Connecting an *RLC* Circuit to a Diode

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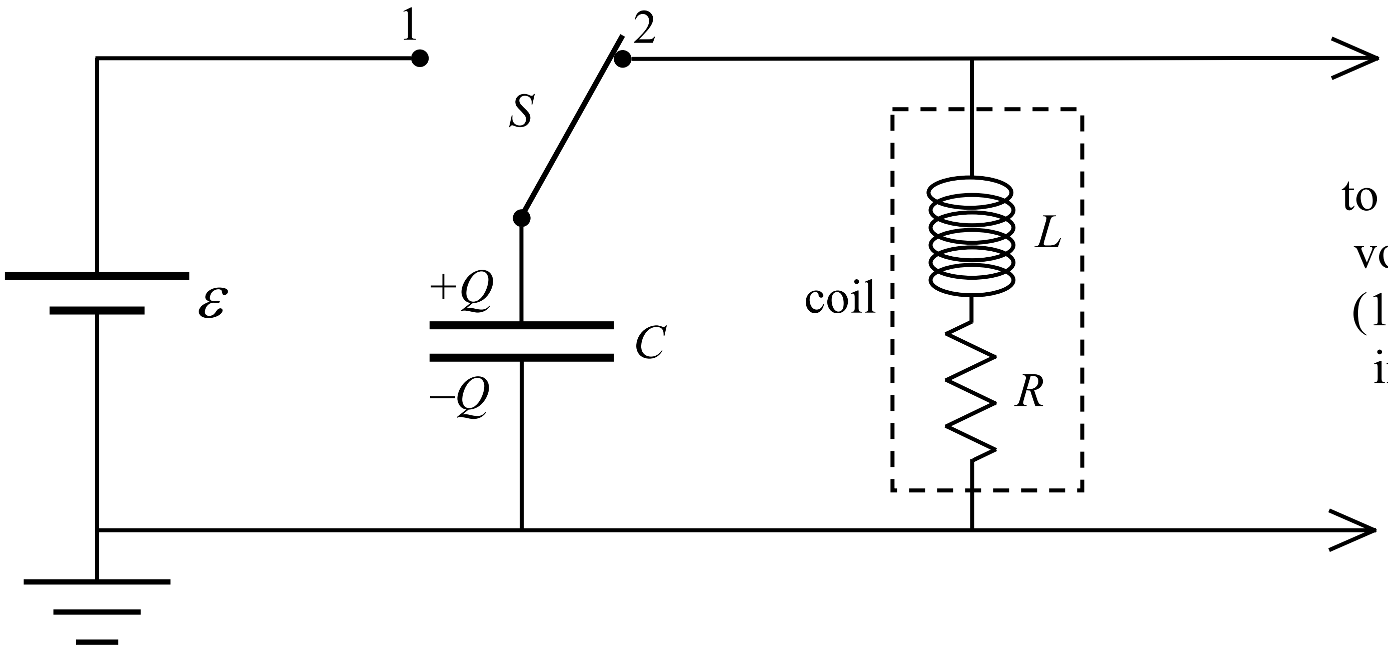
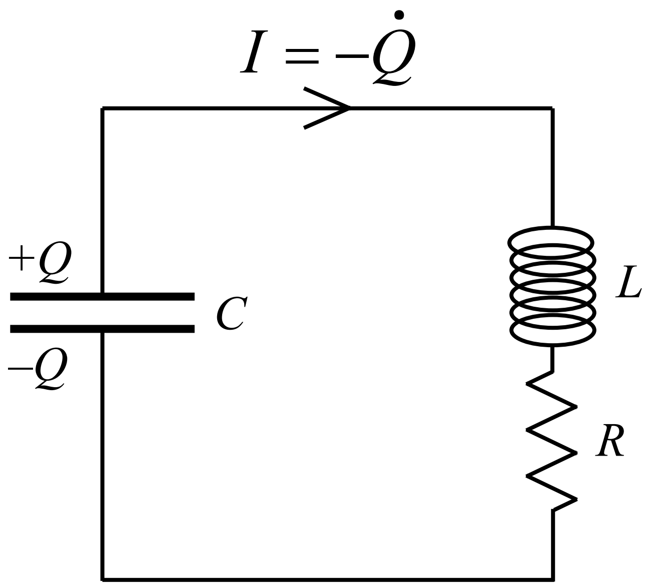
The standard undriven series *RLC* circuit:



experimental parameters:
$L = 0.81$ H
$R = 62.7\ \Omega$
$C = 2.96\ \mu$F
$\varepsilon = 25.0$ V

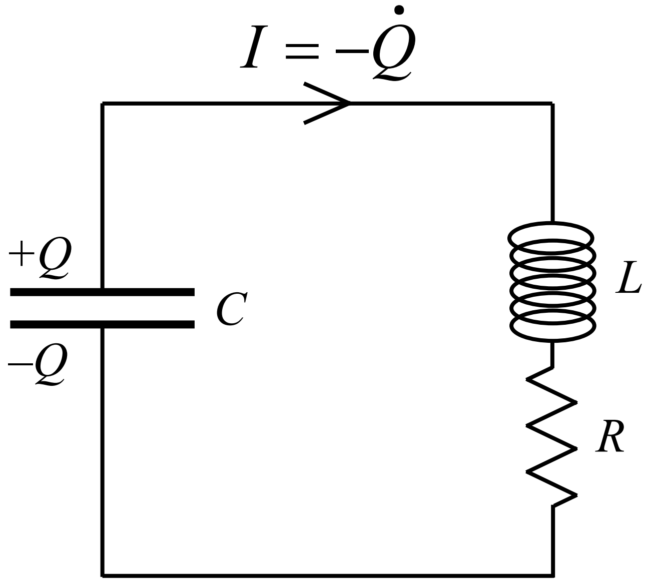
Switch S has been in position 1 for a long time so that $Q_0 = C\varepsilon$.
Then at $t = 0$ it is flipped to position 2.



$$\frac{Q}{C} - L\frac{dI}{dt} - IR = 0$$

$$\div L \Rightarrow \boxed{\ddot{Q} + \frac{\dot{Q}}{\tau} + \omega^2 Q = 0}$$

where $\omega \equiv \dfrac{1}{\sqrt{LC}}$ and $\tau \equiv \dfrac{L}{R}$



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$$\text{where } \omega \equiv \frac{1}{\sqrt{LC}} \text{ and } \tau \equiv \frac{L}{R}$$

$$\text{solution: } \boxed{Q = A_0 e^{-t/2\tau}\cos(\omega' t - \phi)}$$

$$\text{where } \omega' \equiv \sqrt{\omega^2 - \frac{1}{(2\tau)^2}} \text{ which is underdamped if } \omega > \frac{1}{2\tau} \Rightarrow R << \sqrt{\frac{L}{C}} \approx 500\ \Omega$$

Suggestion: Ask students to verify this soln by subbing it back into the above DE.

$$Q = A_0 e^{-t/2\tau} \cos(\omega' t - \phi)$$

fit A_0 and ϕ to IC:

$$Q(0) = Q_0 = C\varepsilon$$

$$I(0) = 0$$

The solution is NOT $A_0 = Q_0$ and $\phi = 0$.

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fit A_0 and ϕ to IC:

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$$I(0) = 0$$

The solution is NOT $A_0 = Q_0$ and $\phi = 0$.

