Relativity on Rotated Graph Paper

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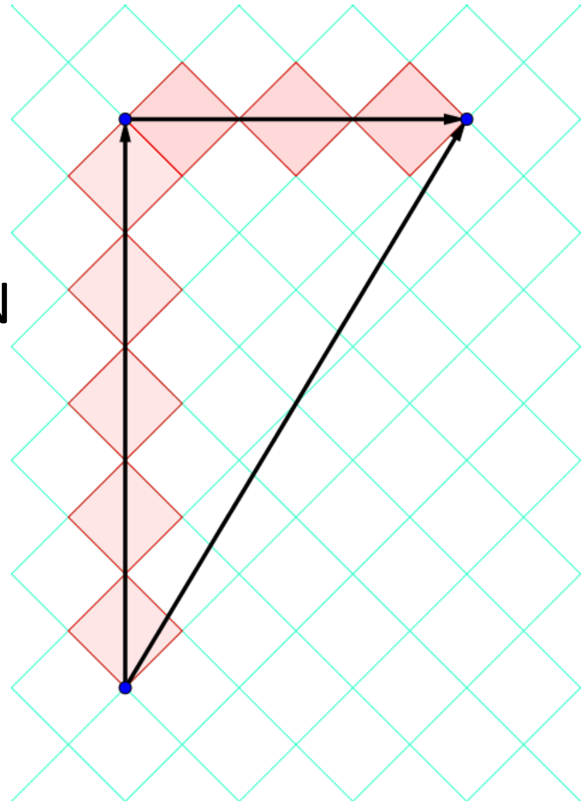

“Relativity on rotated graph paper”
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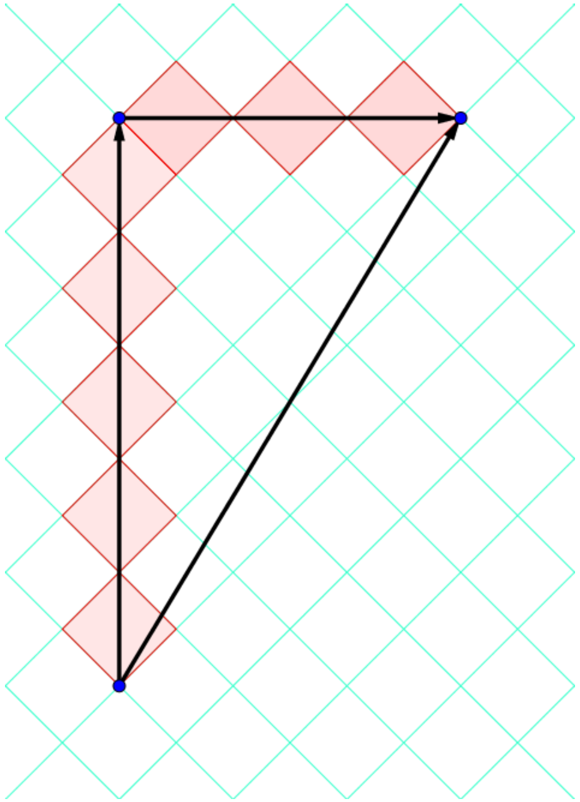
https://doi.org/10.1119/1.4943251

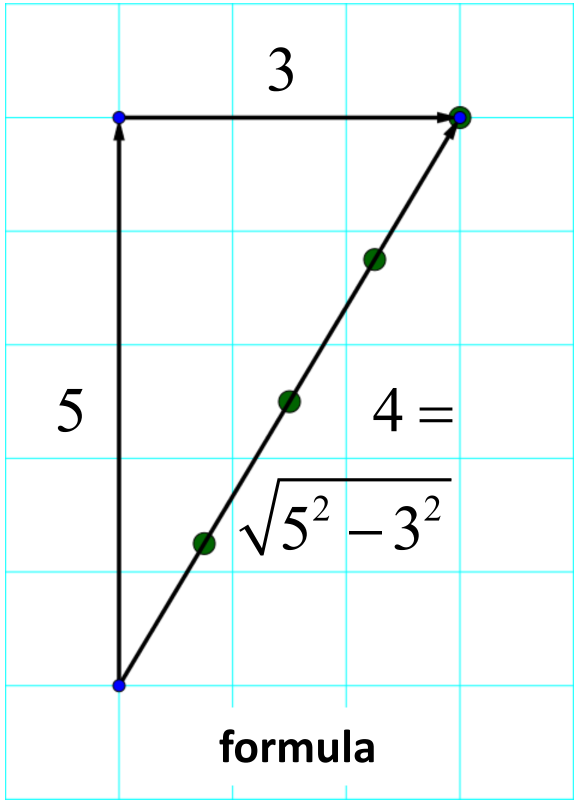
Relativity on Rotated Graph Paper

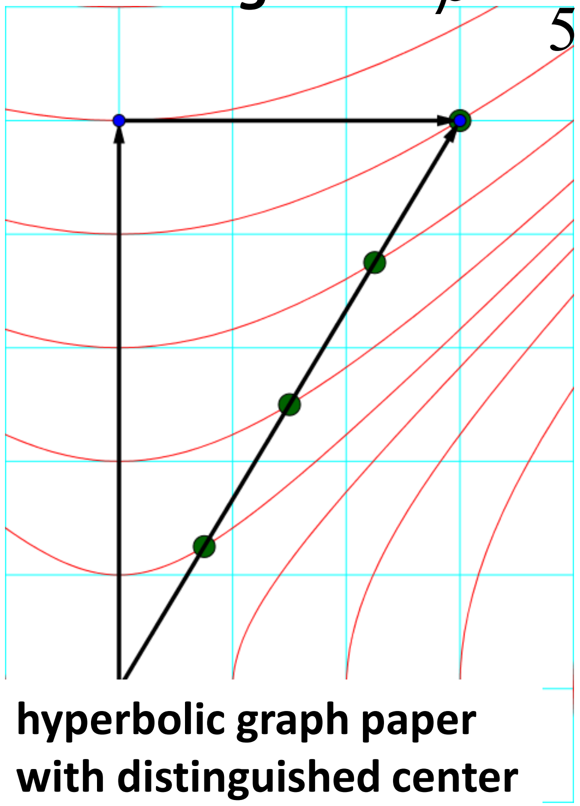
- an **ordinary Minkowski spacetime diagram**
  emphasizing light-signals
  allowing "ticks of a light-clock" to be visualized
- *physically motivated:*
  - traced out by the **light-signals in a ticking light-clock;**
  - based on the **Relativity** and **Speed-of-Light postulates**
- *method of calculation:*
  count boxes ("clock diamonds") and do **simple algebra**
- the visualization **encodes many relativistic effects**
  and lends itself to numerous physical interpretations
- first developed for use in **algebra-based introductory** courses
- *new methods more appropriate for more advanced students*

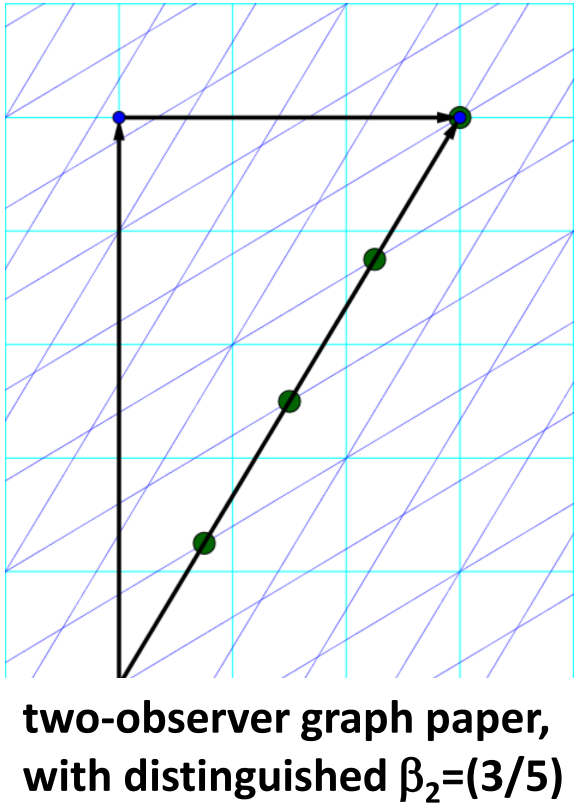


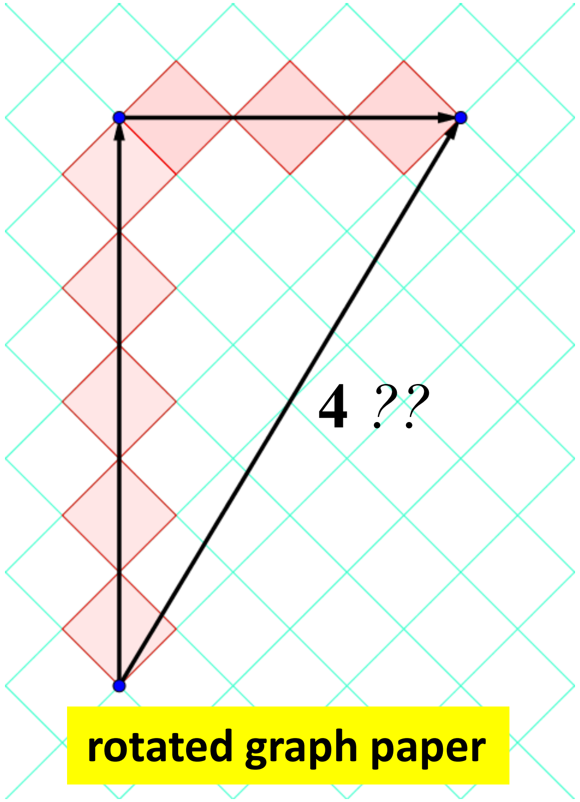
Can you see the "4 ticks" on a spacetime diagram?

