighted by this unit that some even read ahead (!).

Book Review

Thomas Bernatowicz
Washington University
Am. J. Phys. 2006

The active learning aspect of *Six Ideas* is greatly facilitated by the "Two-Minute Problems" at the end of each chapter, which consist of multiple-choice and true/false exercises designed for in-class participation.

volunteers explain the reasoning behind their choice. Often this leads to constructive discussion as other students object to an explanation or offer an alternative one. The students are uniformly enthusiastic about the two-minute problems, since they obviously alleviate boredom, provide a convenient venue for questions (and talking in general), and present an opportunity to find out how others have thought about the physics.

Thomas Bernatowicz
Washington University
Am. J. Phys. 2006

Book Review

How does a *Six Ideas* class differ from the standard, lecture-based course in introductory physics? The most important difference is that the instructor's role is more that of a coach than a sage. Beyond that, the nature of the class really depends on the instructor's stylistic choices, as well as on the physical classroom environment.

At first glance, it may seem that all of this interaction would lead to a superficial treatment of topics compared to a standard lecture-based format, because of the necessary minimization of lecture time. This is not so. It is instead a very effective and very efficient procedure overall. Because of the text reading and problem solving done prior to the class meeting, the students are already well acquainted with the topic *du jour* before they come to class, and their minds are prepared to augment and deepen an initial understanding by exposure to and participation in the various classroom activities.

Conclusions

- Thomas Moore's Unit R (Six Ideas That Shaped Physics)
  - graphical presentation of spacetime diagrams
  - emphasis on concepts (in addition to math)
  - ideal for active-learning classroom
- Basic feedback from students
  - transition to regular Modern Physics textbook is a bit rough
  - students performed best on special relativity in final exam!
- Personal conclusions (my own…)
  - excellent way to present special relativity!
  - potentially the basis for a full-semester course
  - Thomas Moore also has a general relativity workbook

R5M.3 The Federation space cruiser *Execrable* is floating in Federation territory at rest relative to the border of Klingon space, which is 6.0 min away in the $+x$ direction. Suddenly, a Klingon warship flies past the cruiser in the direction of the border at a speed of $\frac{3}{5}$. Call this event A, and let it define time zero in both the Klingon and cruiser reference frames. At $t_B = 5.0$ min according to cruiser clocks, the Klingons emit a parting disrupter blast (event B) that travels at the speed of light back to the cruiser. The disrupter blast hits the cruiser and disables it (event C), and a bit later (according to cruiser radar measurements) the Klingons cross the border into Klingon territory (event D).

The Klingon–Federation Treaty states that it is illegal for a Klingon ship in Federation territory to damage Federation property. When the case comes up in interstellar court, the Klingons claim that they are within the letter of the law: according to measurements made in their reference frame, the damage to the *Execrable* occurred *after* they had crossed back into Klingon territory, so they were *not* in Federation territory at the time. Did event C (disrupter blast hits the *Execrable*) *really* happen after event D (Klingons cross the border) in the Klingons' frame? Answer this question by using your two-observer diagram, and check your work with the Lorentz transformation equations.

Chapter R5

Coordinate Transformations

Question 1: Two guns are mounted a distance of 40 ns apart on an embankment beside some railroad tracks. The barrels of the guns project outward toward the track so that they almost brush against a speeding train as it passes by. The train is moving at a speed of $v = 0.6$ with respect to the ground.