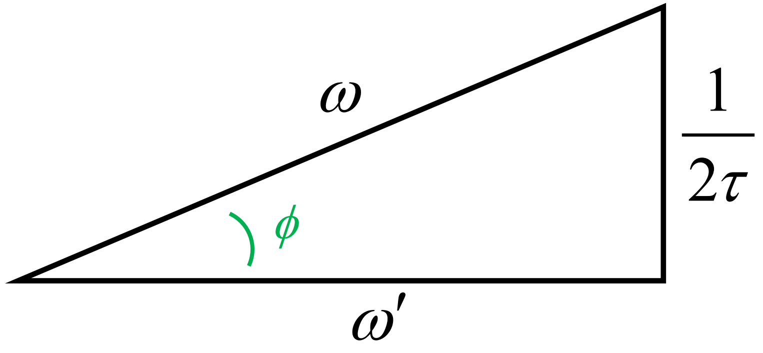
Instead it is

$$A_0 = Q_0 \frac{\omega}{\omega'}$$

$$\phi = \cos^{-1} \frac{\omega'}{\omega}$$

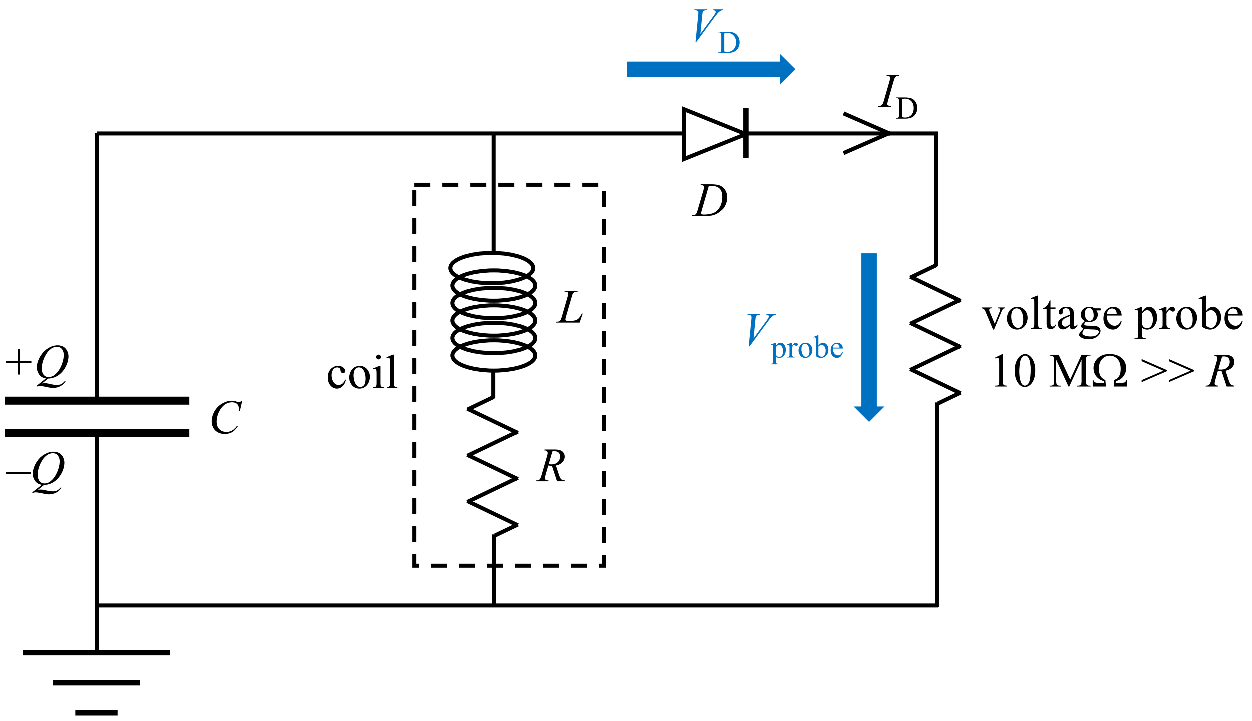
as you may also wish

to ask students to verify.



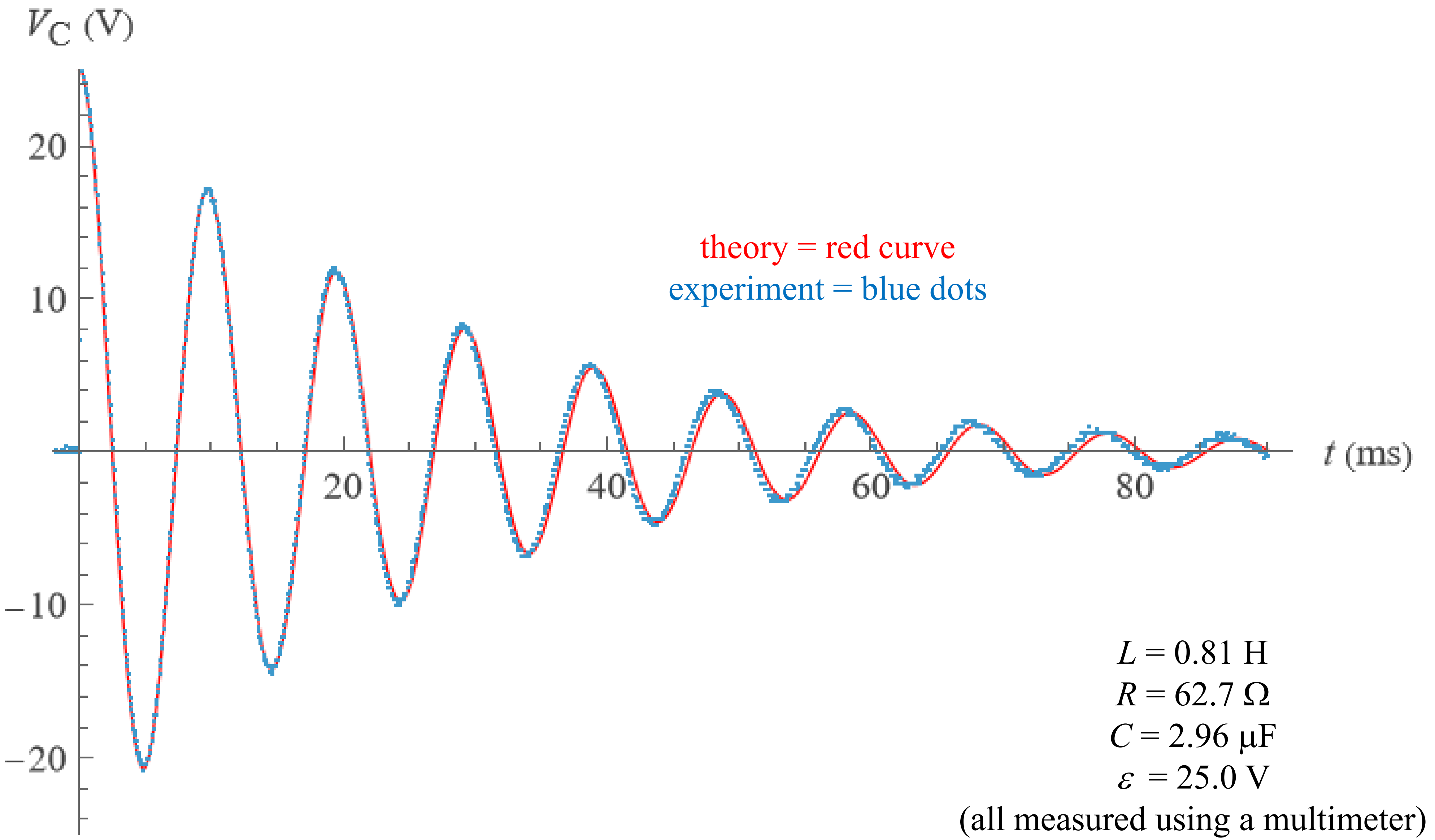
consistent with $\omega' \equiv \sqrt{\omega^2 - \frac{1}{(2\tau)^2}}$

Next let's add a diode *D* into the circuit:



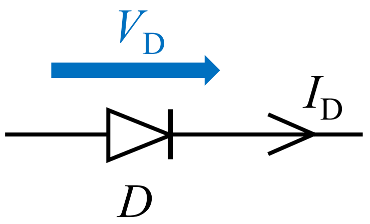
Start without the diode present. Then the voltage probe measures the underdamped *RLC* oscillations:

$$V_{probe} = V_C = Q/C \quad \text{where} \quad Q = Q_0 \frac{\omega}{\omega'} e^{-t/2\tau} \cos\left(\omega' t - \cos^{-1}\frac{\omega'}{\omega}\right)$$

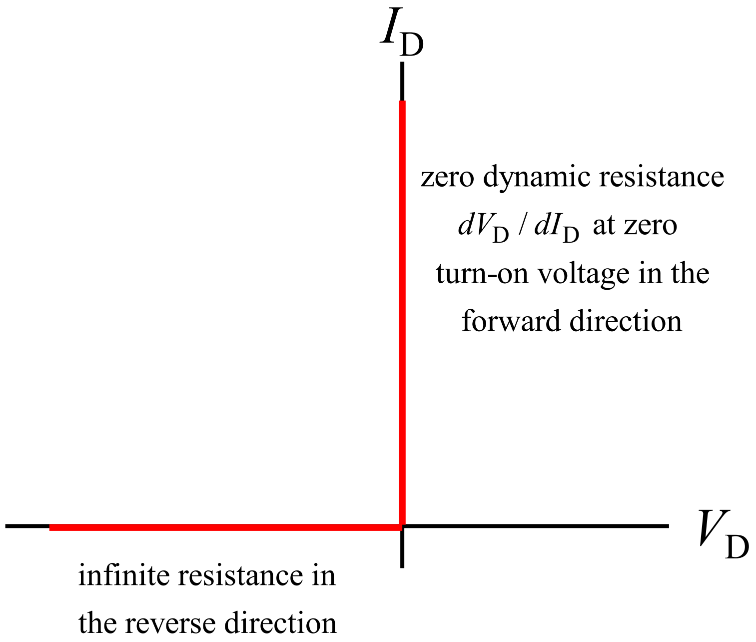


V_C (V)
20
10
−10
−20
20
40
60
80
t (ms)
theory = red curve
experiment = blue dots
L = 0.81 H
R = 62.7 Ω
C = 2.96 μF
ε = 25.0 V
(all measured using a multimeter)