Inertial observer Alice at rest and
inertial observer Bob traveling with $\beta = \frac{3}{5}$







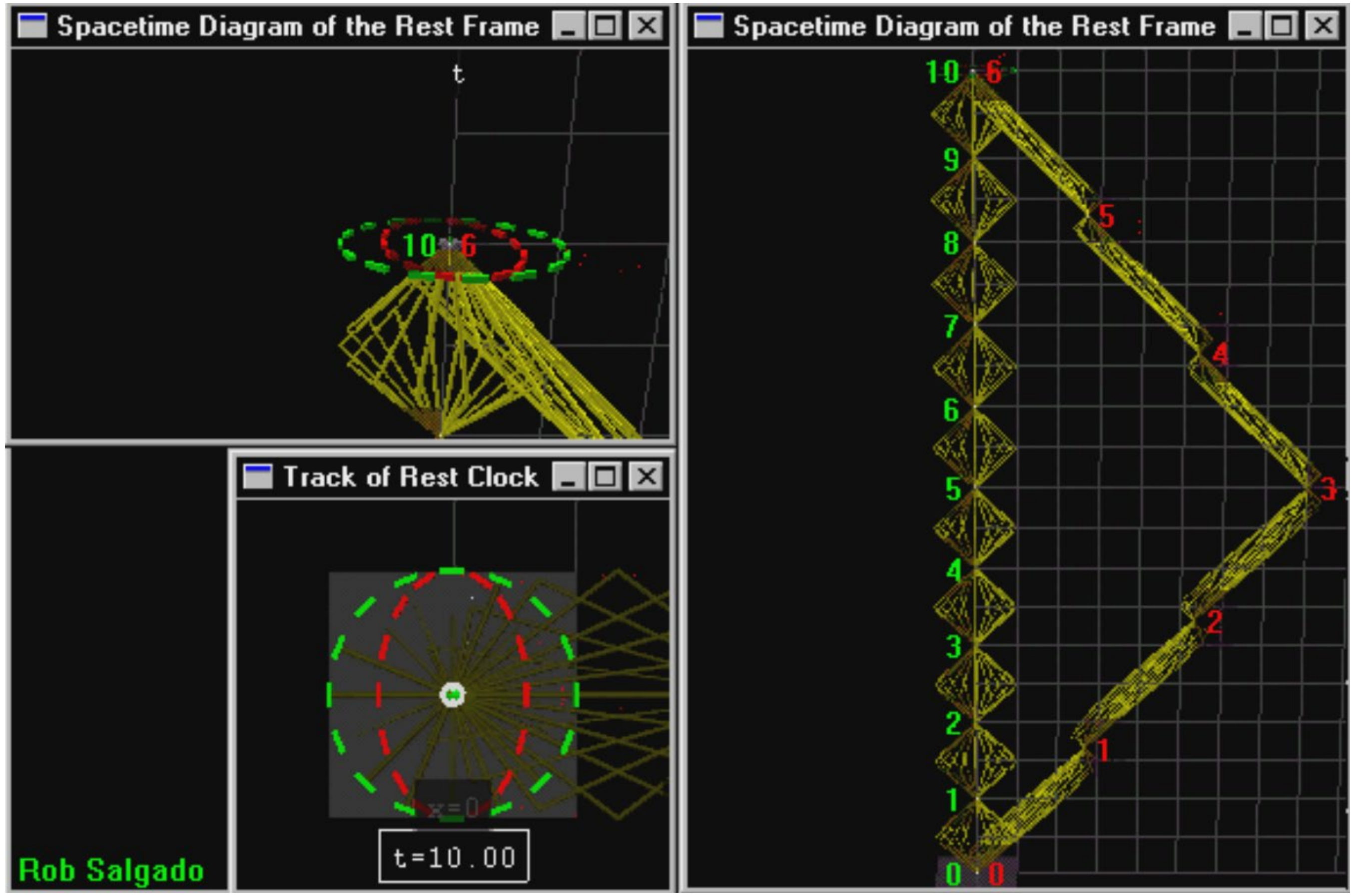


"Ticks" (a.k.a. "clock diamonds")
are constructed using the
light-signals in a longitudinal light-clock

Visualizing Proper Time in Special Relativity

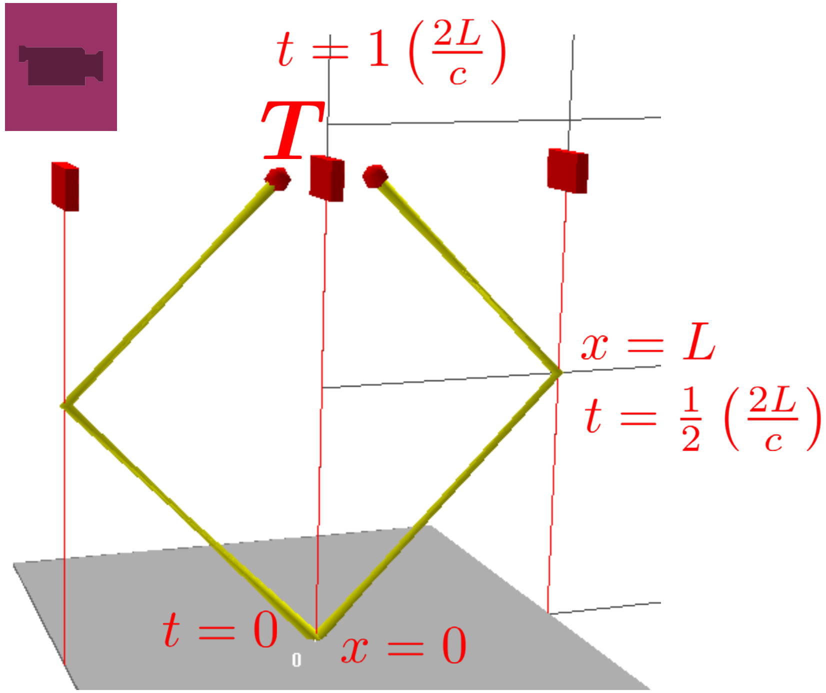
43sec

- https://www.youtube.com/watch?v=**NqjAOyGR82s**

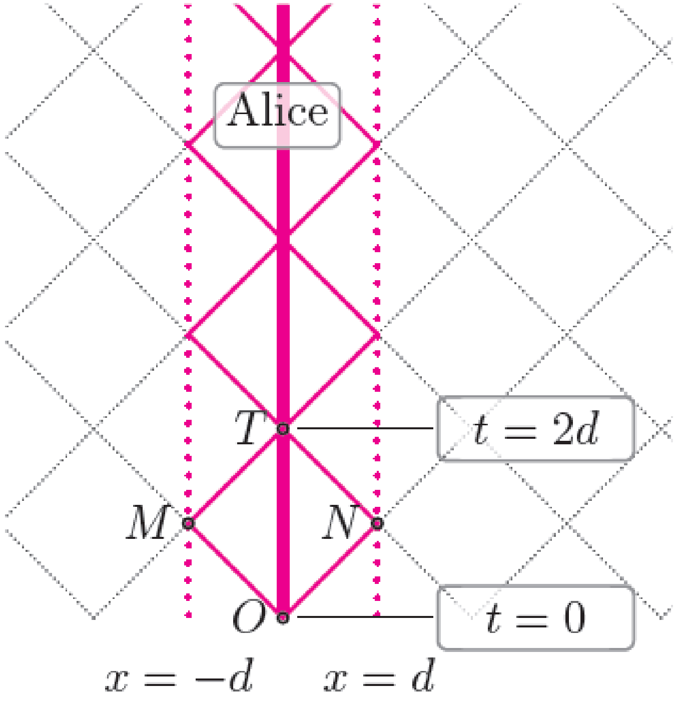


$$\boldsymbol{\beta} = \frac{\boldsymbol{v}}{\boldsymbol{c}} = \frac{\mathbf{4}}{\mathbf{5}}$$

Light Clocks on a Spacetime Diagram

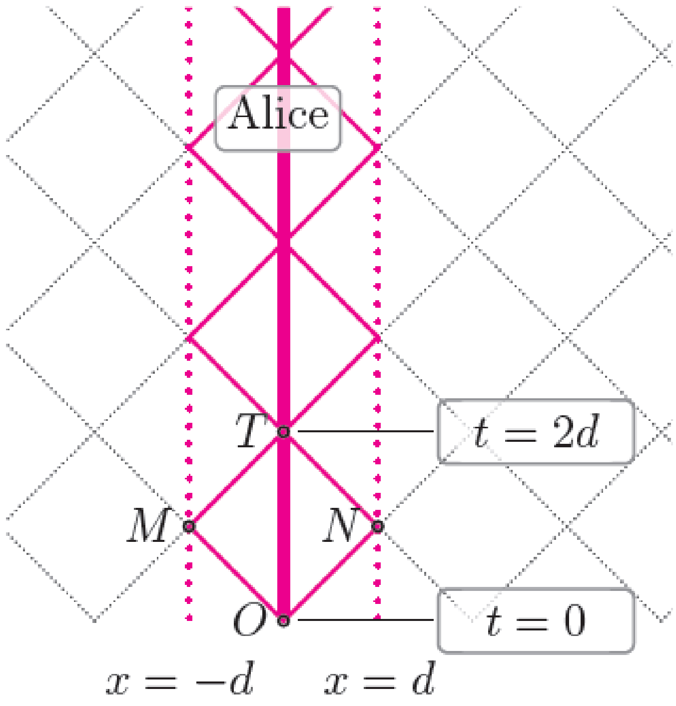


Alice and
her distant mirrors
a length *L* away (in Alice's frame)



"light-clock diamonds"

Light-Clocks on a spacetime diagram



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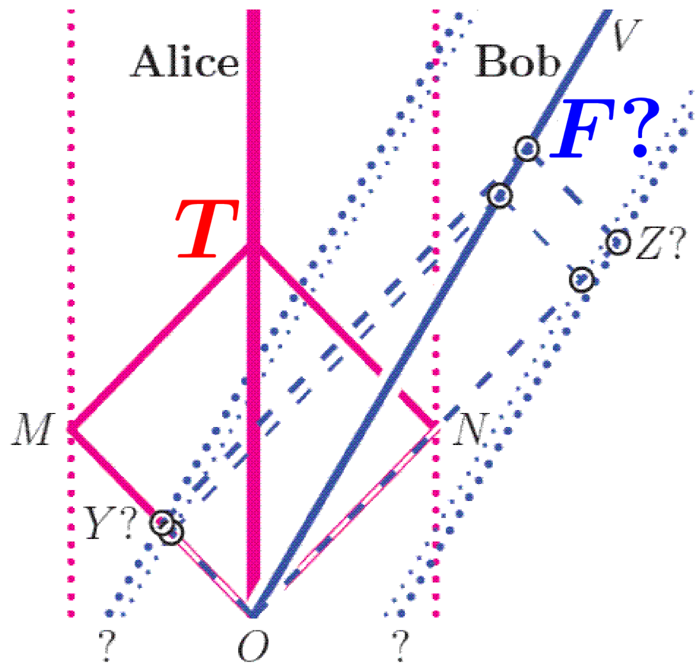
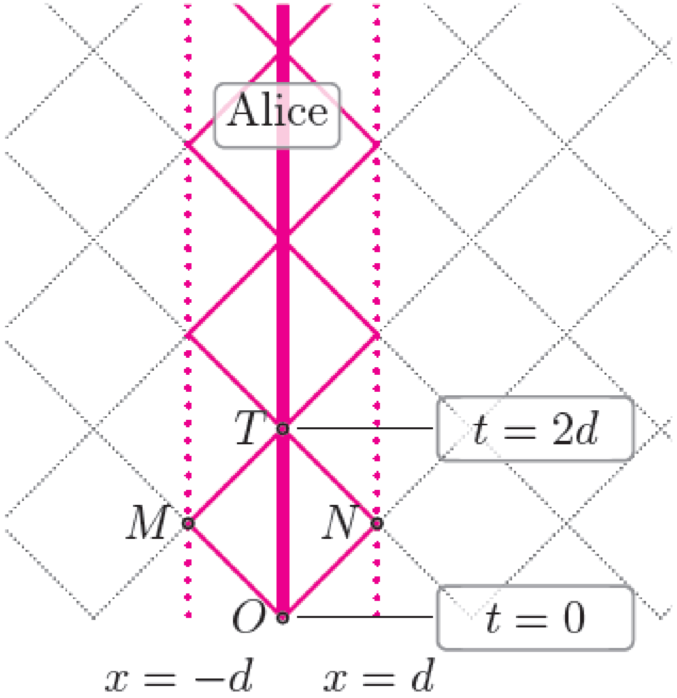


Calibrating Bob's Light-Clock….