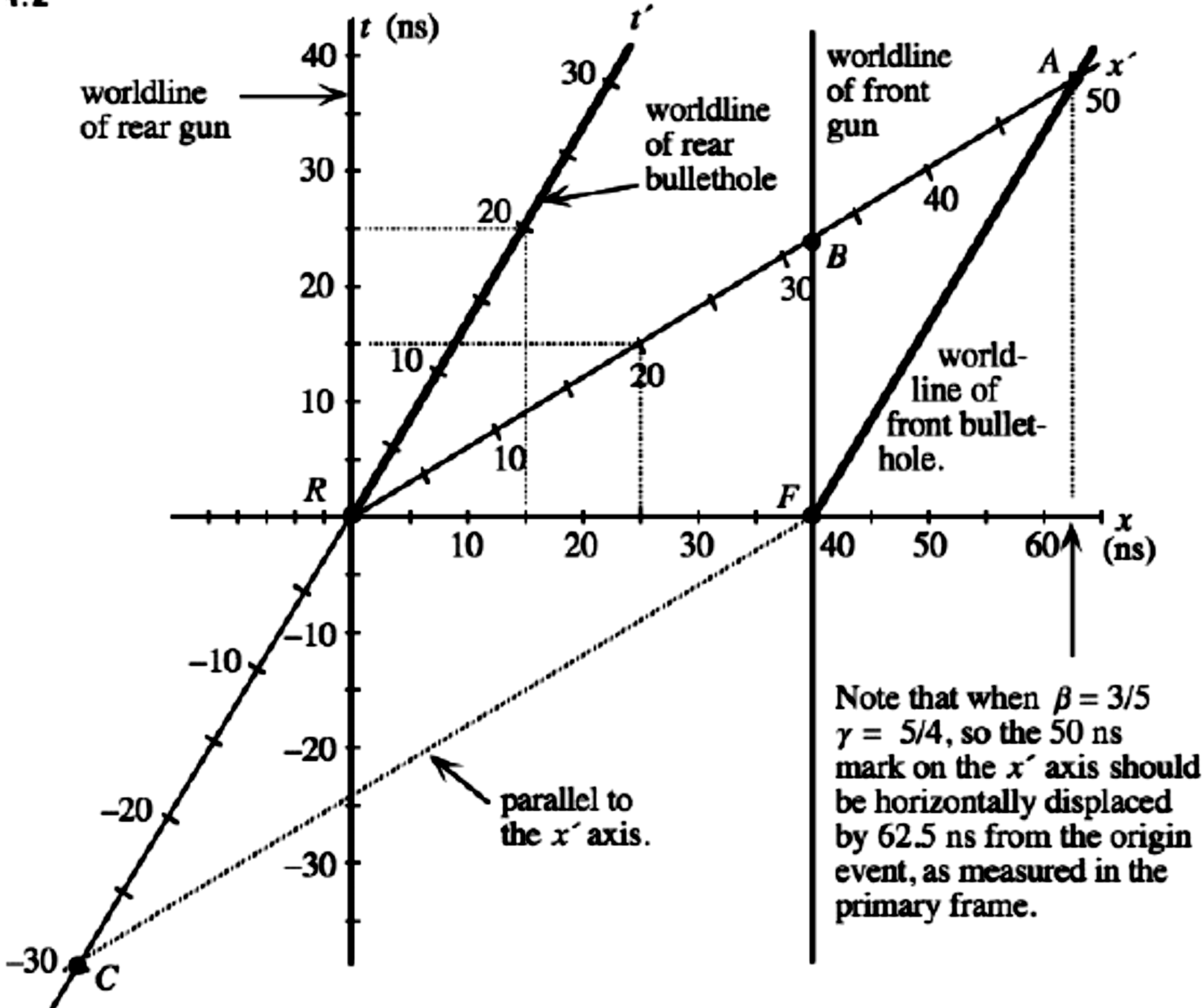
Now suppose the two guns fire *simultaneously* (as measured in the ground frame) leaving two bullet holes in the train.

(2) Using your spacetime diagram, determine the ***spacing of the bullet holes in the train frame***. Identify this spacing by a labeled line segment on your figure. Explain in words how you knew where to pick the two endpoints of your line segment.

(3) Using your spacetime diagram, find the ***gun separation as measured in the train frame***. Identify this separation by a labeled line segment on your figure. Explain in words how you knew where to pick the two endpoints of your line segment.

(4) We know that the guns both fire at $t = 0$ in the ground frame. Do the guns fire simultaneously in the train frame? If not, and given that the rear gun fired at $t' = 0$ (by definition, from the construction of your spacetime diagram), use your spacetime diagram to find ***the time at which the front gun fired in the train frame***.

(5) You should find that your answer to #2 is larger than your answer to #3 above! How is that possible? Use your answer to #4 to explain how it is in fact possible.


t (ns)
t´
x´
x (ns)
worldline of rear gun
worldline of rear bullethole
worldline of front gun
world-line of front bullet-hole.
A
B
C
R
F
parallel to the x´ axis.
Note that when $\beta = 3/5$ $\gamma = 5/4$, so the 50 ns mark on the x' axis should be horizontally displaced by 62.5 ns from the origin event, as measured in the primary frame.

(b) Events R and A occur along the bullet-hole worldlines and at the same time ($t' = 0$) in the train frame. These events therefore define the distance between those bullet holes in that frame. From the diagram it is easy to see that the separation $\Delta x'$ between these events in the train frame is indeed about 50 ns.

(c) The length L between the guns in the train frame can be found from the rest length $L_R = 40$ ns between them in the ground frame using the usual Lorentz contraction formula:

$$L = L_R\sqrt{1-\beta^2} = L_R\sqrt{1-9/25} = L_R\sqrt{16/25} = \tfrac{4}{5}L_R = \tfrac{4}{5}(40\text{ ns}) = 32\text{ ns}.$$

Events R and B occur along the gun worldlines and at the same time ($t' = 0$) in the train frame. These events therefore define the distance between the guns in that frame. From the diagram it is easy to see that $\Delta x'$ between these events is indeed about 32 ns.

(d) The guns fire simultaneously in the ground frame, but *not* in the train frame, because events that are simultaneous in one frame will not generally be simultaneous in any other. In fact, by drawing a parallel from event F to the t' axis, you can easily see from the diagram that the front gun fires roughly 30 ns before the rear gun fires, as measured in the train frame. In that intervening time, the guns move backward about (3/5)(30 ns) ≈ 18 ns, just the distance required for guns separated by 32 ns to leave holes separated by 50 ns. In short: what actually is observed in the train frame is that the front gun fires, the guns move backward 18 ns in 30 ns of time, and then the rear gun fires leaving bullet holes 32 ns + 18 ns = 50 ns apart. There is *no* contradiction: everything can be logically explained in both frames.

bullet-hole paradox
prob. R7R.2