Diode IV Characteristic



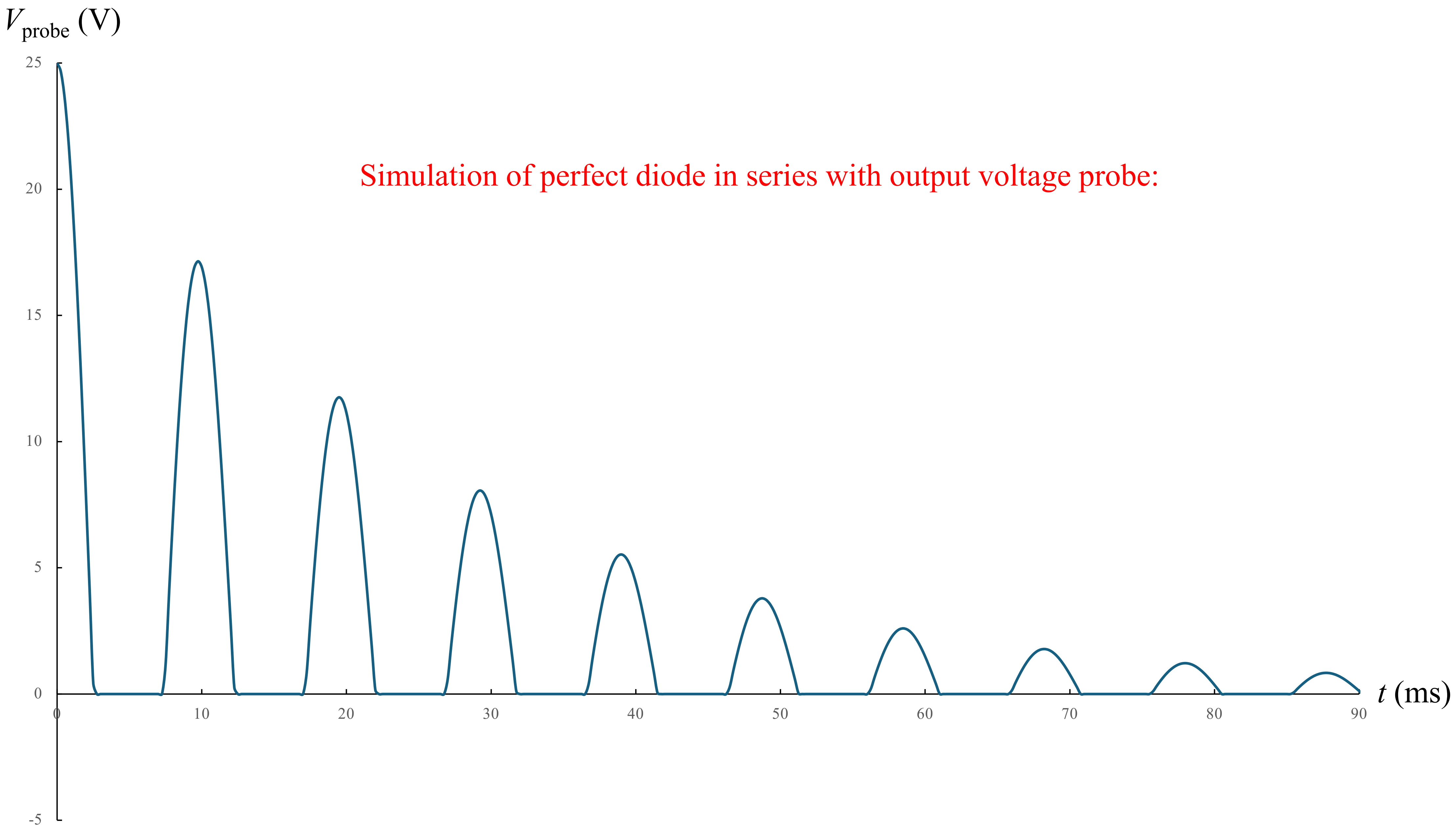
Perfect diode:



acts like a one-way valve

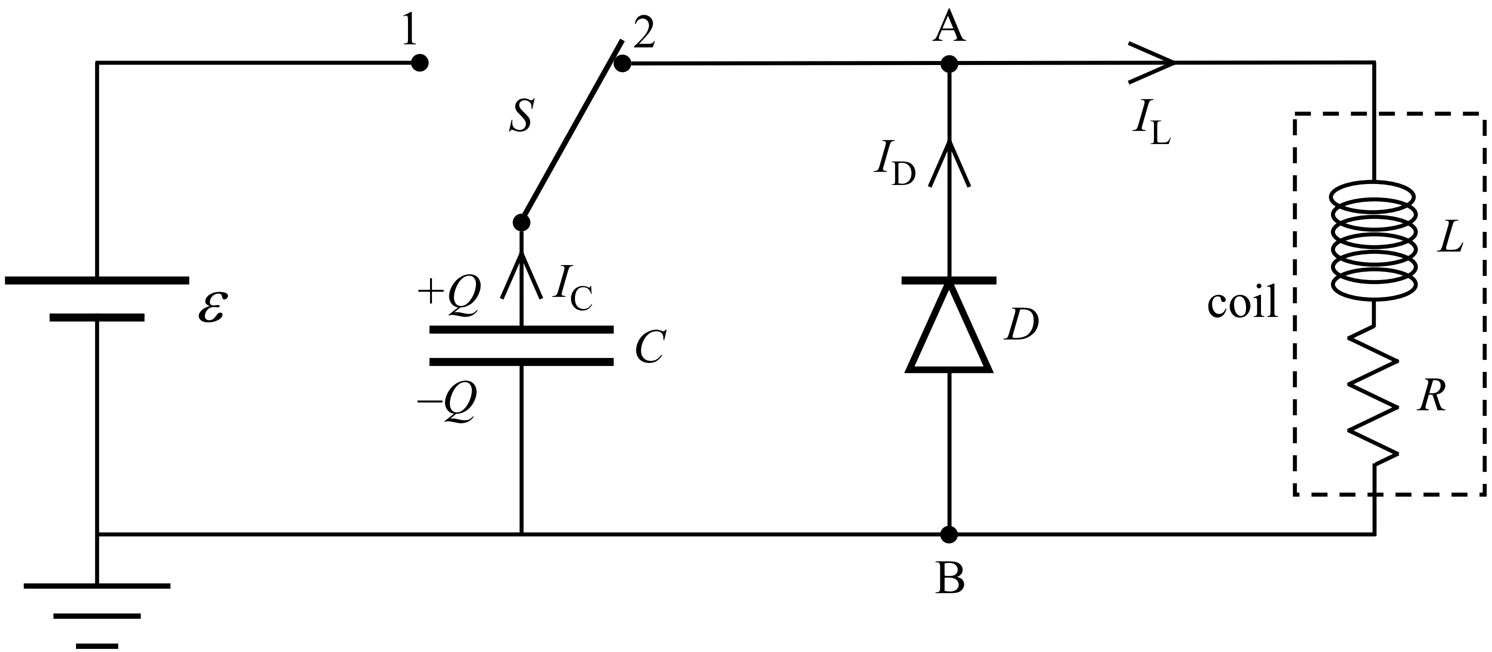
Ideal silicon diode:

Real silicon diode:


V_{probe} (V)
Simulation of perfect diode in series with output voltage probe:
25
20
15
10
5
0
-5
0
10
20
30
40
50
60
70
80
90
t (ms)

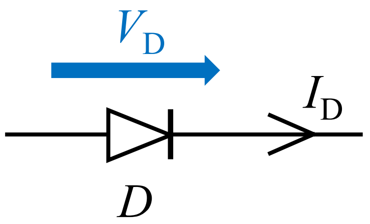
That's not a very smooth rectified current.

What happens if instead of putting the diode in series with the output probe, we put it in parallel with it?



the voltage probe is connected between points A and B

Find DE for I_L using junction rule at A, loop rule for capacitor-diode loop, and loop rule for diode-coil loop.



Diode *IV* Characteristic

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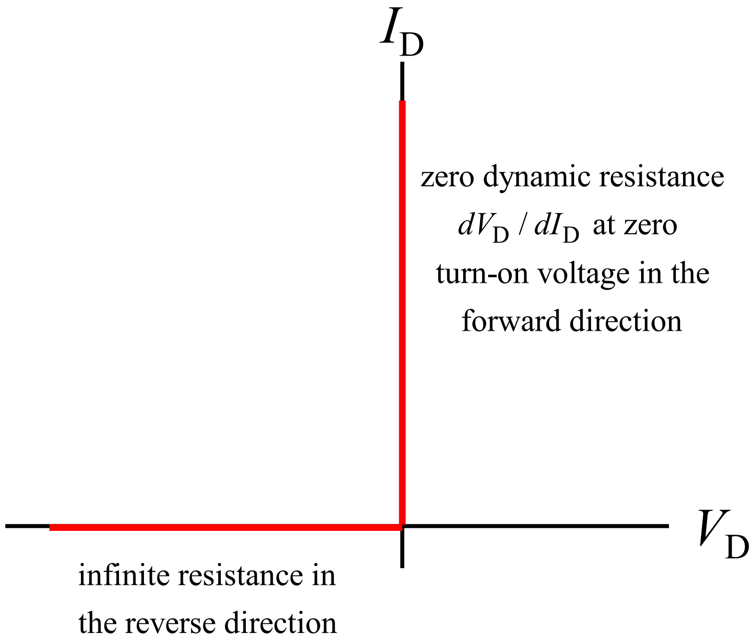