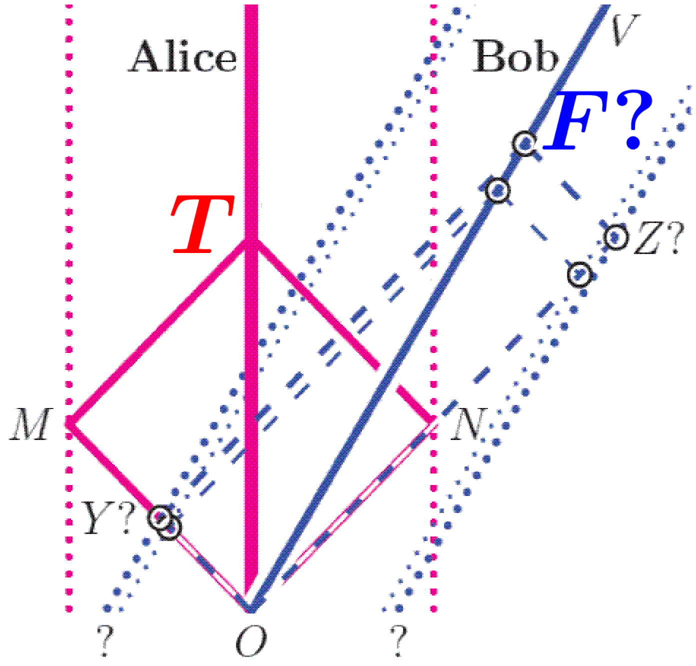
Where is Bob's first tick event F?

(What is the separation of Bob's mirrors?)

Light-Clocks on a spacetime diagram



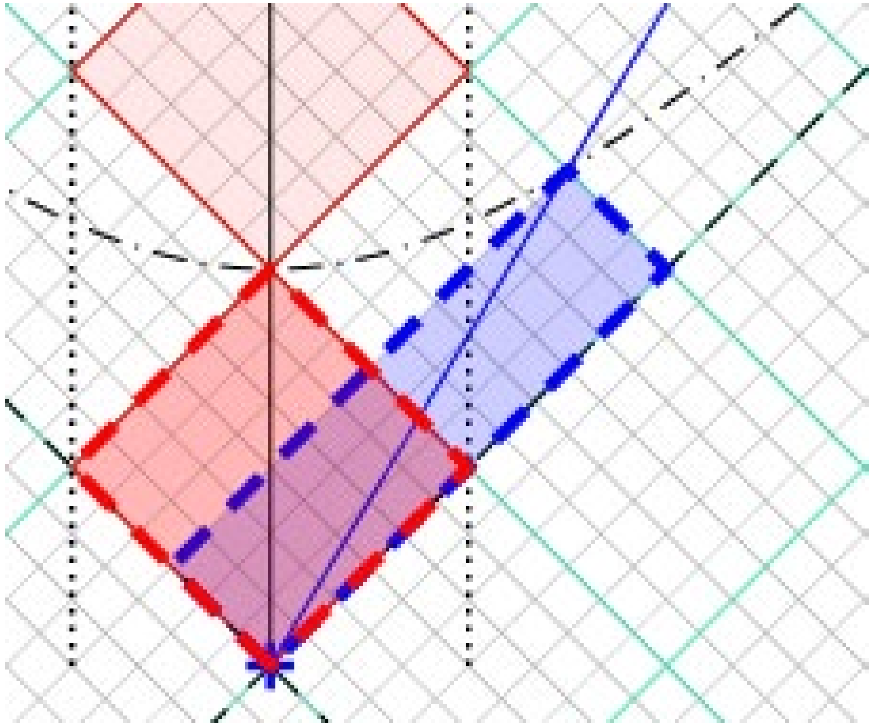
“light-clock diamonds”



Calibrating Bob’s Light-Clock….

Where is Bob’s first tick event $F?$

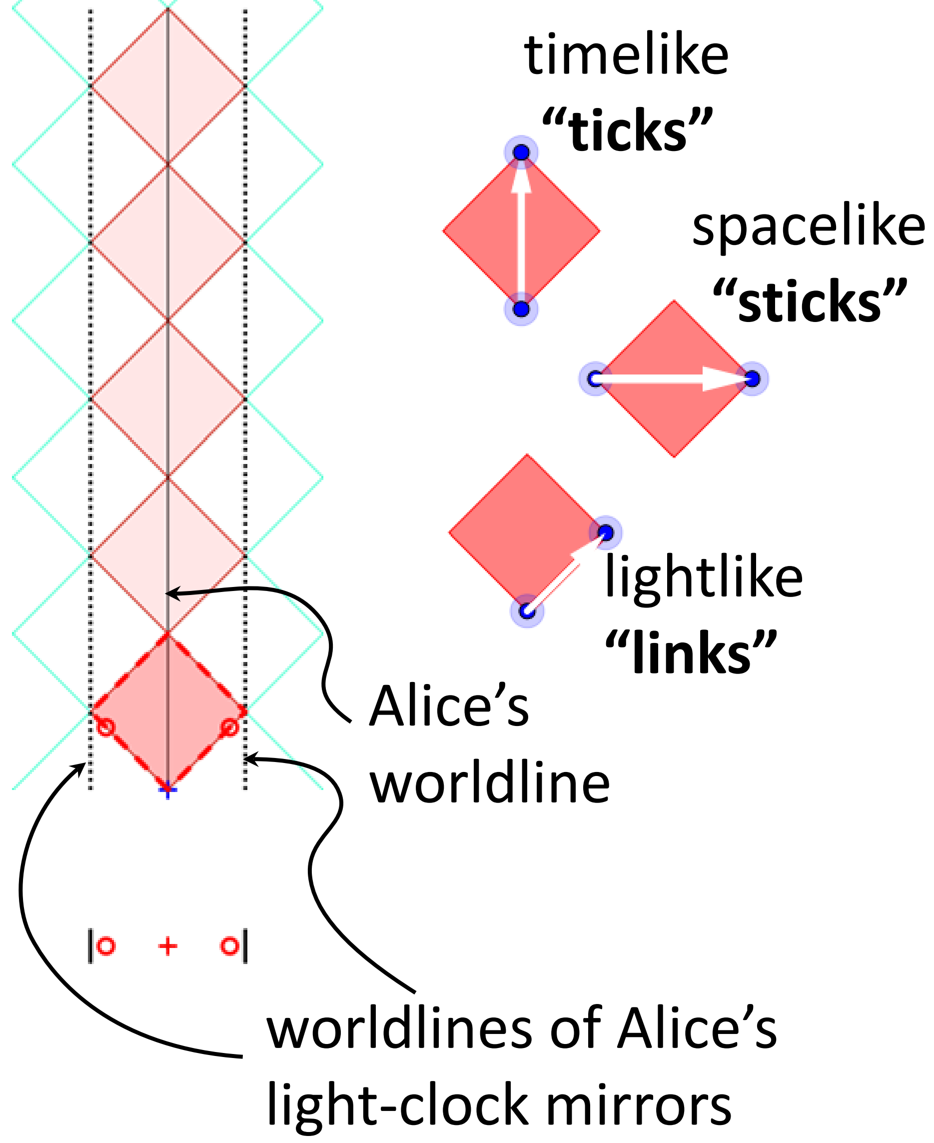
(What is the separation of Bob’s mirrors?)