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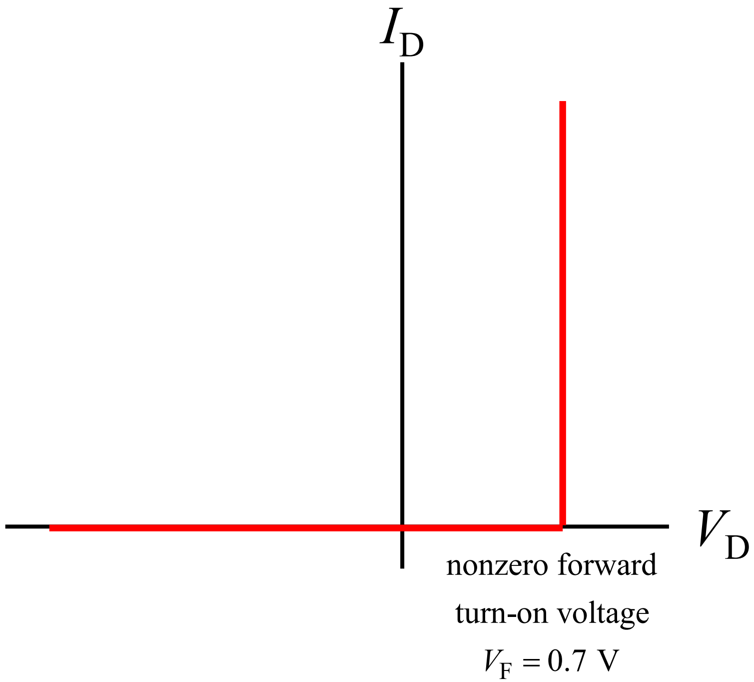

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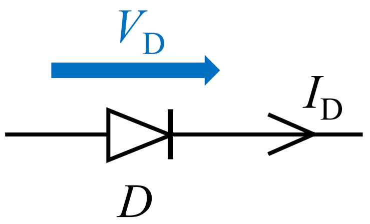

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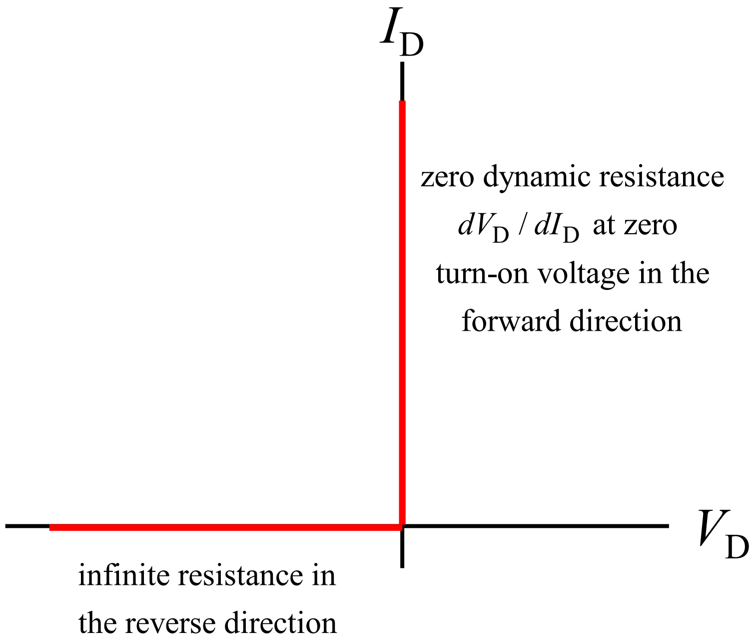

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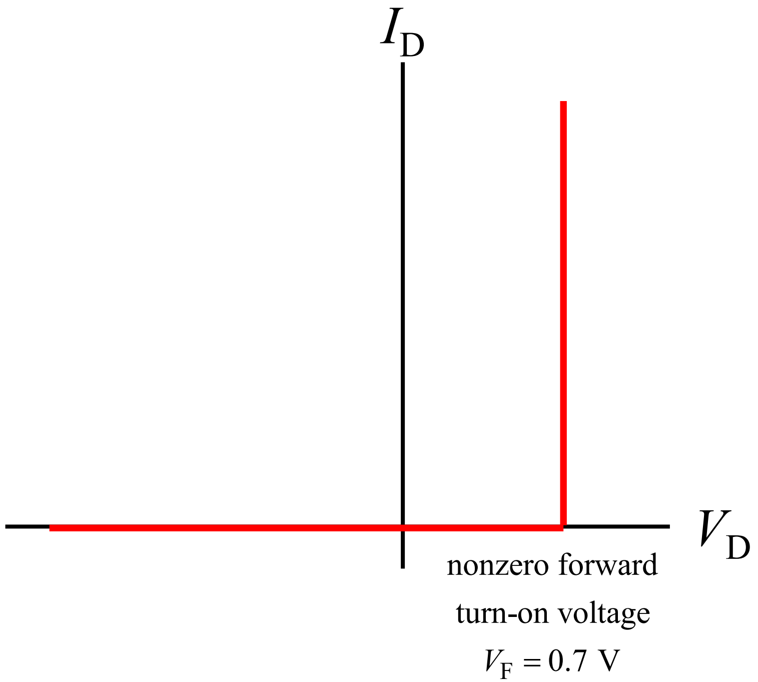

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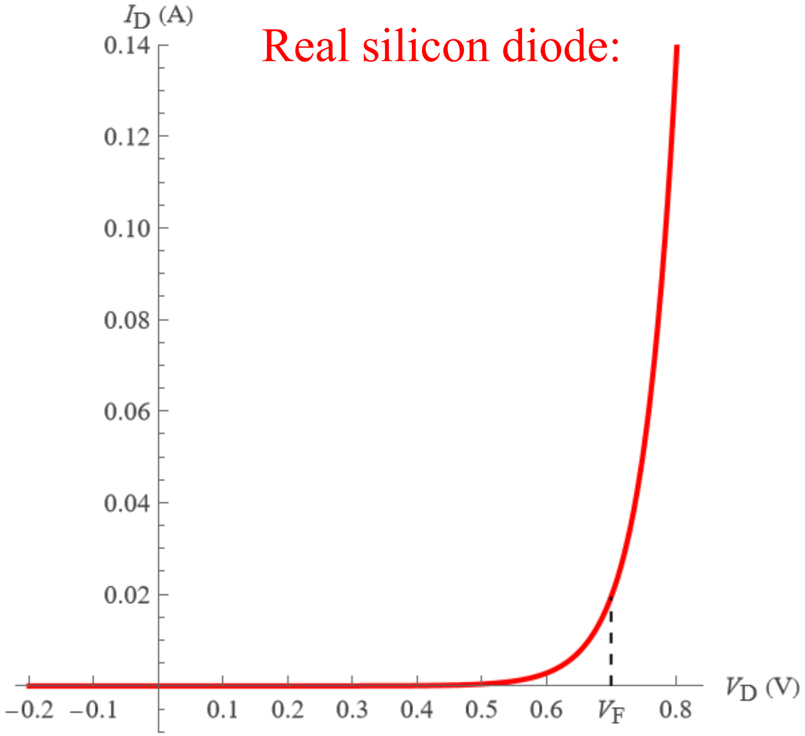

$$I_D = I_S[\exp(V_D / V_T) - 1]$$

where $V_T = 2kT / e$ is the thermal voltage

and $I_S = 25$ nA is the reverse saturation current

for a 1N4148 silicon diode

Resulting DE for the current through the coil:

$$LC\ddot{I}_{\mathrm{L}} + RC\dot{I}_{\mathrm{L}} + I_{\mathrm{L}} = I_{\mathrm{S}}\left[\exp\left(-\frac{L\dot{I}_{\mathrm{L}} + RI_{\mathrm{L}}}{V_{\mathrm{T}}}\right) - 1\right]$$