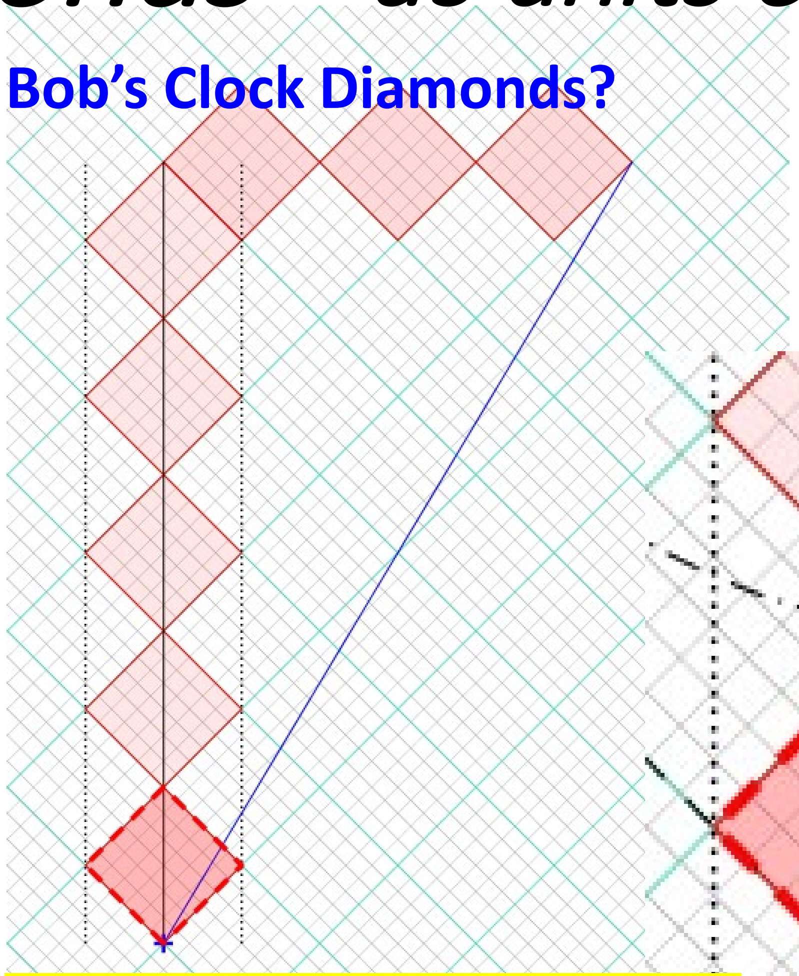
Events T and F lie on a hyperbola

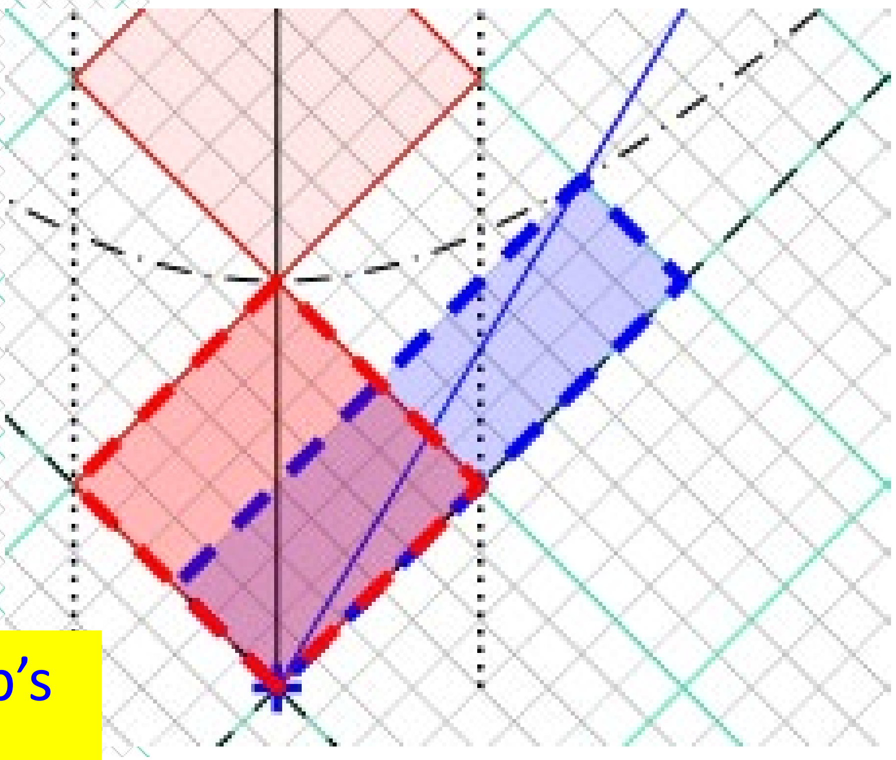
Light-Clock Diamonds – as units of displacement

Alice's Clock Diamonds



Bob's Clock Diamonds?

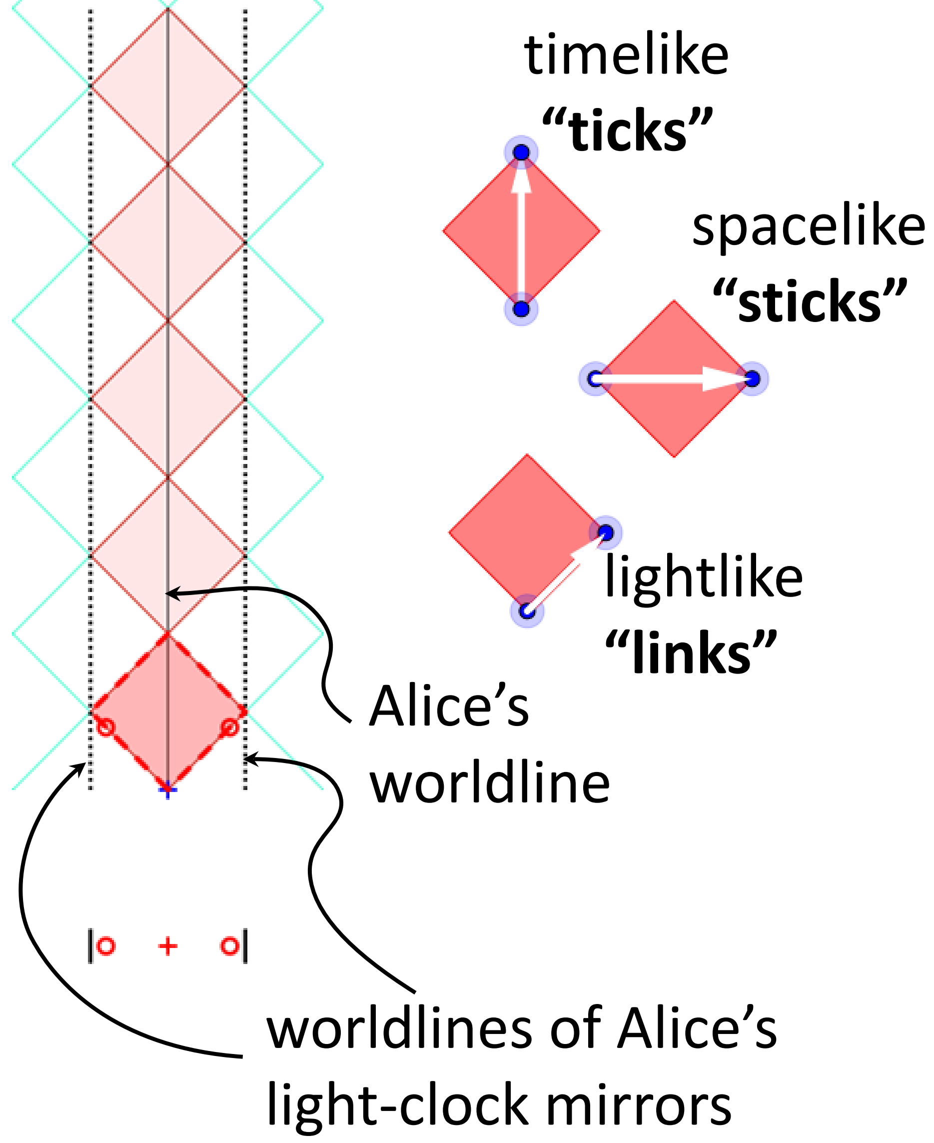




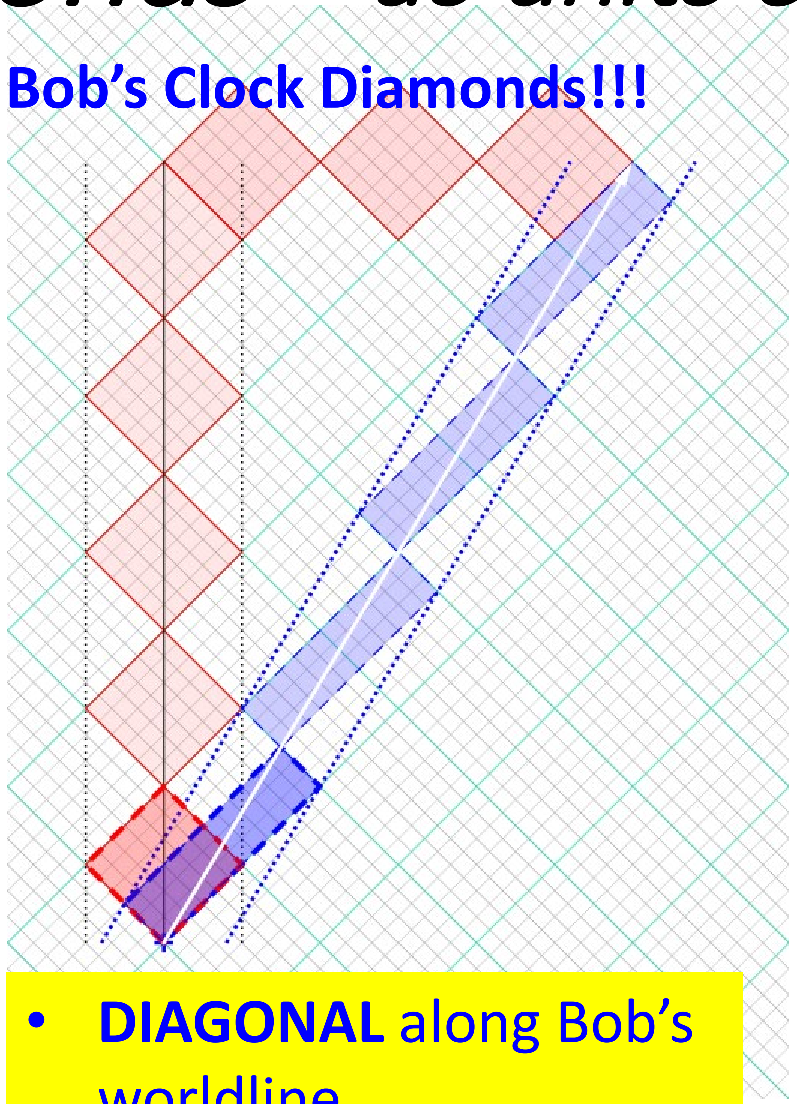
- **DIAGONAL** along Bob's worldline
- **SAME AREA** as Alice's Clock Diamond

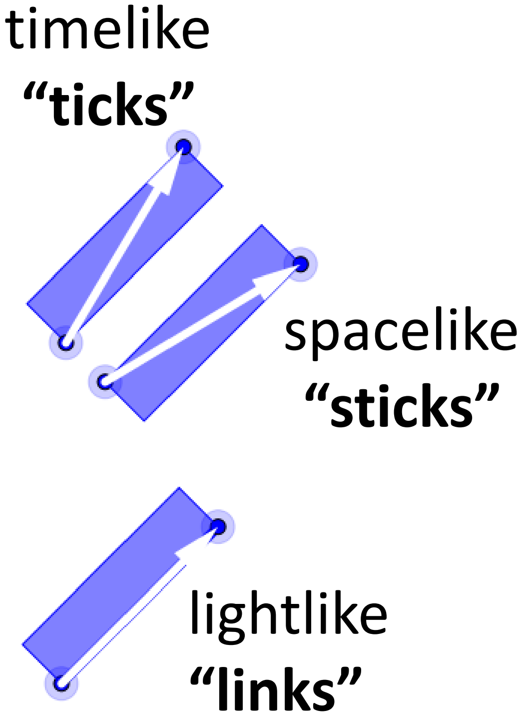
Light-Clock Diamonds – as units of displacement

Alice's Clock Diamonds



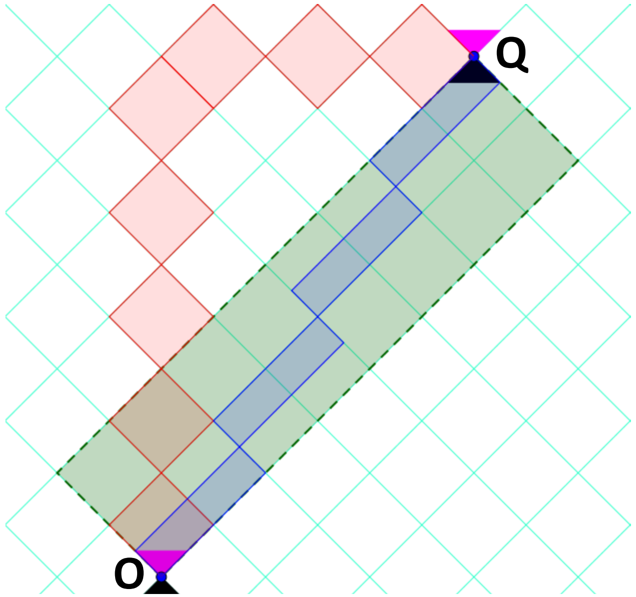
Bob's Clock Diamonds!!!