(check: get the series *RLC* equation if $I_{\mathrm{S}} = 0$ in the absence of the diode)

which is solved numerically in Mathematica and then used to compute the voltage at the probe (or equivalently across the capacitor) as:

$$V_{\mathrm{C}} = L\dot{I}_{\mathrm{L}} + RI_{\mathrm{L}}$$

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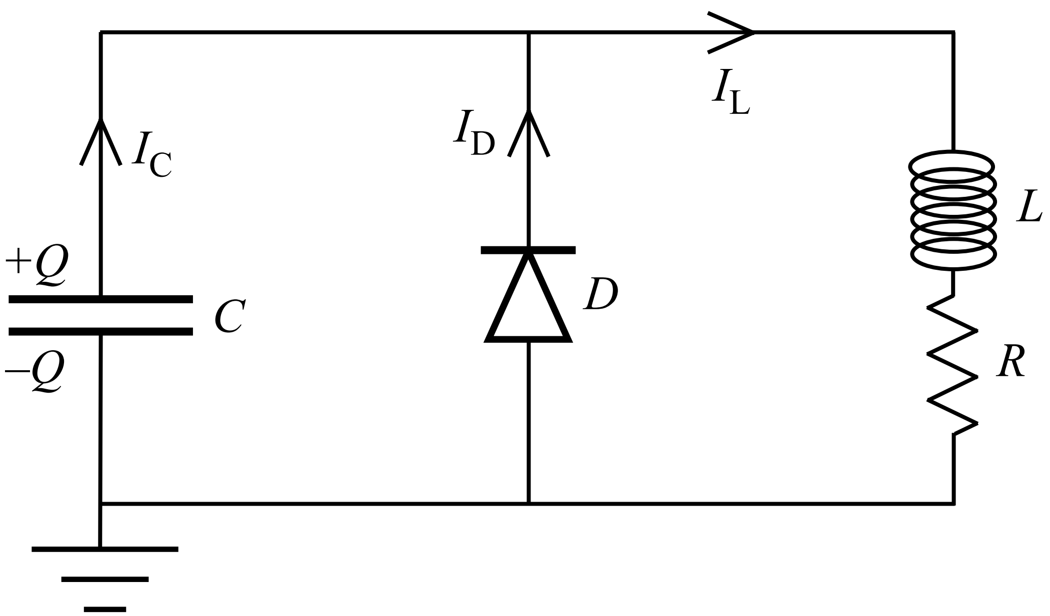
But before I show you the result for this voltage as a function of time, can you predict what it will look like?

(The oscilloscope is set to record a single trace when the switch is flipped from position 1 to 2.)



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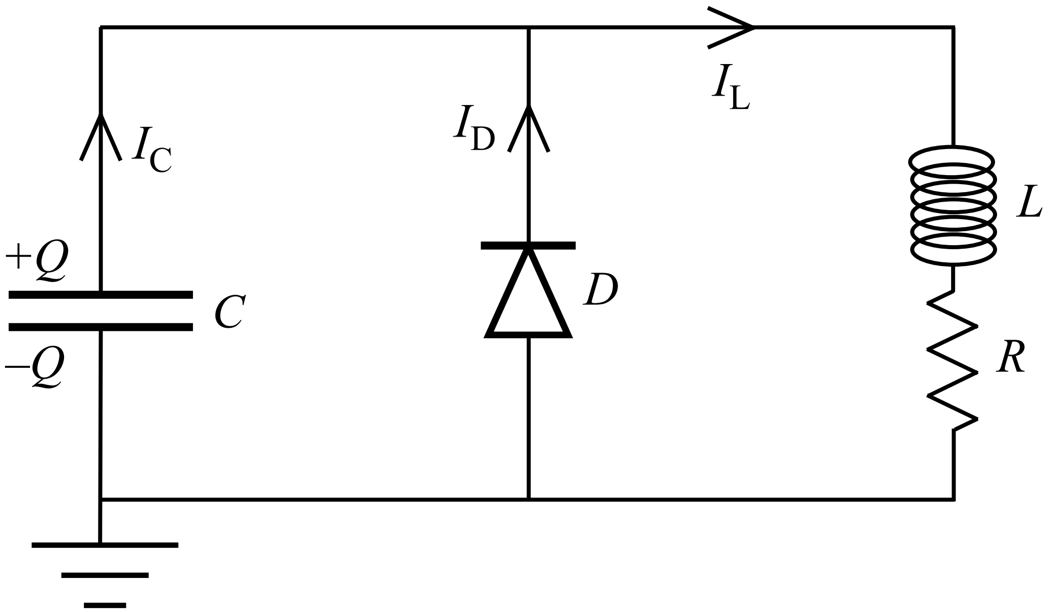

The diode is initially reverse biased by 25 V so it is off.

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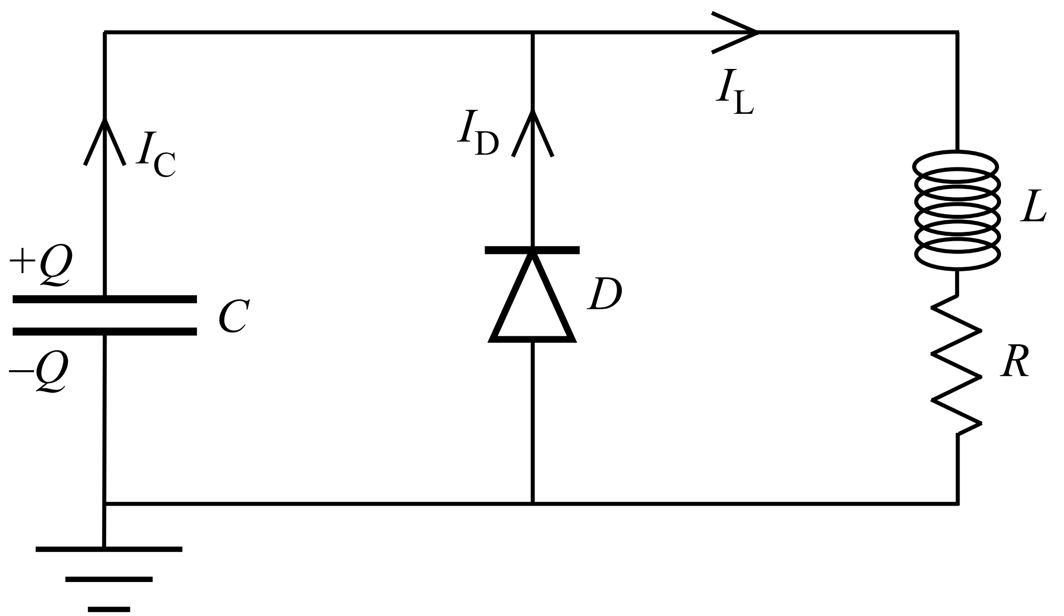

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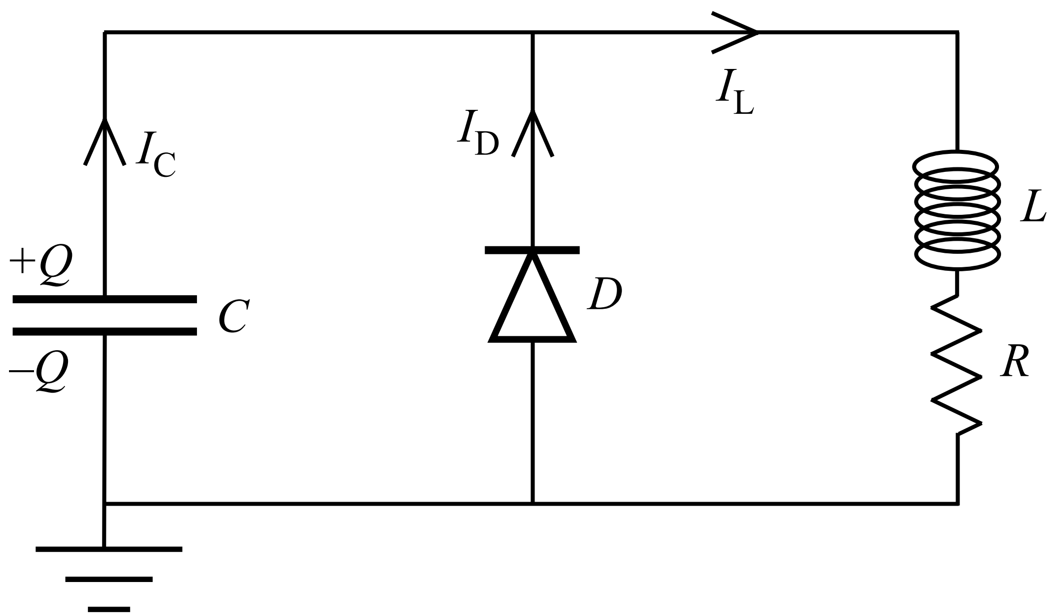
Thus we get a bit more than a quarter cycle of an underdamped *RLC* oscillation through the capacitor and coil until $V_C = -0.7$ V and I_L is just past maximum.



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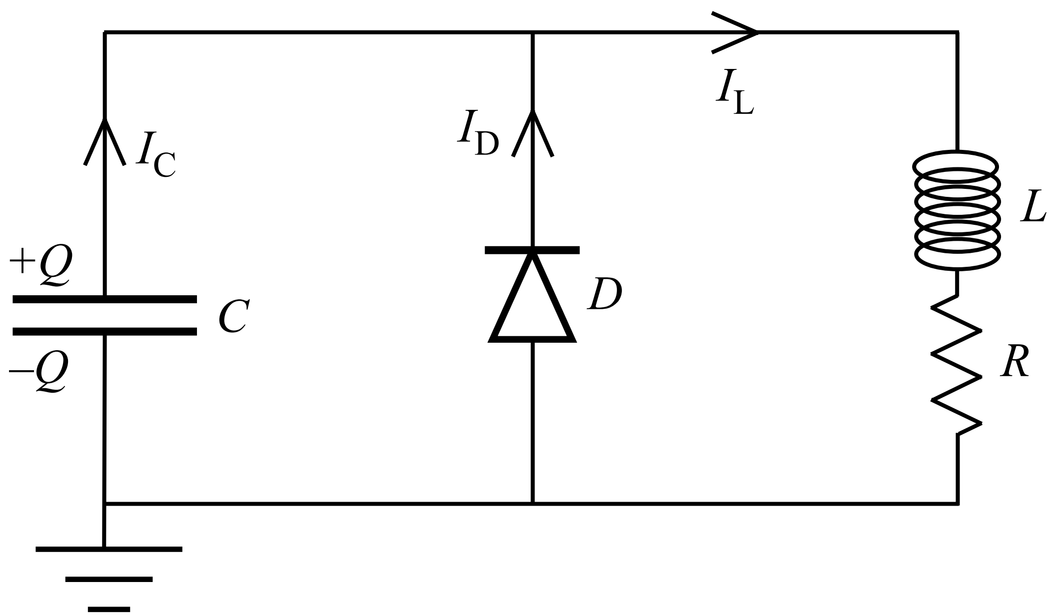
At that point the diode switches on and shorts out the capacitor. But the inductor keeps I_L going.

APPLICATION: For a perfect diode and ideal inductor, we would have a long-lasting nearly constant current looping through the diode and coil.



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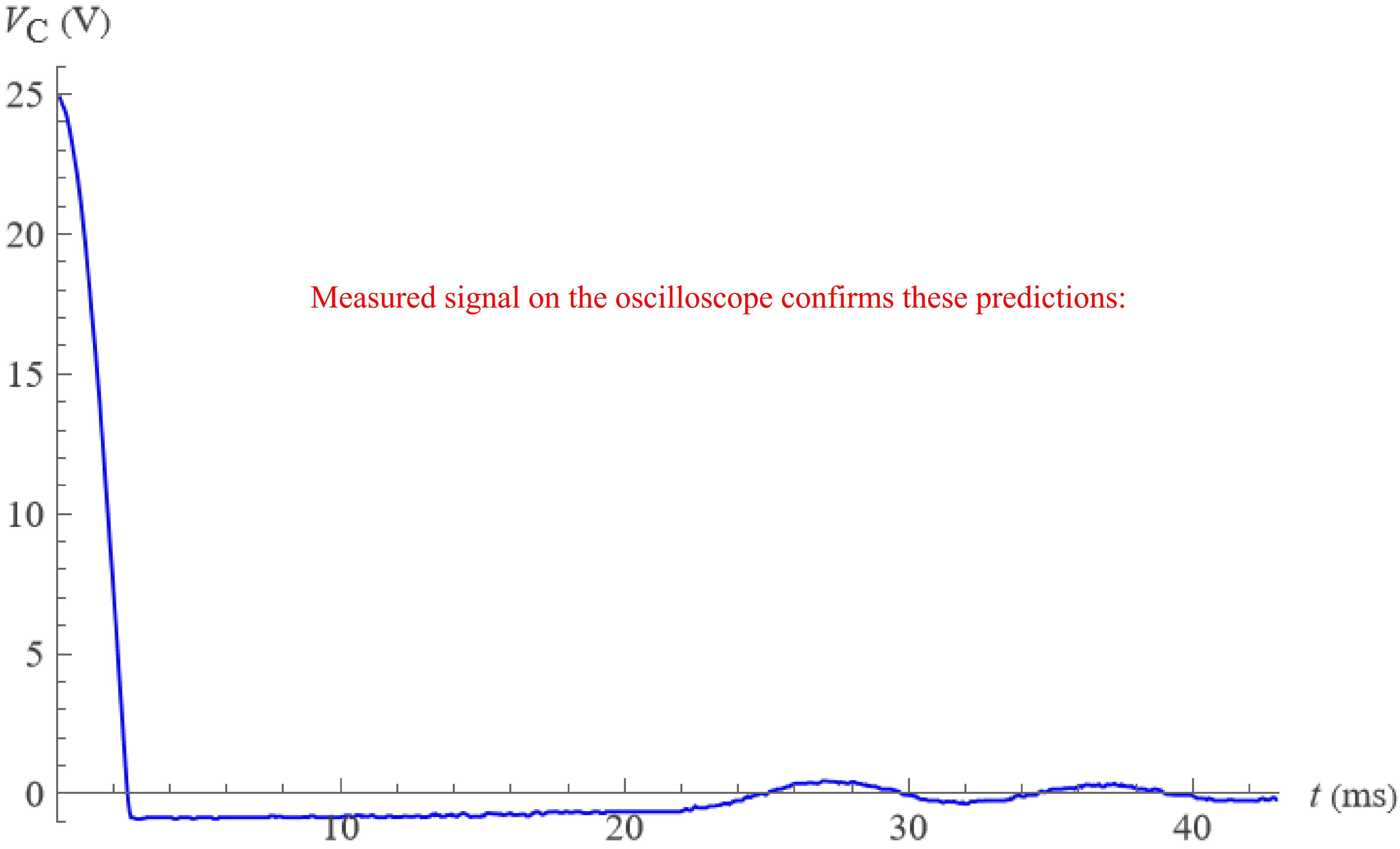
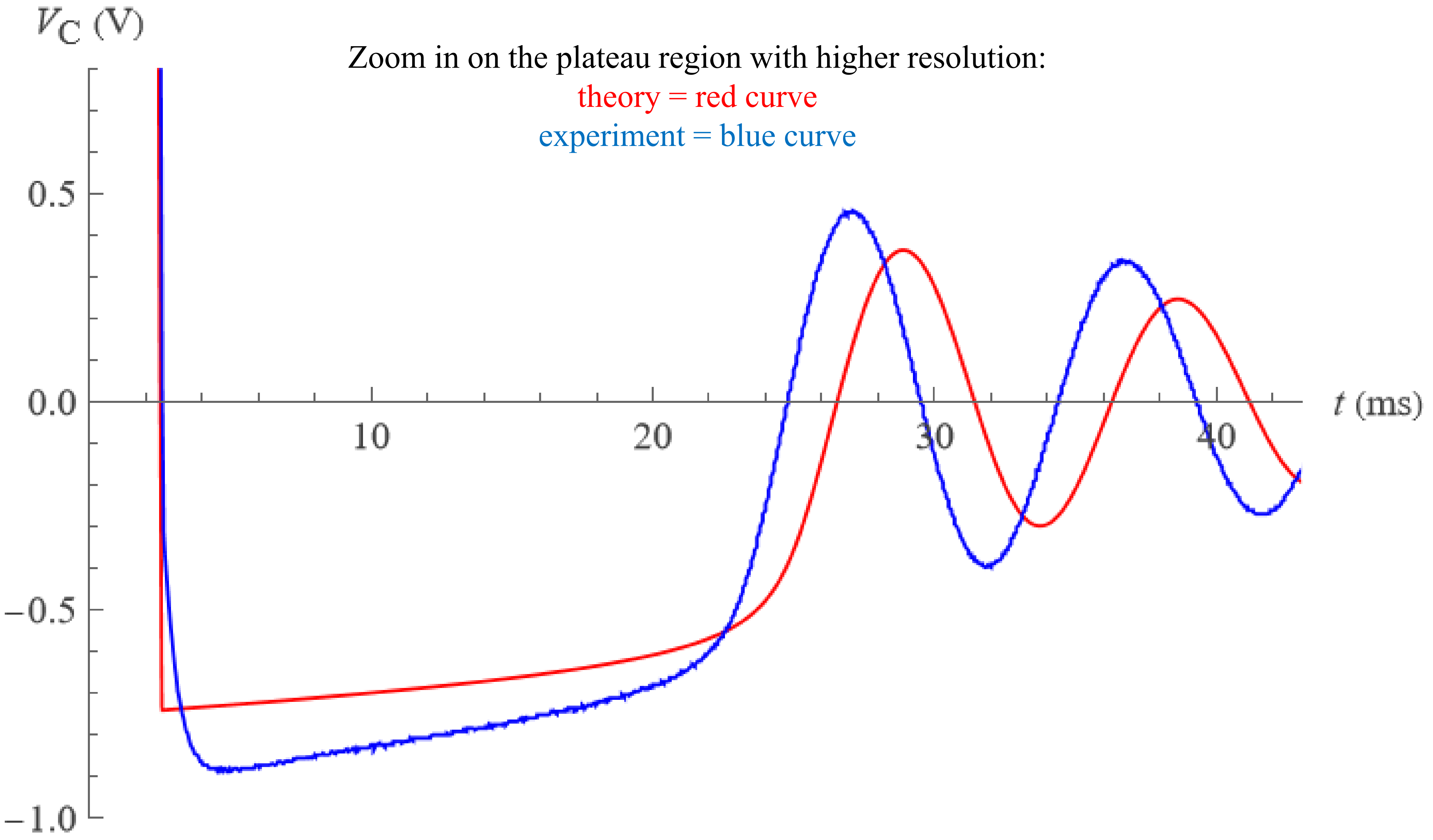
However in the real world, the resistance of the coil and diode causes that current to slowly decay. The diode will then fall below its turn-on voltage and underdamped *RLC* oscillations will resume until the circuit is fully discharged.