- **DIAGONAL** along Bob's worldline
- **SAME AREA** as Alice's Clock Diamond

Causal Diamonds *(Mermin's "Light Rectangles")*

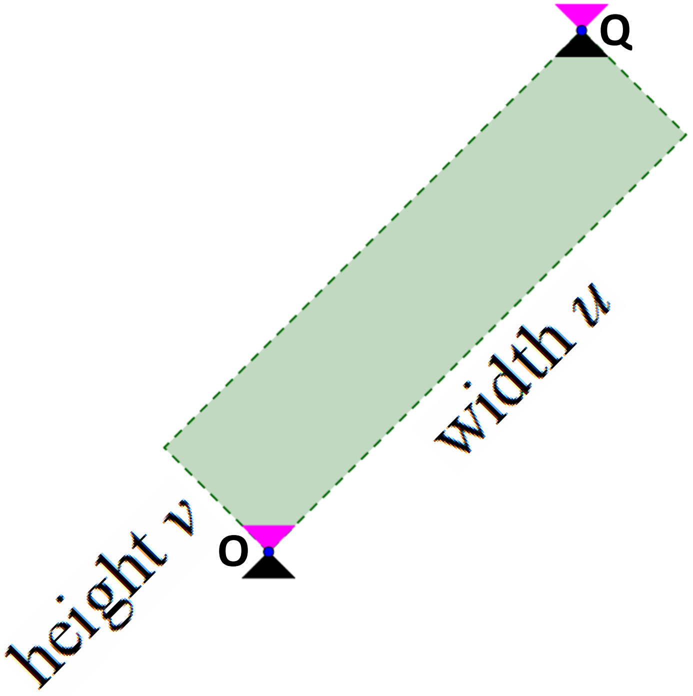


- **Intersection** of (the **future** light cone of event O) and (the **past** light cone of event Q).

  "events that can be influenced by O and can then influence Q"

- **area** of the diamond (in units of clock diamonds)
  = **squared-interval**
  = **(proper time)²**

$$s^2 = (\text{width } u)(\text{height } v)$$



- **aspect ratio** of the diamond
  = **square of the Doppler Factor**
  (encodes velocity $\beta = (V/c)$)

$$k^2 = \frac{(\text{width } u)}{(\text{height } v)}$$

$$\text{Doppler } k = \sqrt{\frac{1+\beta}{1-\beta}}$$

$$\beta = \frac{k^2 - 1}{k^2 + 1}$$

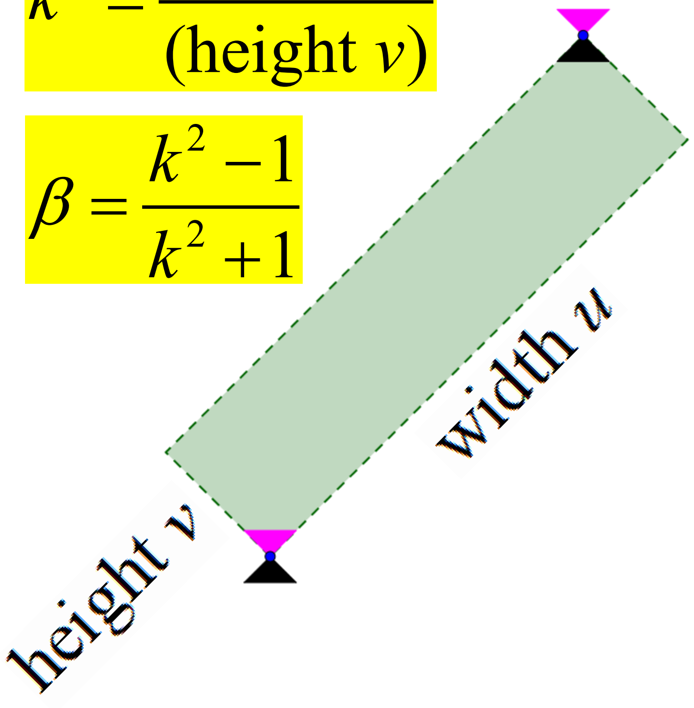
Causal Diamonds

with clock-diamond components

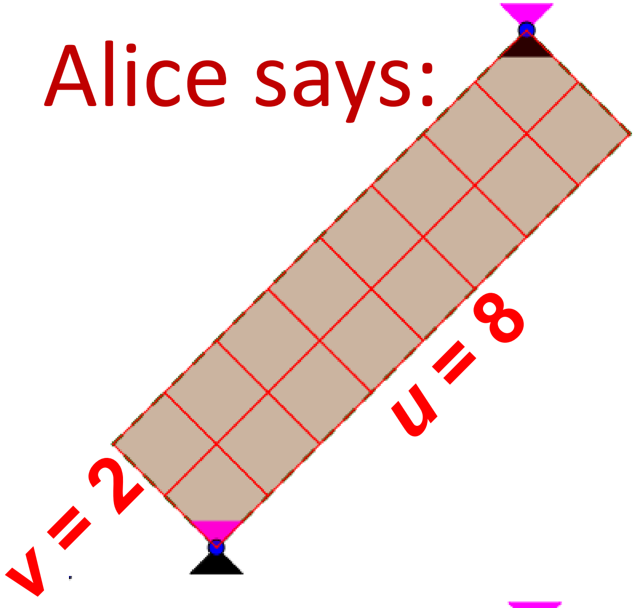
$$s^2 = (\text{width } u)(\text{height } v)$$

$$k^2 = \frac{(\text{width } u)}{(\text{height } v)}$$

$$\beta = \frac{k^2 - 1}{k^2 + 1}$$



Alice says:



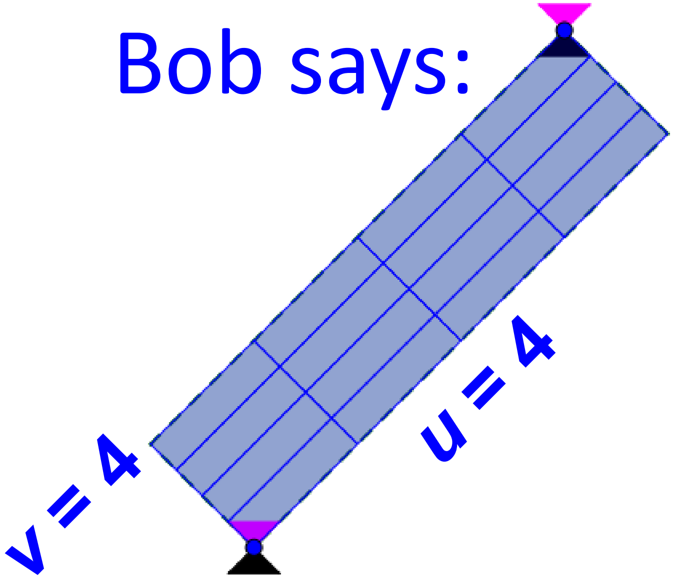
$$s^2 = (8)(2) = 16 = (\mathbf{4})^2$$

$$k^2 = \frac{(8)}{(2)} = 4 = (2)^2$$

$$\beta = \frac{(4) - 1}{(4) + 1} = \frac{3}{5}$$

The "4"!

Bob says:



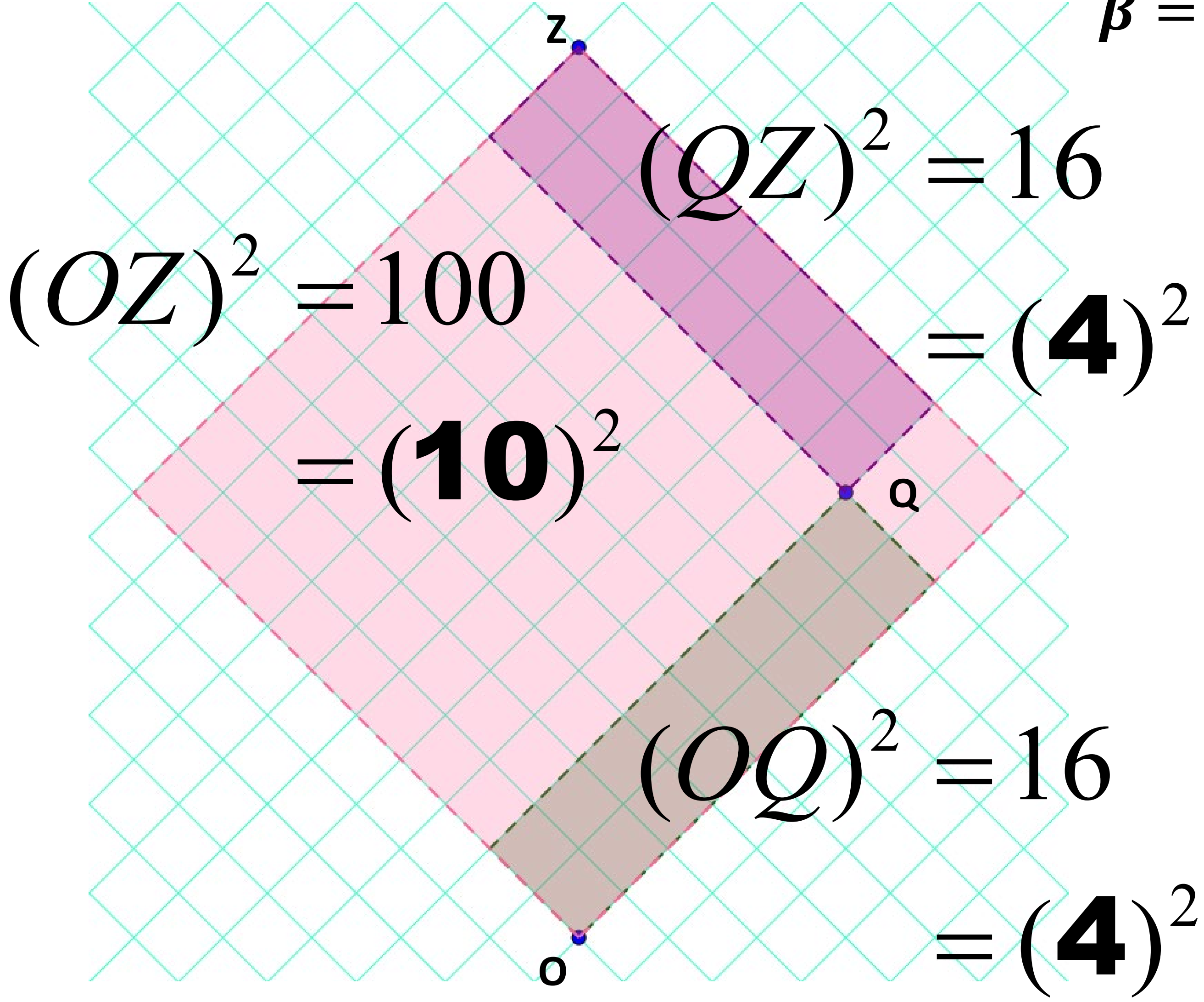
$$s^2 = (4)(4) = 16 = (\mathbf{4})^2$$

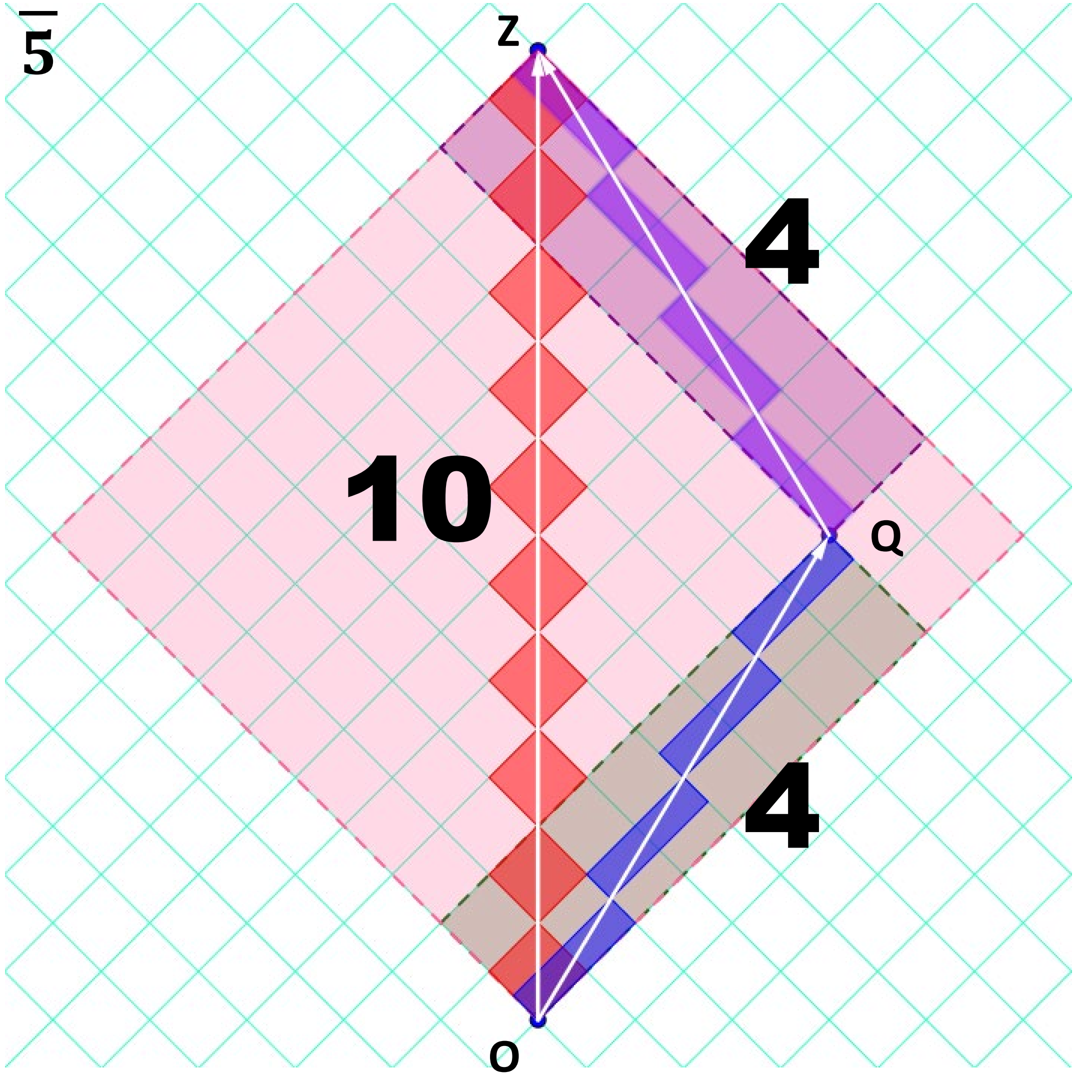
$$k^2 = \frac{(4)}{(4)} = 1 = (1)^2$$

$$\beta = \frac{(1) - 1}{(1) + 1} = 0$$

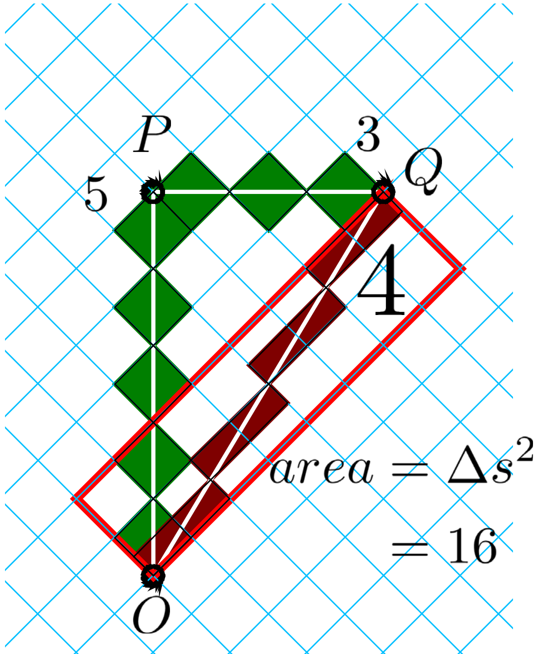
The Clock Effect/Twin Paradox

$$\beta = \frac{v}{c} = \frac{3}{5}$$





time-dilation and γ



$$\Delta s^2 = (\Delta t)^2 - (\Delta x)^2$$
$$(4)^2 = (5)^2 - (3)^2$$

Observer-Bob travels at (3/5)c.
How much of Bob's proper-time
has elapsed from O to Q?

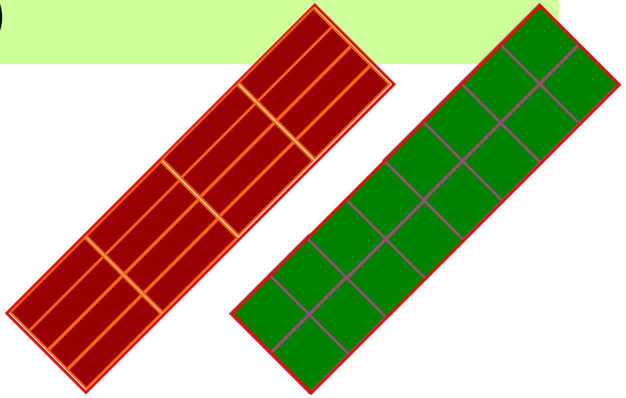
Determine Bob's ticks as follows:

- (using Alice's ticks)
Draw OP and PQ so that $\dfrac{(PQ)}{(OP)} = \dfrac{3}{5}c$

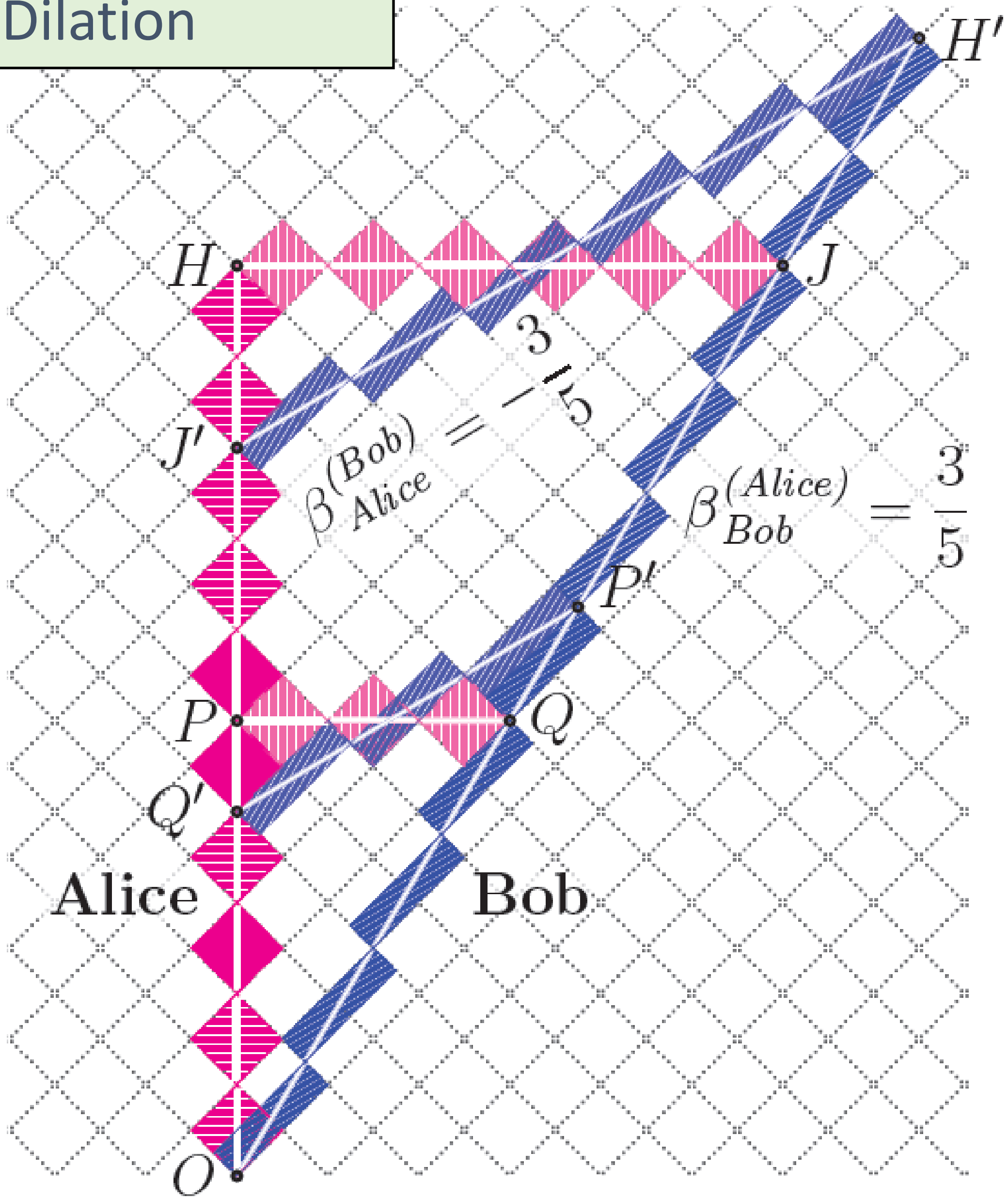
- Determine OQ as the
square-root of the area of the causal diamond
between O and Q.
(Note that Alice and Bob, using their own ticks, will determine the same area.)