Measured signal on the oscilloscope confirms these predictions:

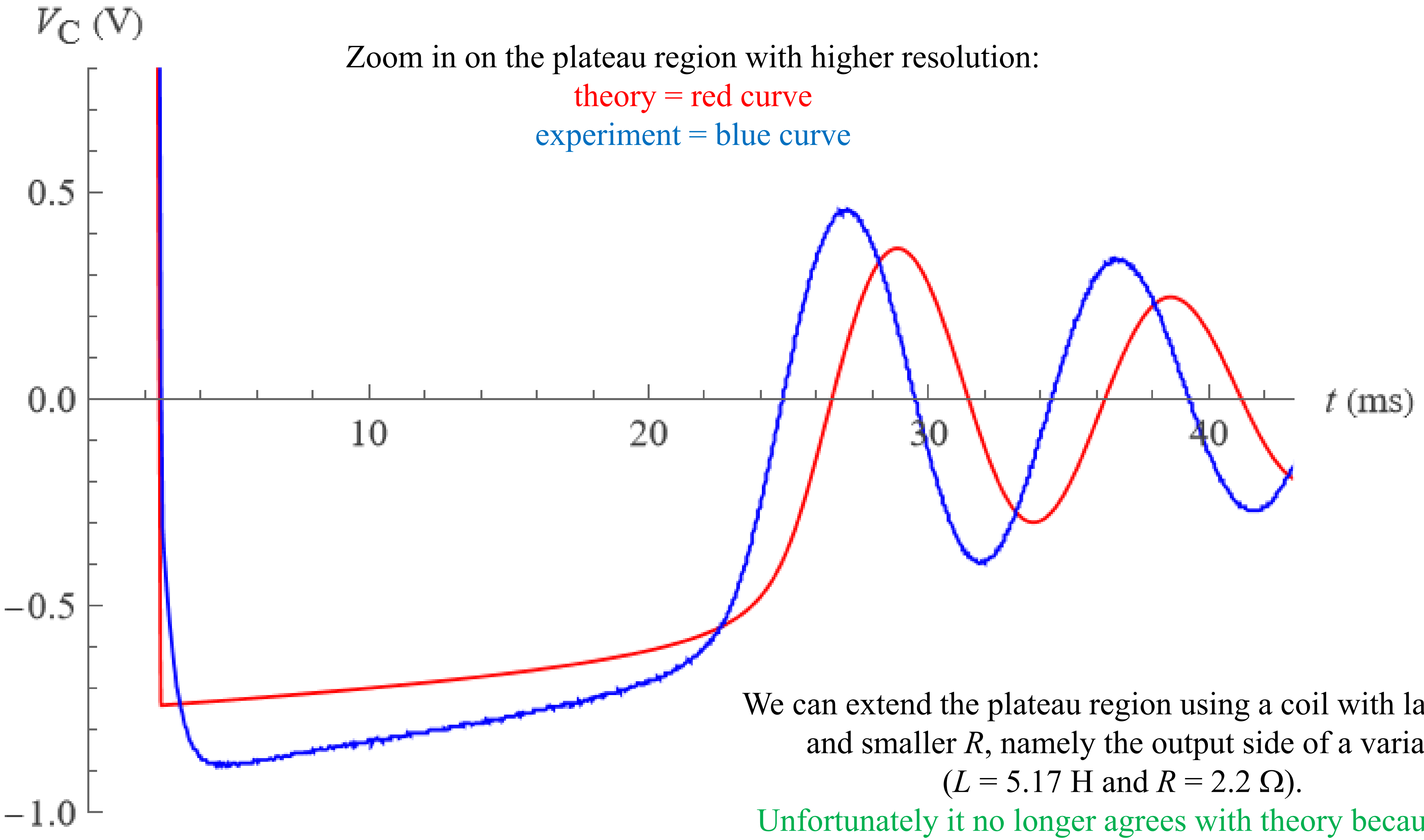



Zoom in on the plateau region with higher resolution:
theory = red curve
experiment = blue curve
V_C (V)
0.5
0.0
−0.5
−1.0
10
20
30
40
t (ms)

Zoom in on the plateau region with higher resolution:

theory = red curve

experiment = blue curve



We can extend the plateau region using a coil with larger L and smaller R, namely the output side of a variac (L = 5.17 H and R = 2.2 Ω).

Unfortunately it no longer agrees with theory because of frequency-dependent losses due to hysteresis in the ferromagnetic core!

Comments or questions?

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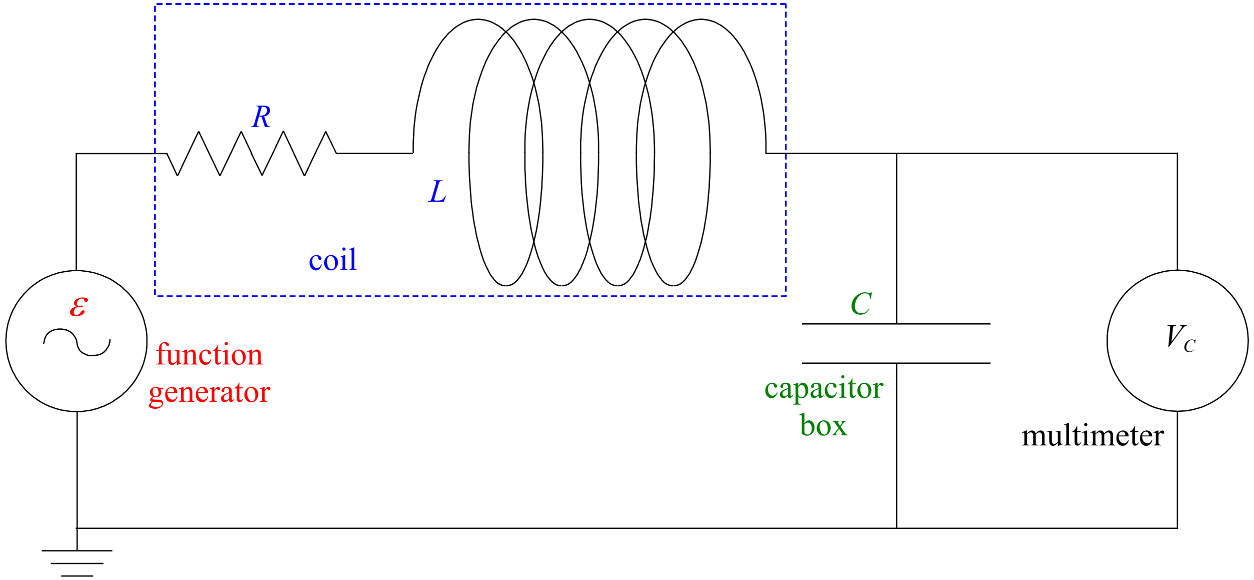
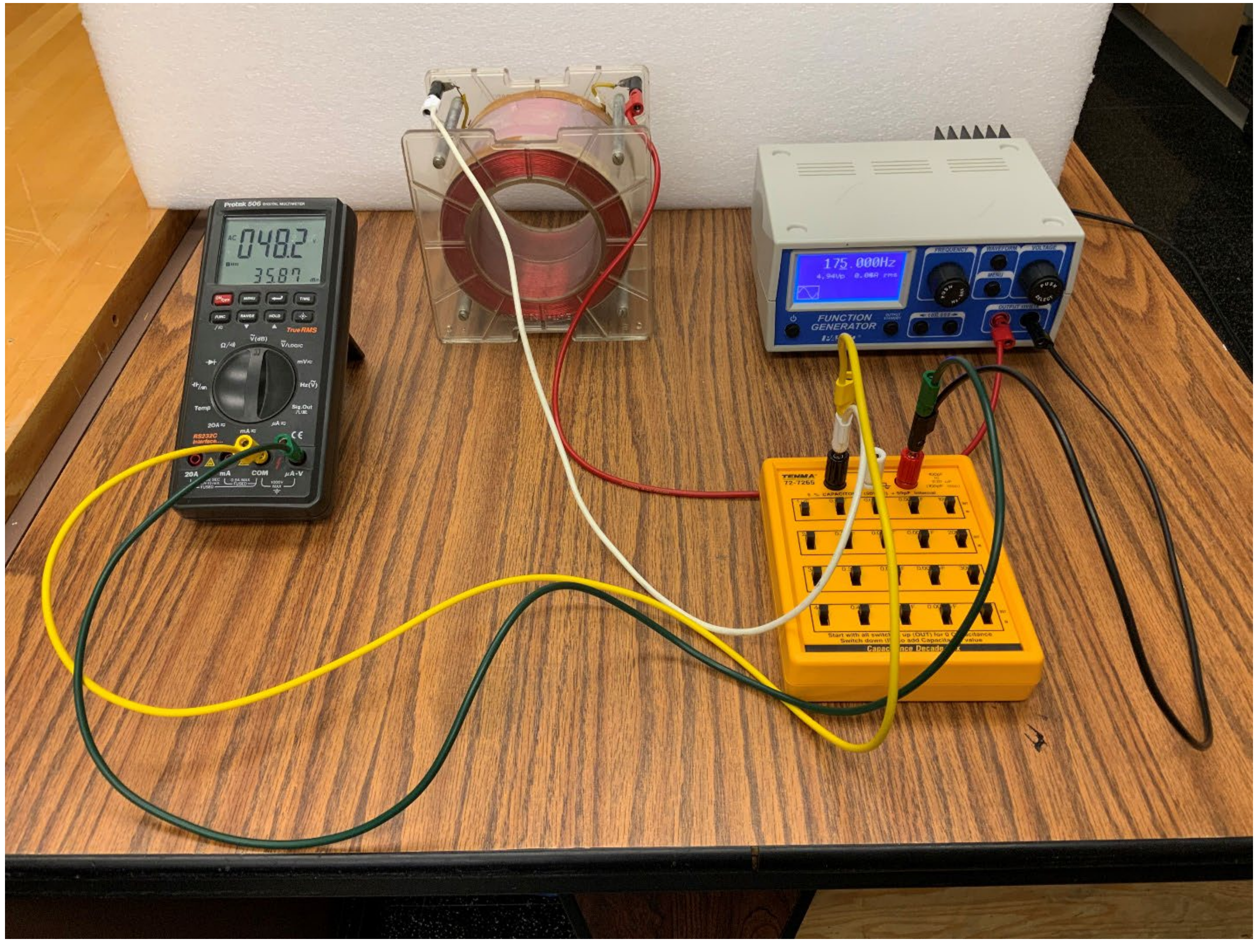
webpage: usna.edu/Users/physics/mungan

where you can find the article on which this presentation is based:
"Flattening the oscillations of an underdamped RLC circuit using a diode,"
European Journal of Physics 47, 025207 (2026)

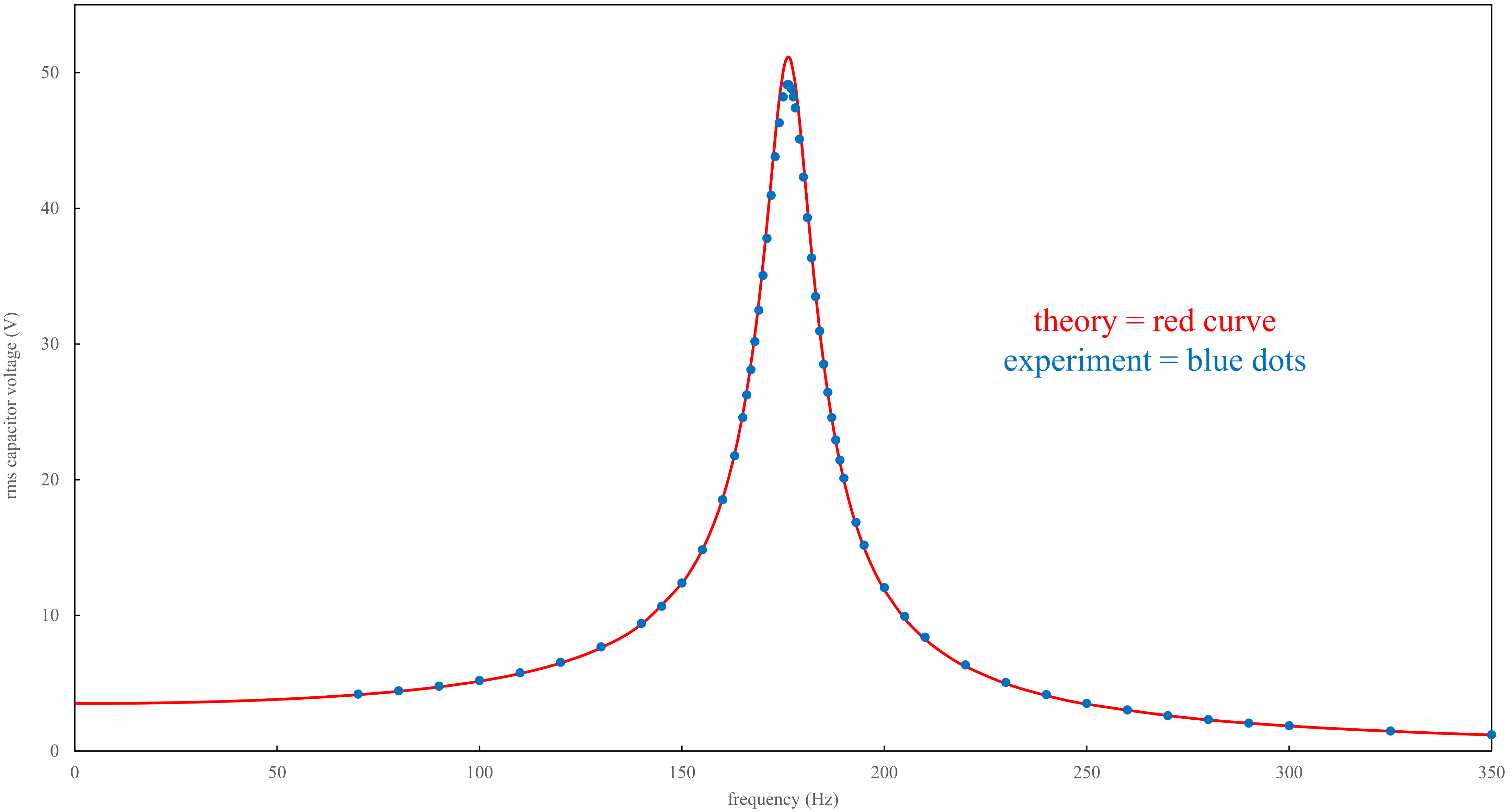
Extra slides

experiment: *Simple but accurate driven RLC experiment,* Phys. Educ. **57**, 053002 (2022)



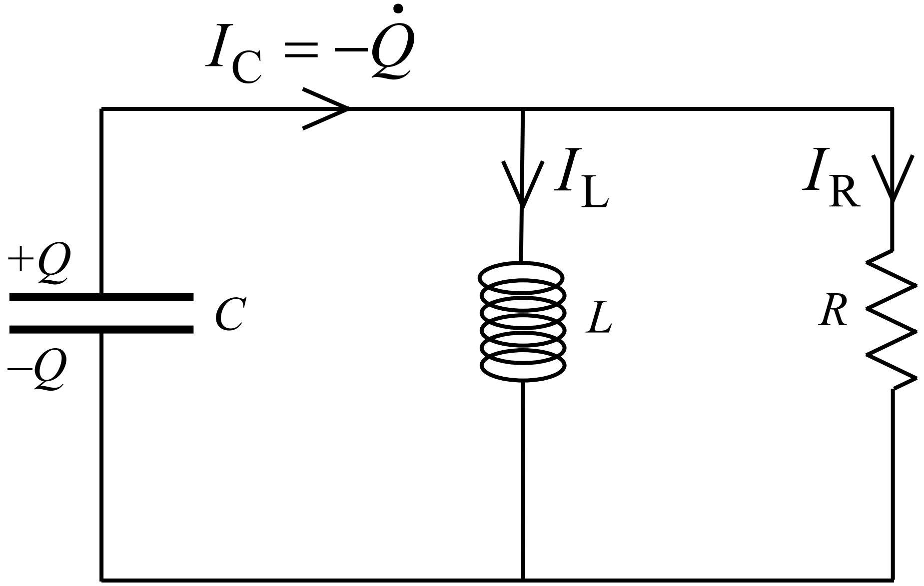


fix the function generator output voltage at 3.5 V



theory = red curve
experiment = blue dots
rms capacitor voltage (V)
frequency (Hz)
0
10
20
30
40
50
0
50
100
150
200
250
300
350

Consider a parallel *RLC* circuit:



junction rule $I_C = I_L + I_R \quad \Rightarrow \quad \dot{Q} + I_L + I_R = 0$ (by subbing in I_C)

outer loop $\dfrac{Q}{C} = RI_R \quad \Rightarrow \quad \ddot{Q} + \dot{I}_L + \dfrac{\dot{Q}}{\tau} = 0$ ($\div R$, sub in I_R, $\tau \equiv RC$, time deriv)

left loop $\dfrac{Q}{C} = L\dfrac{dI_L}{dt} \quad \Rightarrow \quad \boxed{\ddot{Q} + \dfrac{\dot{Q}}{\tau} + \omega^2 Q = 0}$ ($\div L$, sub in $\dot{I}_L$, $\omega \equiv \dfrac{1}{\sqrt{LC}}$)

We get the SAME equation as for a series *RLC* circuit!

The only differences are now $\tau = RC$ instead of $\tau = L/R$ and the oscillations are underdamped (i.e., real ω') when

$$\omega > \frac{1}{2\tau} \quad \Rightarrow \quad R >> \sqrt{\frac{L}{C}} \approx 500\ \Omega \text{ instead of } R << \sqrt{\frac{L}{C}}.$$

It makes sense that R needs to be small for the series circuit and large for the parallel circuit, so that the resistor only makes a small disturbance to the LC oscillations.