using a formula:

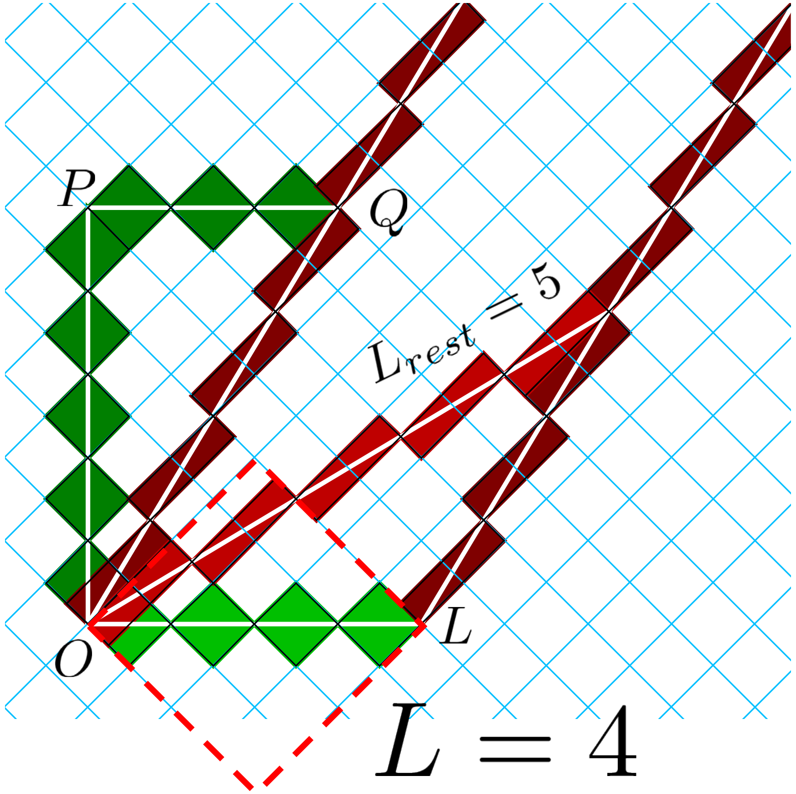
$$\gamma = \frac{1}{\sqrt{1-(\frac{3}{5})^2}}$$
$$= \sqrt{\frac{5^2}{5^2-3^2}} = \sqrt{\frac{25}{16}} = \frac{5}{4}$$



example: Symmetry of Time-Dilation



length contraction



using a formula:

$$L = \frac{L_{rest}}{\gamma} = \frac{5}{\frac{5}{4}} = 4$$

Observer-Bob traveling at 0.6c
carries a ladder that is 5 units long.
How long is that ladder according to Alice?

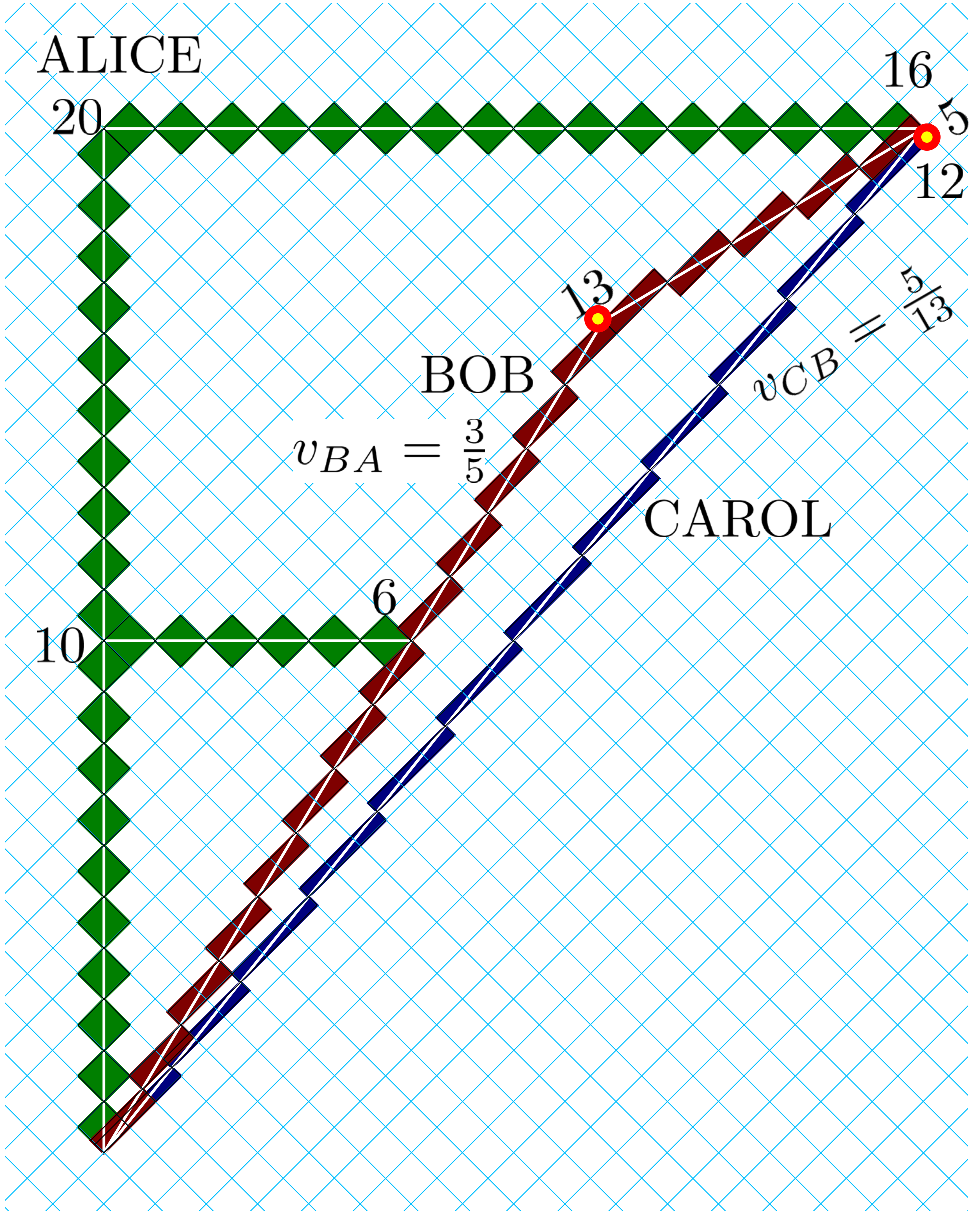
Determine Bob's ticks as follows:

•(using Alice's ticks)
Draw OP and PQ so that $\frac{(PQ)}{(OP)} = \frac{3}{5}c$

•(using Bob's ticks)
Count L_{rest}=5 spatial-ticks from O.
Construct a parallel to Bob.

•Determine OL (as the square-root of the area of the spatial diamond between O and L).
(Note that Alice and Bob, using their own ticks, will determine the same area.)

velocity composition

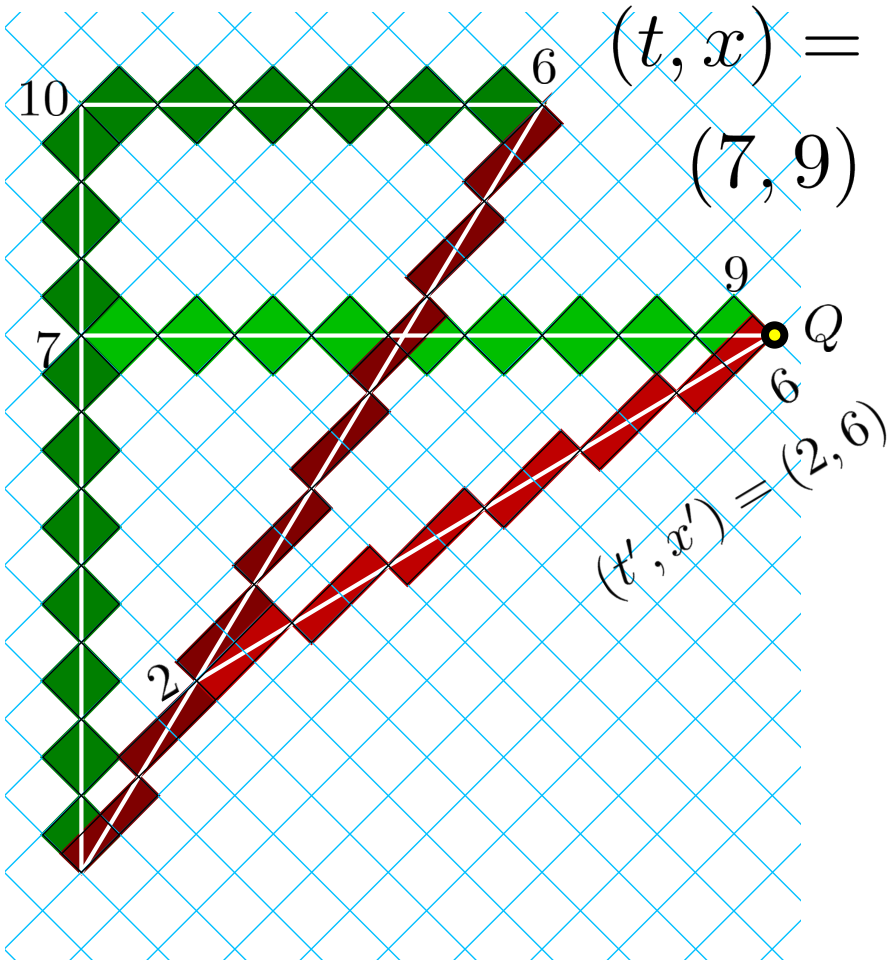


Observer-Bob,
traveling at 0.6c with respect to Alice,
observes Carol moving forward at (5/13)c.

What is the velocity of
Carol with respect to Alice?

$$v_{CA} = \frac{v_{CB} + v_{BA}}{1 + v_{CB}v_{BA}} = \frac{\frac{5}{13} + \frac{3}{5}}{1 + (\frac{5}{13})(\frac{3}{5})} = \frac{39 + 25}{65 + 15} = \frac{64}{80} = \frac{4}{5} = (16/20)$$

Lorentz Boost transformation



Observer-Bob traveling at 0.6c
assigned Q coordinates using his ticks
as $(t', x') = (2, 6)$

What coordinates would Alice assign using her ticks?

$v = \frac{3}{5}$ implies $\gamma = \frac{5}{4}$.

$$t = \gamma(t' + vx') = \frac{5}{4}(2 + \frac{3}{5}6)$$
$$= \frac{1}{4}(10 + 18) = 7$$

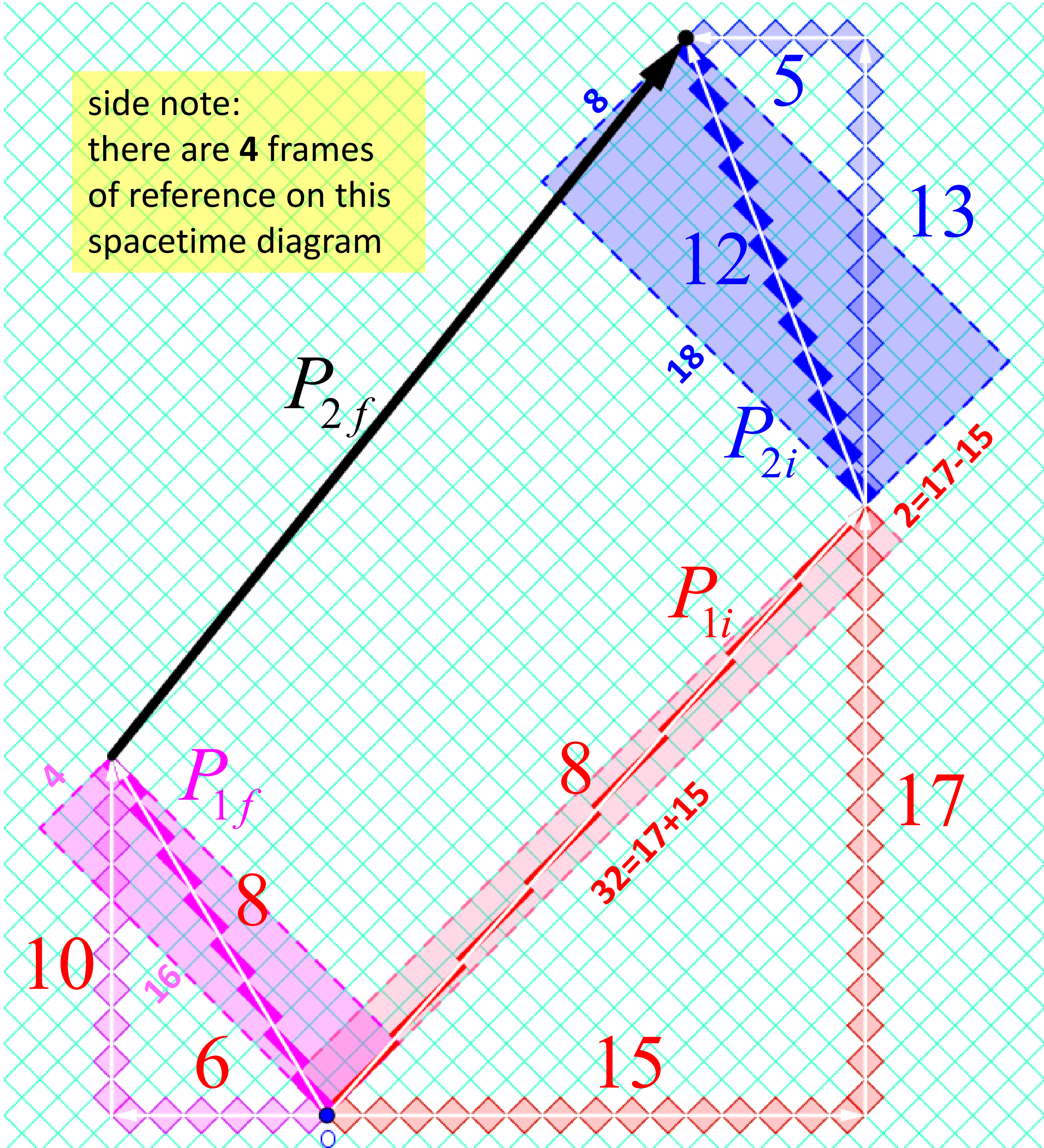
$$x = \gamma(t' + vx') = \frac{5}{4}(6 + \frac{3}{5}2)$$
$$= \frac{1}{4}(30 + 6) = 9$$

Collision
(in Energy-Momentum Space)

$m_1 = 8 \quad \beta_{1i} = \frac{15}{17} \quad \beta_{1f} = -\frac{3}{5}$

$m_2 = 12 \quad \beta_{2i} = -\frac{5}{13} \quad \beta_{2f} = ?$

- verify $m_{2f} = 12$
- compute β_{2f}



Moore (Six Ideas that Shaped Physics) R10S.3

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$m_2 = 12 \qquad \beta_{2i} = -\dfrac{5}{13} \qquad \beta_{2f} = ?$

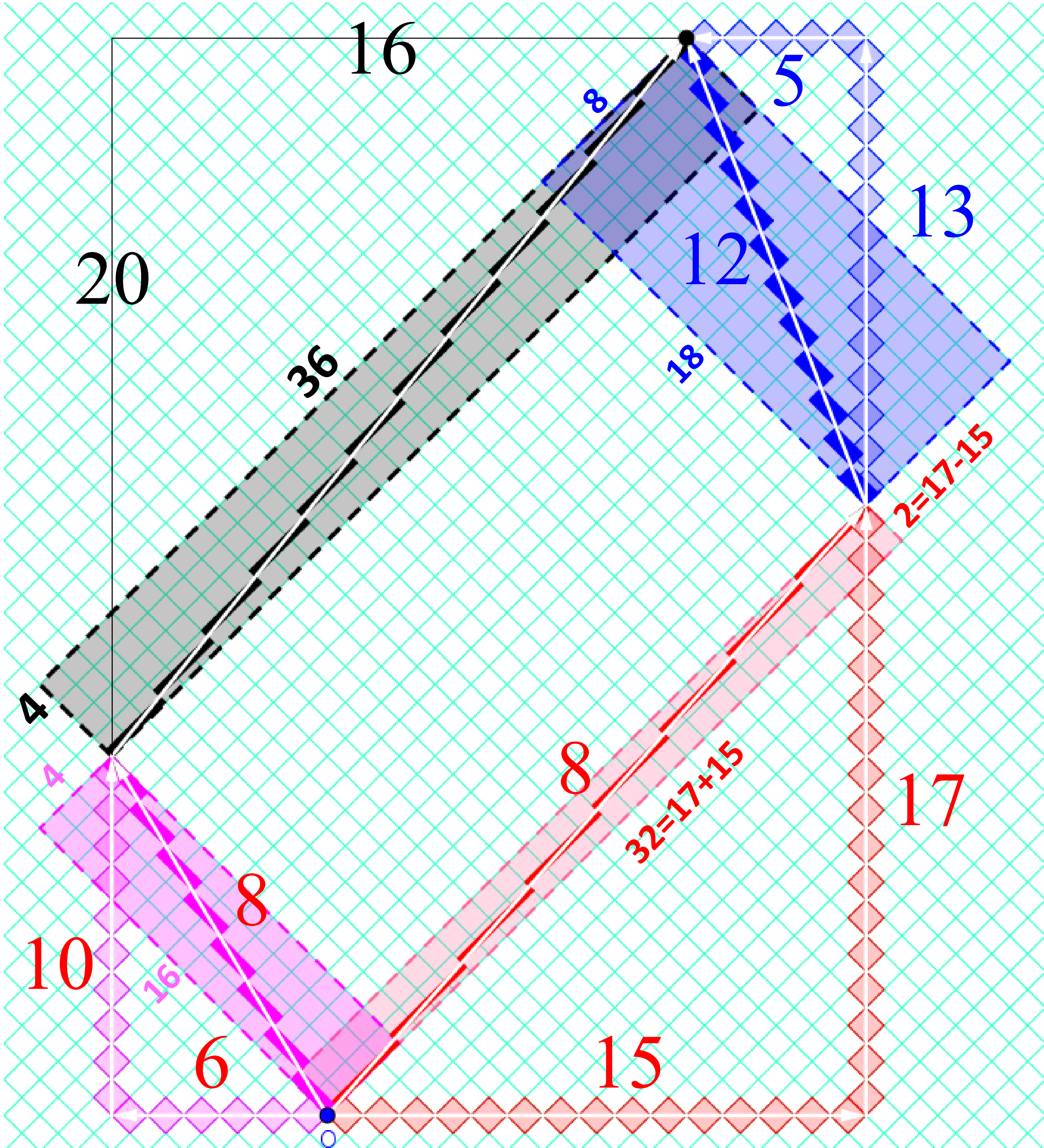
- verify $m_{2f} = 12$

$$s^2 = (36)(4) = 144 = (\mathbf{12})^2$$

- compute β_{2f}

$$k^2 = \frac{(36)}{(4)} = 9$$

$$\beta = \frac{(9)-1}{(9)+1} = \frac{8}{10}$$



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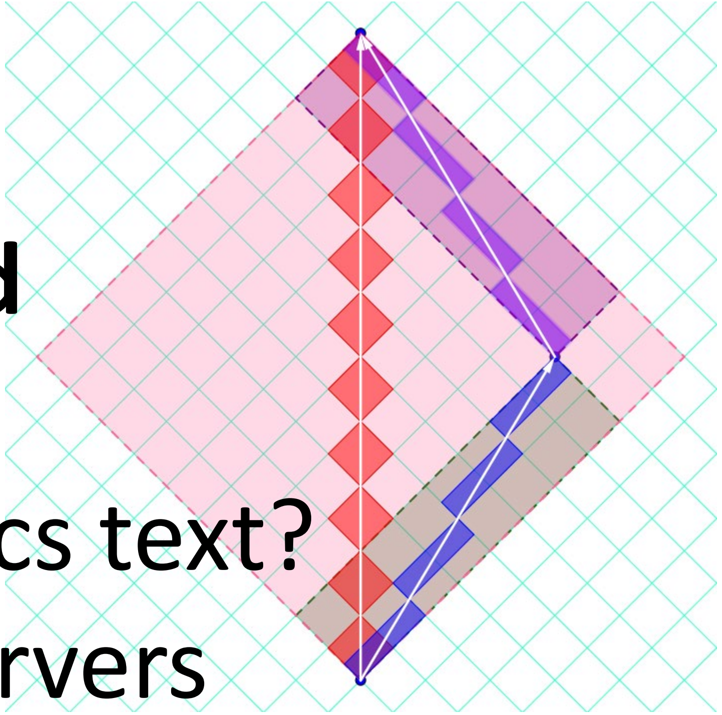
Relativity on Rotated Graph Paper

- **get folks to use the method**

- chapter for a Modern Physics text?
- uniformly-accelerated observers
- mathematical properties



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